(1) The Delta IV Heavy launch vehicle can inject 9900 kg of payload into a lunar transfer orbit. From this point, it will require 3500 m/sec of ∆V to land on the moon. Assuming that the DIVH payload consists of a propulsion stage with an exhaust velocity of 3150 m/sec and an inert mass fraction of 0.08, along with a lunar payload, find the payload that the Delta IV Heavy can deliver to the lunar surface.

\[ r = e^{-\frac{\Delta V}{V_e}} = e^{-\frac{3500}{3150}} = 0.3292 \]

\[ \lambda = r - \delta = 0.3292 - 0.08 = 0.2492 \]

\[ M_{pl} = \lambda M_o = 0.2494(9900) = 2467 \text{ kg} \]

(2) Repeat the analysis of question (1), but assume that over and above the traditional “inert mass” of the lunar landing vehicle is the landing gear, which has a mass of 5% of the mass landed on the moon. Calculate the payload to the lunar surface in this case, and the mass of the landing gear. (Hint: In this case, the total vehicle mass should be written as \( M_o = M_{payload} + M_{inert} + M_{propellant} + M_{landing \text{ gear}} \).)

The landed mass is \( M_{payload} + M_{inert} \). The landing gear mass is 0.05 \times \text{ this quantity, so}

\[ r = \frac{M_{payload} + M_{inert} + M_{landing \text{ gear}}}{M_o} = \frac{1.05(M_{payload} + M_{inert})}{M_o} = 1.05(\lambda + \delta) \]

\[ \lambda = \frac{r}{1.05} - \delta = \frac{0.3292}{1.05} - 0.08 = 0.2335 \]

\[ M_{pl} = \lambda M_o = 0.2335(9900) = 2312 \text{ kg} \]

(3) One proposed approach to lunar operations is to stage lunar missions out of the Earth-Moon L1 libration point, which is directly between the Earth and the Moon at a distance of 38,200 km from the center of the Moon. You want to transfer from the L1 point to a circular lunar orbit at an altitude of 100 km. Find the ∆V for this maneuver, assuming the L1 point is the same as being in a circular lunar orbit at that distance.

\[ r_1 = 38,200 \text{ km}; \quad r_2 = h_{LLO} + r_{moon} = 100 + 1738 = 1838 \text{ km} \]

\[ \Delta V_1 = \sqrt{\frac{\mu_{moon}}{r_1}} \left( 1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right) = \sqrt{\frac{4667.9}{38200}} \left( 1 - \sqrt{\frac{2 \times 1838}{38200 + 1838}} \right) = 0.2437 \text{ km/sec} \]

\[ \Delta V_2 = \sqrt{\frac{\mu_{moon}}{r_2}} \left( \sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right) = \sqrt{\frac{4667.9}{1838}} \left( \sqrt{\frac{2 \times 38200}{38200 + 1838}} - 1 \right) = 0.6078 \text{ km/sec} \]

\[ \Delta V_{Total} = \Delta V_1 + \Delta V_2 = 0.2437 + 0.6078 = 0.8514 \text{ km/sec} \]

(Note that the order of subtraction is reversed because your initial circular velocity is larger than the apogee velocity, and ∆Vs must be positive.)
(4) Repeat (3) incorporating a 90° plane change in the maneuver to go into a lunar polar orbit. Where is the preferred place to perform the plane change?

*The best place to do a single plane change maneuver is where the velocity is lowest, i.e., at apoapsis.*

\[
V_{\text{circ}} = \sqrt{\frac{\mu_{\text{moon}}}{r_1}} = \sqrt{\frac{4667.9}{38200}} = 0.3956 \text{ km/sec}
\]

\[
V_{\text{ap}} = \sqrt{\frac{\mu_{\text{moon}}}{r_1}} \sqrt{\frac{2r_2}{r_1 + r_2}} = \sqrt{\frac{4667.9}{38200}} \sqrt{\frac{2 \times 1838}{38200 + 1838}} = 0.1059 \text{ km/sec}
\]

Since the two velocities are orthogonal, Pythagoras says:

\[
\Delta V_1 = \sqrt{V_{\text{circ}}^2 + V_{\text{ap}}^2} = \sqrt{(0.3956)^2 + (0.1059)^2} = 0.3653 \text{ km/sec}
\]

\[
\Delta V_2 \text{ is unchanged from before, so}
\]

\[
\Delta V_{\text{Total}} = \Delta V_1 + \Delta V_2 = 0.3653 + 0.6078 = 0.9731 \text{ km/sec}
\]

(5) It actually takes 27.322 days for the L1 point to travel a full circle around the moon. Repeat question (3) based on the real motion of the L1 point, rather than assuming it is in a classical orbit. \(\mu_{\text{Moon}} = 4667.9 \text{ km}^3/\text{sec}^2\); \(r_{\text{Moon}} = 1738 \text{ km}\)

Rather than starting from circular orbital velocity, the actual velocity of a spacecraft at L1 with respect to the moon would be

\[
V_{L1} = \omega r_1 = \frac{2\pi}{(27.322 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})} \times 38200 \text{ km} = 0.1017 \text{ km/sec}
\]

\[
\Delta V_{\text{Total}} = V_{\text{ap}} - V_{L1} = 0.1059 - 0.1017 = 0.0042 \text{ km/sec}
\]

