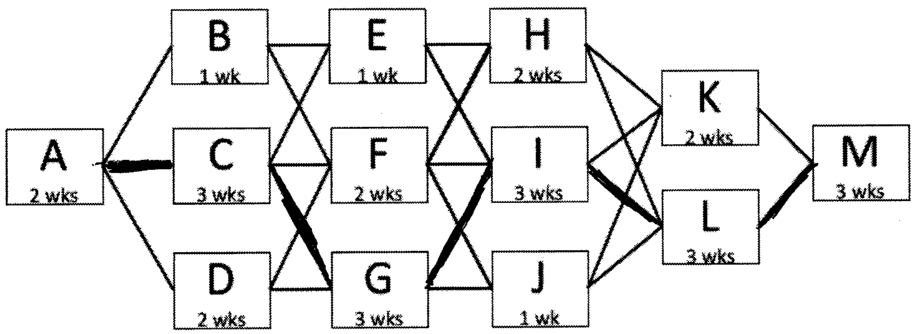
**ENAE 483/788D MIDTERM – FALL, 2016 – *NAME:***

*One 8.5" x 11" piece of paper allowed for notes (both sides). No Internet-enabled devices allowed. Put your name on the cover page, and on each page if you disassemble the quiz package. Please write neatly, and put boxes around your answers.*

A project scheduling activity results in a PERT chart as shown in the following graphic.




(1) What is the critical path?

A-C-G-I-L-M

(2) How long does it take to complete the project?

17 weeks

(3) What is the slack time for task K?

1 week

The components of your SEP tug are launched into a circular low Earth orbit with an altitude of 300 km. It has been decided to perform the assembly of the tug at the Earth-Moon L5 point, which is a circular Earth orbit with a radius of 384,400 km. ($\mu_{Earth} = 398,604\ km^3/sec^2$; $r_{Earth} = 6378\ km$)

(4) Assuming the LEO orbit is coplanar with the L5 orbit, what is the $\Delta v$ required for a Hohmann trajectory from LEO to L5?

$$r_1 = h_1 + r_E = 300 + 6378 = 6678\ km$$

$$V_{c_1} = \sqrt{\frac{\mu}{r_1}} = 7.726\ km/sec \qquad \Delta V_1 = V_{c_1}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right) = 3.106\ km/sec$$

$$V_{c_2} = \sqrt{\frac{\mu}{r_2}} = 1.018\ km/sec \qquad \Delta V_2 = V_{c_2}\left(1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right) = 0.8301\ km/sec$$

$$\Delta V_{TOT} = \Delta V_1 + \Delta V_2 = 3.936\ km/sec$$

(5) How would your answer to (4) change if there is a 20° plane change required, all of which is performed at apogee?

$\Delta V_1$ unchanged $\qquad \Delta V_{ap} = V_{c_2}\sqrt{\frac{2r_1}{r_1+r_2}} = 0.188\ km/sec$

$$\Delta V_2 = \sqrt{V_{c_2}^2 + V_{ap}^2 - 2V_{c_2}V_{ap}\cos\Delta i} = 0.8438\ km/sec$$

$$\Delta V_{TOT} = \Delta V_1 + \Delta V_2 = 3.950\ km/sec$$

(6) How long would it take to travel from LEO to L5 in this orbit?

$P = 2\pi\sqrt{\frac{a^3}{\mu}}$  $a = \frac{1}{2}(r_1 + r_2) = 195{,}500$ km

$P = 860{,}500$ sec $= 239$ hr

$t = \frac{P}{2} = 119.5$ hr $= 4.98$ days

(7) What would be the required $\Delta v$ if a SEP tug with an $I_{sp}$ of 5000 sec were used to perform the maneuver?

$\Delta V = \sqrt{\frac{\mu}{r_1}} - \sqrt{\frac{\mu}{r_2}} = 6.708$ km/sec

(8) At a constant acceleration of 0.001 m/sec$^2$, what would be the time required for the low-thrust transfer from LEO to L5?

$t = \frac{\Delta V}{a} = 6{,}708{,}000$ sec $= 1863$ hrs $= 77.63$ days

(9) The Van Allen radiation belts stretch from an altitude of 800 km to 72,000 km. This is of concern because the photovoltaic arrays of the SEP suffer accelerated degradation from the radiation in the van Allen belts. How long would the SEP be within the van Allen belts for the LEO-L5 transfer?

