

ENAE 483/788D TECHNICAL SPECIALTY PROBLEMS – FALL, 2015

NOTE: The structures problems are posted separately

- (1) Calculate the decompression R-values for each of the following transitions
 (a) From Earth sea level (14.7 psi, 21% O₂) to Gemini (3.5 psi, 100% O₂)

$$R = \frac{PP_{N_2 \text{ start}}}{P_{\text{final}}} = \frac{0.79(14.7)}{3.5} = 3.318$$

- (b) From Earth sea level (14.7 psi, 21% O₂) to Skylab (5 psi, 80% O₂)

$$R = \frac{PP_{N_2 \text{ start}}}{P_{\text{final}}} = \frac{0.79(14.7)}{5} = 2.323$$

- (c) From Earth sea level (14.7 psi, 21% O₂) to a lunar habitat (10.2 psi, 30% O₂)

$$R = \frac{PP_{N_2 \text{ start}}}{P_{\text{final}}} = \frac{0.79(14.7)}{10.2} = 1.139$$

- (d) From a lunar habitat (10.2 psi, 30% O₂) to a Shuttle-type pressure suit (4.3 psi, 100% O₂)

$$R = \frac{PP_{N_2 \text{ start}}}{P_{\text{final}}} = \frac{0.7(10.2)}{4.3} = 1.660$$

- (e) From a lunar habitat (10.2 psi, 30% O₂) to an Russian Orlan pressure suit (5.8 psi, 100% O₂)

$$R = \frac{PP_{N_2 \text{ start}}}{P_{\text{final}}} = \frac{0.7(10.2)}{5.8} = 1.231$$

- (2) You would like to generate Mars gravity (3.8 m/sec²) in the transit habitat for the trip to and from Mars.

- (a) If the crew can tolerate 3 rpm, what is the required radius of spin for the habitat?

$$\omega = (3 \text{ rpm}) \frac{2\pi \text{ rad/rev}}{60 \text{ sec/min}} = 0.3142 \text{ rad/sec}$$

$$a_{\text{centripetal}} = \omega^2 r \implies r = \frac{a_{\text{centripetal}}}{\omega^2} = \frac{3.8}{0.3142^2} = 38.49 \text{ m}$$

- (b) The habitat has two decks, 2 m apart. If the system is designed to provide Mars gravity at the upper deck, what is the gravity at the lower deck?

$$a_{\text{centripetal}} = \omega^2 r = 0.3142^2(38.49 + 2) = 3.997 \text{ m/sec}^2$$

- (3) A rocket engine on the first stage of a launch vehicle has a combustion chamber pressure of 1500 psi and an exit pressure of 9 psi. The ratio of specific heats γ in the exhaust is 1.2. The exhaust velocity at ideal conditions ($P_e = P_a$) is 3000 m/sec.

- (a) Calculate the expansion ratio of this engine's nozzle

$$\frac{A_t}{A_e} = \left(\frac{\gamma + 1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{p_e}{p_0}\right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

$$\begin{aligned} \frac{A_t}{A_e} &= \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{9}{1500}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2} \left[1 - \left(\frac{9}{1500}\right)^{\frac{0.2}{1.2}}\right]} \\ &= (1.611)(0.01408)(2.512) = 0.05695 \implies \frac{A_e}{A_t} = 17.56 \end{aligned}$$

(b) If you wanted to lower the exit pressure to 1 psi, what expansion ratio would you need?

$$\begin{aligned} \frac{A_t}{A_e} &= \left(\frac{2.2}{2}\right)^{\frac{1}{0.2}} \left(\frac{1}{1500}\right)^{\frac{1}{1.2}} \sqrt{\frac{2.2}{0.2} \left[1 - \left(\frac{1}{1500}\right)^{\frac{0.2}{1.2}}\right]} \\ &= (1.611)(0.002256)(2.784) = 0.01011 \implies \frac{A_e}{A_t} = 98.89 \end{aligned}$$

(c) How would the engine's exhaust velocity change with the new nozzle, if all other conditions (including ideal exit matching conditions $P_e = P_a$) in the engine were held constant?