(6) A lunar rover on level terrain has a wheel base (front wheel-back wheel contact points) of 2 m and the vehicle center of gravity is 1.2 m behind the front wheels and 1.4 meters off the ground. If the rover is traveling at a speed of 4 m/sec and decelerates at a constant rate, what is the minimum stopping distance? Assume there is no delay between recognizing an obstacle and braking the rover. \(g_{\text{moon}} = 1.545 \text{ m/sec}^2\)

\[
s_{\text{stop}} = \frac{1}{2} at^2; \ v = at \implies s_{\text{stop}} = \frac{v^2}{2a}
\]

Based on similar triangles, we can write

\[
a_{\text{crit}} = \frac{\ell_{\text{front wheels}}}{h_{\text{CG}}} \implies a_{\text{crit}} = \frac{1.545}{1.4} = 1.324 \text{ m/sec}^2
\]

\[
s_{\text{stop}} = \frac{v^2}{2a} = \frac{4^2}{2(1.324)} = 6.041 \text{ m}
\]

(7) For the rover in problem (6), you need to redesign to produce a 3 m stopping distance. How far forward would you have to move the front wheels to produce a stable stopping distance of 3 m?

\[
a = \frac{v^2}{2s_{\text{stop}}} = \frac{4^2}{2(3)} = 2.667 \text{ m/sec}^2
\]

Again using similar triangles, we can write

\[
\frac{\ell_{\text{front wheels}}}{h_{\text{CG}}} = \frac{a_{\text{crit}}}{g_{\text{moon}}} \implies \ell_{\text{front wheels}} = \frac{1.545 \times 2.667}{1.4} = 2.416 \text{ m}
\]
(8) The current design for the Ares V heavy-lift launch vehicle is for the first stage to consist of six SSME rocket engines working in parallel. If the required overall reliability is 0.99, find the required reliability for each engine assuming no failures are permitted.

\[ R_{\text{system}} = 0.99 = R_{\text{single engine}}^6 \implies R_{\text{single engine}} = 0.99^{1/6} = 0.9983 \]

(9) If an individual engine has a reliability of 0.995, find the overall reliability of the propulsion system if it is okay for one of the six engines to fail.

\[ R_{\text{system}} = R_{\text{engine}}^6 + 6R_{\text{engine}}^5(1 - R_{\text{engine}}) = 0.995^6 + 6(0.995)^5(0.005) = 0.9996 \]

(10) What intercorrelated failure rate \( f \) is acceptable to produce an overall stage propulsion reliability of 0.99?

This refers to Question 9.

\[ R_{\text{system}} = R_{\text{engine}}^6 + (1 - f)6R_{\text{engine}}^5(1 - R_{\text{engine}}) = 0.995^6 + 6(0.995)^5(0.005) = 0.9996 \]

\[ f = \frac{1}{6} \left( \frac{0.9996 - 0.995^6}{0.995^5(0.005)} \right) = 0.3291 \]

(11) If a rocket engine has a first unit production cost of $100,000,000 and a learning curve rate of 76%, what is the production cost of the 100th unit produced?

\[ \text{LC} = 76\% = \ln 0.76 = -0.3959 \]

\[ C_{100} = C_1(n)^p = 100,000,000(100)^{-0.3959} = $16,150,000 \]

(12) In the rocket performance lecture, there was a graph showing payload fraction as a function of the velocity ratio \( \nu = \frac{\Delta V}{V_e} \). For a sample value of inert mass fraction \( \delta \), the graph showed that the optimum number of stages varied as a function of \( \nu \). I would like you to find an analytical expression for the value of \( \nu \) where the overall payload fractions of a single stage and two stage rocket are equal. Assume all vehicles/stages have the same exhaust velocity \( V_e \) and inert mass fraction \( \delta \), and that the \( \Delta V \) for a two-stage rocket is split evenly between the stages. (Large hint: Rather than work directly in terms of \( \nu \) or \( \Delta V \), work out the equations in terms of \( r \) for as long as possible, then substitute to find \( \nu_{\text{crossover}} = f(\delta) \).)

Let \( \Delta V \) be that required to reach orbit, and \( \nu \) be the dimensionless form of it. For a single stage vehicle, \( \lambda = r_o - \delta \) where \( r_o = e^{-\nu} \). For two stages, \( \lambda = (r_1 - \delta)(r_2 - \delta) \) where \( r_1 = r_2 = e^{-\frac{\nu}{2}} = \sqrt{r_o} \). Setting the two values of \( \lambda \) equal to each other, and dropping the subscript,

\[ r - \delta = (\sqrt{r} - \delta)^2 = r - 2\sqrt{r}\delta + \delta^2 \implies -\delta = -2\sqrt{r}\delta + \delta^2 \]

\[ 2\sqrt{r}\delta = \delta^2 + \delta \implies 2\sqrt{r} = \delta + 1 \implies r = \left( \frac{\delta + 1}{2} \right)^2 \]

\[ e^{-\nu} = \left( \frac{\delta + 1}{2} \right)^2 \implies -\nu = 2\ln \left( \frac{\delta + 1}{2} \right) = 2\ln(\delta + 1) + \ln \frac{1}{4} \]

\[ \nu = \ln(4) - 2\ln(\delta + 1) \]

So at any given value of \( \delta \), this equation will produce the critical value of \( \nu \) at which the optimum solution switches from a single stage to a two-stage vehicle.