Mass Estimating Relations

- Review of iterative design approach
- Mass Estimating Relations (MERs)
- Sample vehicle design analysis
Design is an iterative process. The necessary number of iterations is one more than the number you have currently done. This is true at any point in time.
Vehicle-Level Prelim Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- $\Delta V=9200$ m/sec
- 5000 kg payload
- LOX/LH2 propellants
  - $\text{Isp}=430$ sec
  - $\text{Ve}=4214$ m/sec
  - $\delta=0.08$

\[
\begin{align*}
    r &= e^{-\frac{\Delta V}{\text{Ve}}} = 0.1127 \\
    \lambda &= r - \delta = 0.0327 \\
    M_o &= \frac{M_\ell}{\lambda} = 153,000 \text{ kg} \\
    M_i &= \delta M_o = 12,240 \text{ kg} \\
    M_p &= M_o(1 - r) = 135,800 \text{ kg}
\end{align*}
\]
System-Level Estimation

• Start with propellant tanks (biggest part)
• LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  – LH2: 19,390 kg
  – LOX: 116,400 kg
• Propellant densities

\[ \rho_{LOX} = 1140 \ \frac{kg}{m^3} \quad \rho_{LH2} = 71 \ \frac{kg}{m^3} \]
Propellant Tank Regression Data

\[ y = 9.0911x \]
\[ R^2 = 0.9896 \]

\[ y = 12.158x \]
\[ R^2 = 0.9328 \]

[Graph showing regression lines for LH2 Tanks, LOX Tanks, and RP-1 Tanks with respective equations and R-squared values.]
Propellant Tank MERs (Volume)

- LH\(_2\) tanks
  \[ M_{LH_2\ Tank} \langle kg \rangle = 9.09 V_{LH_2} \langle m^3 \rangle \]

- All other tanks
  \[ M_{Tank} \langle kg \rangle = 12.16 V_{prop} \langle m^3 \rangle \]
Propellant Tank MERs (Mass)

- LH$_2$ tanks
  \[ \rho_{LH_2} = 71 \frac{kg}{m^3} \implies M_{LH_2 \ Tank} \langle kg \rangle = 0.128 M_{LH_2} \langle kg \rangle \]

- LOX tanks
  \[ \rho_{LOX} = 1140 \frac{kg}{m^3} \implies M_{LOX \ Tank} \langle kg \rangle = 0.0107 M_{LOX} \langle kg \rangle \]

- RP-1 tanks
  \[ \rho_{RP_1} = 820 \frac{kg}{m^3} \implies M_{RP_1 \ Tank} \langle kg \rangle = 0.0148 M_{RP_1} \langle kg \rangle \]
Cryogenic Insulation MERs

\[ M_{LH_2 \text{ Insulation}} \langle kg \rangle = 2.88 A_{tank} \langle \frac{kg}{m^2} \rangle \]

\[ M_{LOX \text{ Insulation}} \langle kg \rangle = 1.123 A_{tank} \langle \frac{kg}{m^2} \rangle \]
LOX Tank Design

• Mass of LOX=116,400 kg
  \[ M_{LOX\ Tank} = 0.0107(116,400) = 1245 \ kg \]

• Need area to find LOX tank insulation mass - assume a sphere

  \[ V_{LOX\ Tank} = \frac{M_{LOX}}{\rho_{LOX}} = 102.1 \ m^3 \]

  \[ r_{LOX\ Tank} = \left( \frac{V_{LOX}}{4\pi/3} \right)^{\frac{1}{3}} = 2.90 \ m \]

  \[ A_{LOX\ Tank} = 4\pi r^2 = 105.6 \ m^2 \]

  \[ M_{LOX\ Insulation} = 1.123\left(\frac{kg}{m^2}\right)(105.6\langle m^2 \rangle) = 119 \ kg \]
LH₂ Tank Design

