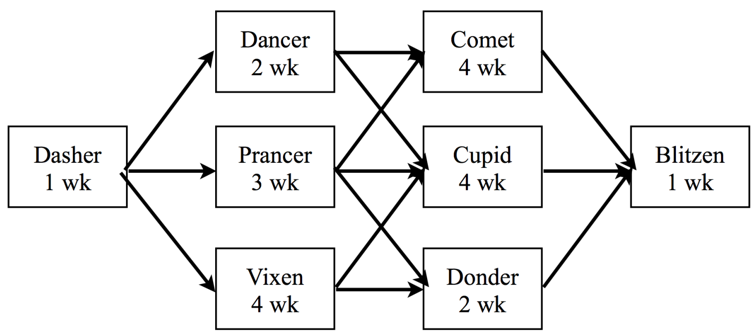
ENAE 483/788D FINAL EXAMINATION – FALL, 2015

No phones, computers, or internet-enabled devices. Use the spaces following the questions to write your answers; you can also use the backs of the pages as necessary, but be sure to label which problem you are working on. Do not get stuck on a problem - if the solution method isn't apparent, move on and come back to it as time allows. Print your name neatly!!! (For that matter, please print everything neatly, and it wouldn't hurt to draw boxes around your answers, either.)

(1) A program plan can be represented by the following graphic:



(a) What is the critical path?

Dasher-Vixen-Cupid-Blitzen

(b) What is the shortest possible time for completion of the program?

10 weeks

(c) What is the slack time for task Dancer?

2 weeks

(d) What is the general name for this type of diagram?

PERT chart

(2) You are in the International Space Station in orbit around the Earth (μ=398,604 $\frac{km^3}{sec^2}$, r_E=6378 km) at an altitude of 400 km. You have been informed of a potential collision with orbital debris in a geostationary transfer orbit with a perigee identical to your orbital altitude, and an apogee radius of 42,240 km. The plane of the debris' orbit is perpendicular to your orbital plane. When the debris has its closest approach, what will its velocity be with respect to ISS?

Your velocity:

$$V_{circ} = \sqrt{\frac{\mu}{r_{circ}}} = \sqrt{\frac{398604}{6378 + 400}} = 7.669 \frac{km}{sec}$$

Debris velocity at perigee:

$$V_p = \sqrt{\frac{\mu}{r_p}}\sqrt{\frac{2r_a}{r_p + r_a}} = \sqrt{\frac{398604}{6778}}\sqrt{\frac{2(42240)}{6778 + 42240}} = 10.07 \frac{km}{sec}$$

Since the two velocities are orthogonal,

$$V_{relative} = \sqrt{V_{circ}^2 + V_p^2} = \sqrt{7.669^2 + 10.07^2} = \boxed{12.66 \frac{km}{sec}}$$

(3) Exploration of Phobos will entail a great deal of extravehicular activity. The space suits that will be used will have a 4.3 psi internal pressure with 100% oxygen. The Phobos Outpost habitat should have an atmosphere which will allow immediate transition to EVA with a decompression ratio of no more than R=1.4. It must also have a partial pressure of O_2 of 3.0 psi.

(a) Assuming you go EVA directly from the Phobos Outpost habitat, calculate the total pressure and oxygen percentage of the habitat to limit the decompression R to 1.4.

$$R = \frac{ppN_2(hab)}{P_{total}(suit)} \implies ppN_2(hab) = R \times P_{total}(suit) = 1.4(4.3) = 6.02\ psi$$

$$P_{total}(hab) = ppN_2(hab) + ppo_2(hab) = 6.02 + 3.0 = \boxed{9.02 \text{ psi}}$$

$$\%O_2 = \frac{ppO_2(hab)}{P_{total}(hab)} = \frac{3.0}{9.02} = \boxed{33.26\% \text{ O2}}$$