$$\begin{aligned} V_e &\propto \sqrt{1 - \left(\frac{p_e}{p_0}\right)^{\frac{\gamma-1}{\gamma}}} \\ V_{e2} &= V_{e1} \sqrt{\frac{1 - \left(\frac{p_{e2}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}{1 - \left(\frac{p_{e1}}{p_0}\right)^{\frac{\gamma-1}{\gamma}}}} = 3000 \sqrt{\frac{1 - \left(\frac{1}{1500}\right)^{\frac{0.2}{1.2}}}{1 - \left(\frac{9}{1500}\right)^{\frac{0.2}{1.2}}}} = 3000 \sqrt{\frac{0.7044}{0.5737}} = 3324 \frac{m}{sec} \end{aligned}$$

(4) You are designing a base on Phobos to monitor the surface of Mars. Your solar arrays are designed to track the sun, but are eclipsed by both nighttime on Phobos and eclipse each orbit by Mars. The orbital period of Phobos is 7 hrs 32 min (in fact, as viewed from the surface, Phobos rises in the west and sets in the east - almost three times per day!) Starting at dawn on your Phobos base, the solar arrays are illuminated for 89 min, until they are eclipsed by Mars. Darkness on the back side of Mars lasts for 51 min, then sunlight is available for 89 min until local sundown. The nighttime duration is then 230 min. The human base will need a continual power level of 60 kW.

$$r_{Mars} = 3393 \text{ km}, r_{Phobos} = 9376 \text{ km}, \text{ mean radius } 11.27 \text{ km}, P = 7h39.2m$$

(a) How much energy storage do you need in your batteries to maintain power during dark periods?

$$\text{Per orbit, } (60 \text{ kW})(230 + 51 \text{ min}) = 16,860 \text{ kW-min} = 281 \text{ kW-hrs}$$

(b) If the batteries are 90% efficient at charging (i.e., only 90% of the power you supply for recharge will be stored in the battery) and 95% efficient at discharge (i.e., 5% of the capacity of the battery is unavailable for use), how does your answer to (a) change?

$$E_{req} = 281 \text{ kW-hrs} \left(\frac{1}{0.95}\right) \left(\frac{1}{0.9}\right) = 328.7 \text{ kW-hrs}$$

(c) If the batteries have a specific energy of 200 W-hrs/kg, what is the mass of the batteries (as defined by your answer to (b))?

$$m_{batt} = \frac{328.7 \text{ kW-hrs}}{200 \text{ W-hrs/kg}} = 1634 \text{ kg}$$

- (d) How much power is required from the photovoltaic arrays to charge the batteries with a 10% energy margin?

$$\text{Daylight/orbit} = 178 \text{ min} \implies P_{\text{recharge}} = \frac{(1.1)328.7 \text{ kW-hrs}}{178 \text{ min}/60 \frac{\text{min}}{\text{hr}}} = 121.9 \text{ kW}$$

- (e) The PV arrays you have chosen have a conversion efficiency of 18% and a mass of 5 kg/m². What are the area and mass of the PV arrays? What is the total mass of your power system?

$$\begin{aligned} P_{\text{total}} &= P_{\text{ops}} + P_{\text{recharge}} = 60 + 121.9 = 181.9 \text{ kW} \\ r_{\text{Mars}}(\text{worst case}) &= 1.666 \text{ AU} \\ I_{s,\text{Mars}} &= I_{s,\text{Earth}} \left(\frac{r_{\text{Earth}}}{r_{\text{Mars}}} \right)^2 = 1394 \left(\frac{1}{1.666} \right)^2 = 502.2 \text{ W/m}^2 \\ A_{\text{array}} &= \frac{P_{\text{total}}}{I_s \eta} = \frac{181,900 \text{ W}}{502.2 \text{ W/m}^2 (0.18)} = 2012 \text{ m}^2 \\ m_{\text{array}} &= (2012 \text{ m}^2)(5 \text{ kg/m}^2) = 10,059 \text{ kg} \\ m_{\text{total}} &= m_{\text{array}} + m_{\text{batt}} = 10059 + 1634 = 11,700 \text{ kg} \end{aligned}$$

- (f) In the event of a failure of your PV arrays, you need to support minimum base functions at 25 kW for 48 hours to give you time to safely evacuate the crew to orbital spacecraft. How does this change the capacity and mass of the batteries?