$r_1 = 800 + 6378 = 7178 \text{ km} \qquad r_2 = 72{,}000 + 6378 = 78{,}380 \text{ km}$

$$\Delta V = \sqrt{\frac{\mu}{r_1}} - \sqrt{\frac{\mu}{r_2}} = 5.197 \text{ km/sec}$$

$$t = \frac{\Delta V}{a} = 5{,}197{,}000 \text{ sec} = 1444 \text{ hr} = 60.14 \text{ days}$$

The second stage of the Falcon 9 has the following characteristics:

| | |
|---|---|
| Propellant mass | 107,500 kg |
| Inert mass | 4000 kg |
| Specific Impulse | 340 sec |
| Payload | 13,150 kg |

(10) What is the $\Delta v$ for this stage?

$m_f = m_{in} + m_{PL} = 17{,}150 \text{ kg} \qquad m_i = m_f + m_{pr} = 124{,}650 \text{ kg}$

$$\Delta V = -g I_{sp} \ln \frac{m_f}{m_i} = 6609 \text{ m/sec}$$

(11) What is the inert mass fraction $\delta$ for this stage?

$$\delta = \frac{m_{in}}{m_{in} + m_{PL} + m_{pr}} = 0.03209$$

(12) What is the stage inert mass fraction $\epsilon$ for this stage?

$$\varepsilon = \frac{m_{in}}{m_{in} + m_{PR}} = 0.03587$$

(13) You would like to replace this LOX/RP-1 stage with methane (LOX/LCH4), which provides an $I_{sp}$ of 382 seconds, and requires an inert mass fraction $\delta$=0.045. To minimize the modifications to the first stage, the total weight of the second stage plus payload must remain the same as the previous design. Calculate the (i) payload mass, (ii) propellant mass, and (iii) inert mass for this new second stage.

$$m_i = 124{,}650 \text{ kg}$$

$$r = e^{-\frac{\Delta V}{g I_{sp}}} = 0.1711 \qquad r = \frac{m_{PL} + m_{in}}{m_i} = \frac{m_{PL} + \delta m_i}{m_i}$$

$$r m_i = m_{PL} + \delta m_i$$

(i) $m_{PL} = (r - \delta) m_i = 15{,}097 \text{ kg}$

(iii) $m_{in} = \delta m_i = 6233 \text{ kg}$

(ii) $m_{PR} = (1 - \delta) m_i = 103{,}320 \text{ kg}$

(14) The Falcon 9 has (surprisingly enough) nine Merlin 1D engines in the first stage. If each engine is 99.8% reliable for launch, what is the likelihood that all nine engines will work successfully on any given launch?

$R_e = 0.998$

$R_v = R_e^9 = 0.9821$

(15) Since the Falcon 9 was designed to reach orbit with one failed engine, what is the overall probability of successfully reaching orbit based on engine reliability?

$R_v = R_e^9 + 9 R_e^8 (1 - R_e) = 0.9998$

(16) How would your answer to (15) change with an intercorrelated failure rate of 10%?

$R_v = R_e^9 + 9(1-f) R_e^8 (1 \cdot R_e) = 0.9981$

(17) If the production cost for the first space shuttle orbiter was \$2000M, what was the cost for the fifth orbiter at an 85% learning curve?

$$p = \frac{\ln(LC\%)}{\ln(2)} = -0.2345$$

$$C_5 = C_1 (5)^p = \$1371\,M$$

(18) While playing a video game, you observe that the 10th time you complete a specific level it takes you half the time it took you the second time you played. Based on these two data points, what is your learning curve for this game?

$$t_{10} = 0.5\, t_2 \qquad t_1 (10)^p = 0.5\, t_1 (2)^p$$

$$5^p = 0.5 \qquad p \ln 5 = \ln 0.5 \Rightarrow p = -0.4307$$

$$LC\% = 2^p = 0.7419 \Rightarrow 74.19\%\ LC$$

(19) You have been offered three options for payment of a debt owed to you: \$1000 today, \$1750 five years from now, or \$2500 10 years from now. Based on a discount rate of 10%, what is the net present value of each offer? Which is the best choice?

$$NPV(\$1000\ \text{today}) = \$1000$$

$$NPV(\$1750/5\,\text{yrs}) = 1750(1+r)^{-5} = \$1087 \Leftarrow \text{Best}$$

$$NPV(\$2500/10\,\text{yrs}) = 2500(1+r)^{-10} = \$964$$

(20) A previous ENAE 484 class produced *Project Magellan*: a crewed rover which was designed to circumnavigate the moon in a single lunar day. Since you could start at local dawn and return to that point to finish by local sunset, you had 42 days of continual sunlight for the trip, allowing the use of solar arrays for power. If you needed a particular component to have a 99% reliability over this mission, what mean time between failures (in hours) should you establish as a requirement for that component?

42 days = 1008 hrs

$$R = e^{-\frac{t}{MTBF}} \Rightarrow \ln R = -\frac{t}{MTBF}$$

$$MTBF = \frac{-t}{\ln R} \Rightarrow MTBF = 100{,}300 \text{ hrs}$$