(b) Another concept for outpost operations is to have small Space Exploration Vehicles (SEVs) which transport crew to the exploration site, and support EVA at that point. The SEV atmosphere also must have an O_2 partial pressure of 3 psi. Assuming the Outpost habitat has an Earth sea level atmosphere (14.7 psi, 21% O_2), select an atmosphere (total pressure and O_2%) for the SEV such that the R value transitioning from the outpost to the SEV is equal to the R value from the SEV cabin to EVA.

$$R = \frac{ppN_2(hab)}{P_{total}(SEV)} = \frac{ppN_2(SEV)}{P_{total}(suit)}; \quad P_{total}(SEV) = ppN_2(SEV) + ppO_2(SEV)$$

$$\frac{ppN_2(hab)}{ppN_2(SEV) + ppO_2(SEV)} = \frac{ppN_2(SEV)}{P_{total}(suit)}$$

$$[ppN_2(SEV)]^2 + ppO_2(SEV)[ppN_2(SEV)] - ppN_2(hab)P_{total}(suit) = 0$$

$$ppN_2(SEV) = \frac{1}{2}\left(\sqrt{ppN_2(SEV)^2 + 4ppN_2(hab)P_{total}(suit)} - ppO_2(SEV)\right)$$

$$ppN_2(SEV) = \frac{1}{2}\left(\sqrt{3^2 + 4(0.79 \times 14.7)4.3} - 3\right) = 5.724\ psi$$

$$P_{total}(SEV) = ppN_2(SEV) + ppO_2(SEV) = 5.724 + 3 = \boxed{8.724\ psi}$$

$$O_2\% = \frac{ppO_2(SEV)}{P_{total}(SEV)} = \frac{3}{8.724} = \boxed{34.39\%}$$

(4) Design a spherical tank to hold 1.0 m^3 helium gas at a pressure of 20 MPa. Use titanium, assuming a tensile yield strength of 920 MPa and a density of 4430 kg/m^3. Design for a factor of safety of 3. What is the empty mass of the tank?

$$V = \frac{4}{3}\pi r^3 \Longrightarrow r = \left(\frac{3V}{4\pi}\right)^{\frac{1}{3}} = \left(\frac{3(1)}{4\pi}\right)^{\frac{1}{3}} = 0.6204\ m$$

$$\sigma_{allow} = \frac{\sigma_y}{FOS} = \frac{920}{3} = 306.7\ MPa$$

$$t = \frac{Pr}{2\sigma_{allow}} = \frac{20(0.6204)}{2(306.7)} = 0.02023\ m$$

$$M_{tank} = 4\pi r^2 t\rho = 4\pi(0.6204)^2(0.02032)(4430) = \boxed{433.4\ kg}$$

(5) United Launch Alliance recently had the 100^{th} successful launch of an Atlas V, with no failures.

(a) What reliability can they claim at an 80% confidence level?

$$R^{100} + C = 1 \implies R = (1-C)^{1/100} = 0.2^{0.01} = \boxed{98.40\%}$$

(b) If their next flight fails, what confidence level would they have for your estimate from (a)?

$$R^{101} + 101R^{100}(1-R) + C = 1 \implies C = 1 - R^{101} - 101R^{100}(1-R) = 1 - 0.1968 - 0.3221 = \boxed{48.11\%}$$

(6) The Space Launch System will have four RS-25 engines in the core vehicle, and two solid rocket boosters (SRBs). Assume the reliability for an RS-25 is 99.5%, and for an SRB is 99.8%. The upper stage uses four RL-10 engines with a reliability of 99.2%.