- Mass of LH₂ = 19,390 kg
  \[ M_{LH₂\ Tank}(kg) = 0.128(19,390) = 2482 \text{ kg} \]
- Again, assume LH₂ tank is spherical

  \[ V_{LH₂\ Tank} = \frac{M_{LH₂}}{\rho_{LH₂}} = 273.1 \text{ m}^3 \]

\[ r_{LH₂\ Tank} = \left( \frac{V_{LH₂}}{4\pi/3} \right)^{\frac{1}{3}} = 4.02 \text{ m} \]

\[ A_{LH₂\ Tank} = 4\pi r^2 = 203.6 \text{ m}^2 \]

\[ M_{LH₂ \ Insulation} = 2.88 \langle \frac{kg}{m^2} \rangle (203.6 \langle m^2 \rangle) = 586 \text{ kg} \]
Current Design Sketch

- LOX Tank 1245 kg
- LOX Tank Insulation 119 kg
- LH₂ Tank 2482 kg
- LH₂ Tank Insulation 586 kg
Other Structural MERs

- Fairings and shrouds
  \[ M_{fairing} \langle kg \rangle = 4.95 \left( A_{fairing} \langle m^2 \rangle \right)^{1.15} \]

- Avionics
  \[ M_{avionics} \langle kg \rangle = 10 \left( M_o \langle kg \rangle \right)^{0.361} \]

- Wiring
  \[ M_{wiring} \langle kg \rangle = 1.058 \sqrt{M_o \langle kg \rangle \ell^{0.25}} \]
External Fairings - First Cut

\[ A_{cone} = \pi r \sqrt{r^2 + h^2} \]

\[ A_{frustrum} = \pi (r_1 + r_2) \sqrt{(r_1 - r_2)^2 + h^2} \]

\[ A_{cylinder} = 2\pi rh \]
External Fairings - First Cut

- Assumptions
  - P/L fairing \( h \) 7 m
  - P/L fairing \( r \) 2.9 m
  - I/T fairing \( h \) 7 m
  - I/T fairing \( r_1 \) 4.02 m
  - I/T fairing \( r_2 \) 2.9 m
  - Aft fairing \( h \) 7 m
  - Aft fairing \( r \) 4.02 m

- LH2
  - \( r = 4.02 \) m

- LOX
  - \( r = 2.90 \) m

- Aft Fairing/Boattail

- Payload Fairing

- Intertank Fairing
Fairing Analysis

• Payload Fairing
  – Area 69.03 m²
  – Mass 645 kg

• Intertank Fairing
  – Area 154.1 m²
  – Mass 1624 kg

• Aft Fairing
  – Area 176.8 m²
  – Mass 1902 kg
Avionics and Wiring Masses

- Avionics
  \[ M_{avionics} \langle kg \rangle = 10 \cdot (153,000)^{0.361} = 744 \ kg \]

- Wiring
  \[ M_{wiring} \langle kg \rangle = 1.058 \sqrt{153,000} \cdot (21 \ m)^{0.25} = 886 \ kg \]
Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass
  \[ M_{\text{Rocket Engine}} (kg) = 7.81 \times 10^{-4} T(N) + 3.37 \times 10^{-5} T(N) \sqrt{\frac{A_e}{A_t}} + 59 \]

- Solid Rocket Motor
  \[ M_{\text{Motor Casing}} = 0.135 M_{\text{propellants}} \]

- Thrust Structure Mass
  \[ M_{\text{Thrust Structure}} (kg) = 2.55 \times 10^{-4} T(N) \]
Propulsion MERs (continued)

- **Gimbal Mass**

  \[ M_{\text{Gimbals}} (kg) = 237.8 \left( \frac{T(N)}{P_0(Pa)} \right)^{.9375} \]

- **Gimbal Torque**

  \[ \tau_{\text{Gimbals}} (N \cdot m) = 990,000 \left( \frac{T(N)}{P_0(Pa)} \right)^{1.25} \]
Propulsion System Assumptions

• Initial $T/mg$ ratio $= 1.3$
  – Keeps final acceleration low with reasonable throttling

• Number of engines $= 6$
  – Positive acceleration worst-case after engine out
    \[ \frac{5}{6}(1.3) = 1.083 > 1 \]