$$\begin{aligned} E_{\text{req}} &= (25 \text{ kW})(48 \text{ hrs}) = 1200 \text{ kW-hrs} \left(\frac{1}{0.95} \right) \left(\frac{1}{0.9} \right) = 1404 \text{ kW-hrs} \\ m_{\text{batt}} &= \frac{1404 \text{ kW-hrs}}{200 \text{ W-hrs/kg}} = 7018 \text{ kg} \end{aligned}$$

- (g) You need to be able to recharge the total battery pack following a PV failure and evacuation to allow you to reopen the base for operations. if the station needs to run at the 25 kW (continuous) level while recharging the batteries, how long will it take to reach a full charge?

$$\begin{aligned} P_{\text{avg}} &= P_{\text{total}} \frac{t_{\text{daylight}}}{t_{\text{orbit}}} = 181.9 \frac{178}{459} = 70.54 \text{ kW} \\ P_{\text{charge,avg}} &= P_{\text{avg}} - P_{\text{ops}} = 70.54 - 25 = 45.54 \text{ kW} \\ t_{\text{charge}} &= \frac{E_{\text{req}}}{P_{\text{charge,avg}}} = \frac{1404 \text{ kW-hrs}}{45.54 \text{ kW}} = 30.83 \text{ hrs} \end{aligned}$$

- (5) You are designing a solar dynamic power system for a large space station. The goal is to generate 400 kW continuously.

- (a) If the system were 100% efficient, how large would the solar concentrator have to be to collect 400 kW of incoming solar power at 1 AU?

$$A = \frac{P}{I_s} = \frac{400,000 \text{ W}}{1394 \text{ W/m}^2} = 286.9 \text{ m}^2$$

- (b) The best thermodynamic efficiency possible is that of the Carnot cycle, $\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$. If $T_{\text{hot}} = 1900\text{K}$ and $T_{\text{cold}} = 450\text{K}$, what is the efficiency? What is the new required solar concentrator area?

$$\begin{aligned} \eta &= 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}} = 1 - \frac{450}{1900} = 0.7632 \\ A &= \frac{P}{\eta I_s} = \frac{400,000 \text{ W}}{0.7632(1394 \text{ W/m}^2)} = 376.0 \text{ m}^2 \end{aligned}$$

(c) Given your answer to (b) above, how much waste heat do you need to radiate away?

$$P_{waste} = AI_s(1 - \eta) = (376.0 \text{ m}^2)(1394 \text{ W/m}^2)(1 - 0.7632) = 124,100 \text{ W} = 124.1 \text{ kW}$$

(d) If the radiator is a flat plate ($\alpha = 0.4$; $\epsilon = 0.8$) that radiates to deep space on both sides without any other heat loads, what area would the radiator have to be?

$$P_{rad} = A\epsilon\sigma T^4 \implies A = \frac{P_{rad}}{\epsilon\sigma T^4} = \frac{124,100}{0.8(5.67 \times 10^{-8})(450)^4} = 66.73 \text{ m}^2$$

Since a flat plate has two sides, $A_{rad} = 33.36 \text{ m}^2$

A_{rad} is the physical size of the radiator assembly, which radiates out of both sides.

(e) If the same radiator design is in low Earth orbit such that one side radiates to Earth ($T_{Earth} = 280\text{K}$) and the other side is illuminated by the sun, what would be the new required radiator area?

$$A_{rad}\alpha I_s + P_{rad} = A_{rad}\epsilon\sigma T_{rad}^4 + A_{rad}\epsilon\sigma (T_{rad}^4 - T_{Earth}^4)$$

$$A_{rad} = \frac{P_{rad}}{\epsilon\sigma T_{rad}^4 + \epsilon\sigma (T_{rad}^4 - T_{Earth}^4) - \alpha I_s}$$

$$A_{rad} = \frac{124,100}{0.8(5.67 \times 10^{-8})(450)^4 + 0.8(5.67 \times 10^{-8})(450^4 - 280^4) - 0.4(1394)}$$

$$A_{rad} = \frac{124,100}{1860 + 1581 - 558} = 43.04 \text{ m}^2$$