(a) If all the rocket engines must operate for a successful launch, what launch reliability will the SLS have?

$$R_{total} = R_{RS25}^{n_{RS25}} \times R_{SRB}^{n_{SRB}} \times R_{RL10}^{n_{RL10}} = 0.995^4 \times 0.998^2 \times 0.992^4 = \boxed{0.9454}$$

(b) If the vehicle can be successful with the failure of one RS-25, what is the new system reliability?

$$\begin{aligned} R_{total} &= \left[R_{RS25}^4 + 4R_{RS25}^3(1-R_{RS25})\right] \times R_{SRB}^2 \times R_{RL10}^4 \\ &= [0.995^4 + 4(0.995)^3(0.005)] \times 0.998^2 \times 0.992^4 = \boxed{0.9644} \end{aligned}$$

(c) If the vehicle can be successful with the failure of one RS-25 or one RL-10, what is the new system reliability?

$$\begin{aligned} R_{total} &= R_{no\ fail} + R_{(1)RS25\ fail} + R_{(1)RL10\ fail} \\ R_{no\ fail} &= R_{RS25}^{n_{RS25}} \times R_{SRB}^{n_{SRB}} \times R_{RL10}^{n_{RL10}} = 0.995^4 \times 0.998^2 \times 0.992^4 = 0.9454 \\ R_{(1)RS25\ fail} &= 4R_{RS25}^3(1-R_{RS25}) \times R_{SRB}^2 \times R_{RL10}^4 = 4(0.995)^3(0.005) \times 0.998^2 \times 0.992^4 = 0.01900 \\ R_{(1)RL10\ fail} &= R_{RS25}^4 \times R_{SRB}^2 \times 4R_{RL10}^3(1-R_{RL10}) = 0.995^4 \times 0.998^2 \times 4(0.992)^3 0.008 = 0.03050 \\ R_{total} &= R_{no\ fail} + R_{(1)RS25\ fail} + R_{(1)RL10\ fail} = 0.9454 + 0.01900 + 0.03050 = \boxed{0.9949} \end{aligned}$$

(7) At the distance of the Earth from the sun (1 A.U.), the insolation constant I_s is 1394 $\frac{W}{m^2}$. There is a planar photovoltaic array positioned perpendicularly to the sunlight, where the front (sun-facing) surface has the properties α_f=0.9, ϵ_f=0.7, and the back surface characteristics are α_b=0.3, ϵ_b=0.85. Assume the array is isothermal.

(a) Calculate the temperature of the array from radiative equilibrium ($\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$).

$$\alpha_f A I_s = A\epsilon_f \sigma T^4 + A\epsilon_b \sigma T^4 \Longrightarrow T = \left[\frac{\alpha_f I_s}{\sigma(\epsilon_f + \epsilon_b)}\right]^{\frac{1}{4}}$$

$$= \left[\frac{0.9(1394)}{5.67 \times 10^{-8}(0.7 + 0.85)}\right]^{\frac{1}{4}} = \boxed{345.7K}$$

(b) If the maximum operating temperature for the array is 155^{o}C (428 K), what is the closest distance to the sun at which the array is usable?

$$\alpha_f A I_s' = A\epsilon_f \sigma T_{lim}^4 + A\epsilon_b \sigma T_{lim}^4 \Longrightarrow I_s' = \frac{\epsilon_f + \epsilon_b}{\alpha_f}\sigma T_{lim}^4 = \frac{0.7 + 0.85}{0.9} 5.67 \times 10_{-8}(428)^4 = 3277 \frac{W}{m^2}$$

Since the solar insolation constant is inversely proportional to the square of the distance,

$$\frac{I_s}{I_s'} = \left(\frac{r_{lim}}{r_{EO}}\right)^2 \Longrightarrow r_{lim} = \sqrt{\frac{I_s}{I_s'}} r_{EO} = \sqrt{\frac{1394}{3277}}(1\ AU) = \boxed{0.6522\ AU}$$

EmphIf, on the other hand, you remembered that 1 AU=149,500,000 km, an equally good answer is $\boxed{97{,}510{,}000 \text{ km}}$

(8) Which of the following numbers have too many significant figures to be appropriate in an engineering presentation?