• Chamber pressure $= 1000$ psi $= 6897$ kN
  – Typical for high-performance LOX/LH2 engines

• Expansion ratio $A_e/A_t = 30$
  – Compromise ratio with good vacuum performance
Propulsion Mass Estimates

- Rocket Engine Thrust (each)
  \[ T(N) = \frac{m_0 g (T / W)_{0}}{n_{\text{engines}}} = 324,900 \text{ N} \]

- Rocket Engine Mass (each)
  \[ M_{\text{Rocket Engine}} (\text{kg}) = 7.81 \times 10^{-4} (324,900) + 3.37 \times 10^{-5} (324,900) \sqrt{30} + 59 = 373 \text{ kg} \]

- Thrust Structure Mass (each)
  \[ M_{\text{Thrust Structure}} (\text{kg}) = 2.55 \times 10^{-4} (324,900) = 497 \text{ kg} \]
First Pass Vehicle Configuration

LOX
r=2.90 m

LH2
r=4.02 m
Mass Summary - First Pass

Initial Inert Mass Estimate 12,240 kg
LOX Tank 1245 kg
LH2 Tank 2482 kg
LOX Insulation 119 kg
LH2 Insulation 586 kg
Payload Fairing 645 kg
Intertank Fairing 1626 kg
Aft Fairing 1905 kg
Engines 2236 kg
Thrust Structure 497 kg
Gimbals 81 kg
Avionics 744 kg
Wiring 886 kg
Reserve -
Total Inert Mass 13,052 kg
Design Margin -6.22 %
Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis
Effect of Vehicle Diameter on Mass Margin

![Graph showing the effect of vehicle diameter on inert mass margin. The x-axis represents vehicle diameter in meters, ranging from 0 to 8, and the y-axis represents inert mass margin in percent, ranging from 0 to 35. The graph shows a curve that peaks around vehicle diameters of 2 to 3 meters, indicating that the mass margin is maximized in this range.](image-url)
Effect of Mass-Optimal Diameter Choice

- Mass-optimal vehicle has diameter = 1.814 m
- Mass margin goes from -6.22% to +33.1%
- Vehicle length = 155 m
- Length/diameter ratio = 86 – approximately equivalent to piece of spaghetti
- No volume for six rocket engines in aft fairing
- Infeasible configuration
Effect of Diameter on Vehicle L/D

Vehicle Diameter (m) vs. Length/Diameter Ratio
Second Pass Vehicle Configuration
### Mass Summary - Second Pass

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<tr>
<th>Component</th>
<th>Initial Estimate</th>
<th>Revised Estimate</th>
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<tr>
<td>Initial Inert Mass Estimate</td>
<td>12,240 kg</td>
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<td>LOX Tank</td>
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<td>LH2 Tank</td>
<td>2482 kg</td>
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<td>LOX Insulation</td>
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<td>Engines</td>
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<td>Design Margin</td>
<td>-6.22 %</td>
<td>+22.9 %</td>
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Modifications for Iteration 3

• Keep 4 m tank diameter
• Change initial assumption of $\delta$ iteratively, with resulting changes in $m_0$ and $m_i$, to reach 30% mass margin
• Modify diameter to keep $L/D \leq 10$ and iterate again for optimal initial mass estimate
Vehicle-Level Prelim Design - 3rd Pass

• Single Stage to Orbit (SSTO) vehicle
• \( \Delta V = 9200 \) m/sec
• 5000 kg payload
• LOX/LH2 propellants
  – Isp=430 sec
    (Ve=4214 m/sec)
  – \( \delta = 0.08323 \)
• Diameter=4.2 m
• L/D=9.7

\[
\begin{align*}
  r &= e^{-\frac{\Delta V}{V_e}} = 0.1127 \\
  \lambda &= r - \delta = 0.0294 \\
  M_o &= \frac{M_\ell}{\lambda} = 169,800 \ kg \\
  M_i &= \delta M_o = 14,130 \ kg \\
  M_p &= M_o(1 - r) = 150,700 \ kg
\end{align*}
\]
# Mass Summary - Third Pass

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## Mass Budgeting

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References