(a) 148,000
(b) 148.000 *Too many significant figures*
(c) 0.1480
(d) 0.148000 *Too many significant figures*
(e) 0.00148
(f) 0.000000148
(g) 3452.98 *Too many significant figures*
(h) 99.0089 *Too many significant figures*
(i) 385.542 *Too many significant figures*
(j) 3,549,298,492.949104948590285884 *WAYYY too many significant figures*

(9) A lunar base experiences 13 days of continuous solar illumination adequate for the use of photovoltaic arrays, followed by 15 days of darkness or dawn/dusk time where energy storage has to be used. The base uses 10 kW of electrical power during the day, and 15 kW at night. Extra power must be generated in the daytime to recharge the energy storage system. Assuming the energy storage charge/discharge cycle is perfectly efficient, and power generation using sun-tracking photovoltaic arrays with a conversion efficiency of 22%, calculate the required surface area of solar arrays for the base. [Use the value of I_s from above.]

$$Daytime\ energy = (10\ kW)(13\ d)(24\ \frac{hr}{d}) = 3120\ kWhr$$

$$Nighttime\ energy = (15\ kW)(15\ d)(24\ \frac{hr}{d}) = 5400\ kWhr$$

$$E_{total} = E_{day} + E_{night} = 3120 + 5400 = 8520\ kWhr$$

All of that energy must be generated during the daytime, so

$$P = \frac{E_{total}}{t_{day}} = \frac{8520}{13(24)} = 27.31\ kW$$

$$A_{array} = \frac{P}{\eta I_s} = \frac{27.31}{0.22(1394)} = \boxed{89.04\ m^2}$$

(10) If a rocket engine has a first unit production cost of $100,000,000 and a learning curve rate of 76%, what is the production cost of the 100^{th} unit produced?

$$LC = 76\% \Longrightarrow p = \frac{\ln 0.76}{\ln 2} = -0.3959$$

$$C_{100} = C_1(n)^p = 100,000,000(100)^{-0.3959} = \boxed{\$16{,}150{,}000}$$

(11) For maneuvering around Phobos, you are going to use N_2O cold gas system for propulsion, with a specific impulse of 70 sec when used at an ambient temperature of 300K.

(a) If you put heaters in the thrusters to increase the N_2O temperature to 700K, what is the new specific impulse?

$$I_{sp} \propto \sqrt{\frac{T}{\bar{M}}} \implies I'_{sp} = I_{sp}\sqrt{\frac{T'}{T}} = 70\sqrt{\frac{700}{300}} = \boxed{106.9\ sec}$$

(b) If you dissociate the N_2O to form N_2 + 1/2 O_2 at a temperature of 1500K, what would the new specific impulse be? (Hint: the atomic weight of N=14, and O=16.)

$$\bar{M}_{N2O} = 2 \times 14 + 16 = 44;\ \bar{M}_{diss} = \frac{1}{3}(2 \times 28 + 32) = 29.33$$

$$I'_{sp} = I_{sp}\sqrt{\frac{T'}{M'}\frac{M}{T}} = 70\sqrt{\frac{1500}{29.33}\frac{44}{300}} = \boxed{191.7\ sec}$$

(12) If you assume the inflation rate will be constant at 2.3% per year, what will \$1 in 2015 be equal to in 2025 dollars?

$$NFV = NPV(1+r)^n = 1(1.023)^{10} = \boxed{\$1.26}$$

(13) Like Mark Watney, you find yourself left behind on the surface of Mars with no way to communicate. However, there is a Phobos Outpost, so you just have to make it there to be saved. The required Δv is 5000 m/sec. μ_{Mars}=42,970 km^3/sec^2, r_{Mars}=3393 km

(a) You manage to find some stored LOX and methane propellants, which has a specific impulse of 370 sec. The automated spacecraft manufacturing system (which is conveniently available) can produce a launch vehicle with an inert mass fraction δ=0.08. You found an old Mars lander cabin which you will use as your spacecraft, for a total payload mass of 5000 kg. Design a single stage to orbit launch vehicle that will get you into Mars orbit. Specify gross mass, inert mass, and propellant mass for this vehicle.

$$r = e^{-\frac{\Delta v}{gI_{sp}}} = e^{-\frac{5000}{9.8(370)}} = 0.2518$$

(I'm sure everyone remembers that "g" in the preceding equation is a conversion factor from force to mass, and has nothing to do with the local gravitational acceleration...)

$$\lambda = r - \delta = 0.2518 - 0.8 = 0.1718; \quad m_o = \frac{m_{PL}}{\lambda} = \frac{5000}{0.1718} = \boxed{29{,}095\ kg}$$

$$m_{in} = \delta m_o = 0.08(29,095) = \boxed{2328\ kg}; \quad m_{pr} = (1-r)m_o = (1-0.2518)(29095) = \boxed{21{,}769\ kg}$$

(b) It turns out you don't have enough propellant at the site to use a single-stage launch vehicle. Design a two-stage launch vehicle using the same parameters, maximizing the payload fraction to orbit. Again, list gross mass, inert mass, and propellant mass for each stage. How much total propellant mass do you need to make this system viable? What was your distribution of Δv between the two stages?

Since $I_{sp1} = I_{sp2}$ and $\delta_1 = \delta_2$, a reasonable approach would be to assume $\boxed{\Delta v_1 = \Delta v_2}$

$$\Delta v_1 = \Delta v_2 = 2500\ \frac{m}{sec} \implies r_1 = r_2 = 0.5018 \implies \lambda_1 = \lambda_2 = 0.4218$$

$$m_{o2} = \frac{m_{PL}}{\lambda_2} = \boxed{11{,}853\ kg}; \quad m_{in2} = \delta_2 m_{o2} = \boxed{948.2\ kg}; \quad m_{pr2} = (1-r_2)m_{o2} = \boxed{5905\ kg}$$

$$m_{o1} = \frac{m_{o2}}{\lambda_1} = \boxed{28{,}101\ kg}; \quad m_{in1} = \delta_1 m_{o1} = \boxed{2248\ kg}; \quad m_{pr1} = (1-r_1)m_{o1} = \boxed{14{,}000\ kg}$$

Total propellant required = $\boxed{19{,}905\ kg}$

(14) The country of Fictitcioustan has developed the Tsolyllatot satellite navigation system, which uses satellites in elliptical orbits with a 12-hour period. μ_{Earth}=398,604 km^3/sec^2, r_{Earth}=6378 km)

(a) If the perigee altitude for this orbit is 1000 km, what is the apogee altitude?

$$P = 2\pi\sqrt{\frac{a^3}{\mu}} \implies a = \left[\mu\left(\frac{P}{2\pi}\right)^2\right]^{\frac{1}{3}} = \left[398604\left(\frac{24 \times 3600}{2\pi}\right)^2\right]^{\frac{1}{3}} = 26,610\ km$$

$$r_p = h_p + r_E = 1000 + 6378 = 7378\ km$$

$$a = \frac{r_p + r_a}{2} \implies r_a = 2a - r_p = 2 \times 26610 - 7378 = 45,843\ km$$

$$h_a = r_a - r_E = 45843 - 6378 = \boxed{39{,}465\ km}$$

(b) How long will it take for a satellite to go from perigee to a position where the true anomaly θ=120^o?

$$n = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{398604}{26610^3}} = 1.454 \times 10^{-4}\ rad/sec$$

$$e = 1 - \frac{r_p}{a} = 1 - \frac{7378}{26610} = 0.7227$$

$$E = 2\ tan^{-1}\left[\sqrt{\frac{1-e}{1+e}}\tan\left(\frac{\theta}{2}\right)\right] = 2\ tan^{-1}\left[\sqrt{\frac{0.2773}{1.7227}}\tan\left(\frac{120}{2}\right)\right] = 69.59^{O} = 1.215\ rad$$

$$t = \frac{E - e\sin E}{n} = \frac{1.215 - 0.7227\sin 1.215}{1.454 \times 10^{-4}} = \boxed{3695\ sec = 1h1m35s}$$