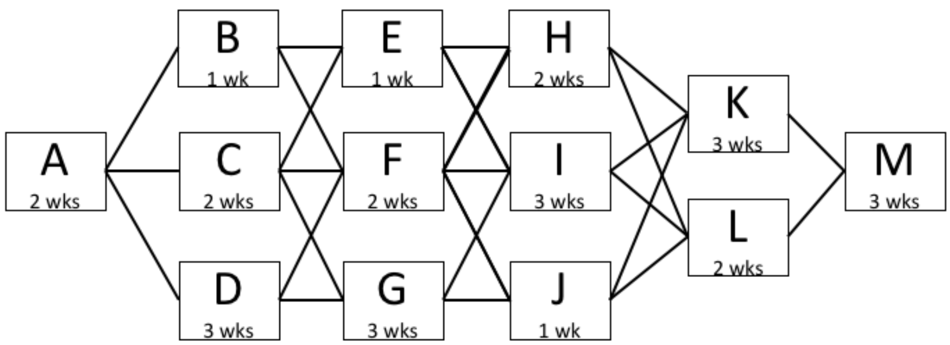
ENAE 483/788D MIDTERM – FALL, 2017 – *NAME:*

One 8.5" x 11" piece of paper allowed for notes (both sides). No Internet-enabled devices allowed. Put your name on the cover page, and on each page if you disassemble the quiz package. Please write neatly, and put boxes around your answers.

Some possibly useful numbers:

$\mu_{Earth} = 398,604\ \frac{km^3}{sec^2}$, $r_{Earth} = 6378\ km$, $\mu_{Mars} = 42,970\ \frac{km^3}{sec^2}$, $r_{Earth} = 3393\ km$

(1) A project scheduling activity results in a PERT chart as shown in the following graphic.



(a) What is the critical path?

A-D-G-I-K-M

(b) How long does it take to complete the project?

17 weeks

(c) What is the slack time for task E?

3 weeks

(2) The requirements for the Solar Electric Propulsion (SEP) stage in RASC-AL have it entering an orbit with a period of five sols around Mars. A sol is 24h39m.

(a) If the Mars orbit is circular, what is the altitude of the orbit above Mars' surface?

$$P = 2\pi\sqrt{\frac{a^3}{\mu_{Mars}}} \Longrightarrow a = \left[\left(\frac{P}{2\pi}\right)^2 \mu_{Mars}\right]^{\frac{1}{3}}$$

$$P = a = \left[\left(\frac{5(88,740)}{2\pi}\right)^2 42970\right]^{\frac{1}{3}} = 59,840\ km\ radius \Longrightarrow \boxed{\text{56,447 km altitude}}$$

(b) Assume instead you are in a circular orbit around Mars with a radius of 60,000 km and you want to transfer into the same orbit as Deimos (r=23,460 km). If you are using a low-thrust stage to do this maneuver, what is the Δv required?

$$\Delta v = |v_{c1} - v_{c2}| = \left|\sqrt{\frac{\mu_{Mars}}{r_1}} - \sqrt{\frac{\mu_{Mars}}{r_2}}\right| = \left|\sqrt{\frac{42970}{60000}} - \sqrt{\frac{42970}{23460}}\right| = \boxed{0.507\ \frac{km}{sec}}$$

(c) If you use a high-thrust stage to do a transfer from your initial orbital radius to Deimos' orbit, what Δv will this require?

$$v_{c1} = \sqrt{\frac{\mu_{Mars}}{r_1}} = \sqrt{\frac{42970}{60000}} = 0.8463 \ \frac{km}{sec}$$

$$v_a = v_{c1}\sqrt{\frac{2r_2}{r_1+r_2}} = 0.8463\sqrt{\frac{2\times 23460}{23460+60000}} = 0.6345 \ \frac{km}{sec}$$

$$\Delta v_1 = v_{c1} - v_a = 0.8463 - 0.6345 = 0.2118 \ \frac{km}{sec}$$

$$v_{c2} = \sqrt{\frac{\mu_{Mars}}{r_2}} = \sqrt{\frac{42970}{23460}} = 1.353 \ \frac{km}{sec}$$

$$v_p = v_{c2}\sqrt{\frac{2r_1}{r_1+r_2}} = 1.353\sqrt{\frac{2\times 60000}{23460+60000}} = 1.622 \ \frac{km}{sec}$$

$$\Delta v_2 = v_p - v_{c2} = 1.622 - 1.353 = 0.2694 \ \frac{km}{sec}$$

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 0.2118 + 0.2694 = \boxed{0.4811 \ \frac{km}{sec}}$$

(d) How would your answer to (c) change if you also need to perform a 15deg plane change? Say whether you will do the plane change maneuver at apoapsis or periapsis and why. *Perform the plane change at apoapsis to minimize* Δv

$$\Delta v_1 = \sqrt{v_a^2 + v_{c1}^2 - 2v_a v_{c1}\cos\Delta i} = \sqrt{0.6345^2 + 0.8463^2 - 2(0.6345)0.8463\cos 15^o} = 0.2854 \ \frac{km}{sec}$$

Δv_2 *is unchanged, so*

$$\Delta v_{total} = \Delta v_1 + \Delta v_2 = 0.2854 + 0.2694 = \boxed{0.5548 \ \frac{km}{sec}}$$

(3) You are evaluating different approaches to reaching low Earth orbit (Δv=9200 m/sec). You are using a propellant combination with an exhaust velocity v_e=4600 m/sec, with an inert mass fraction δ=0.1.

(a) What payload mass is possible using a single-stage vehicle to reach orbit, for a vehicle with an inert mass m_{in}=10 MT?

$$r = e^{-\Delta v/v_e} = e^{-9200/4600} = 0.1353 \Longrightarrow \lambda = r - \delta = 0.1353 - 0.1 = 0.0353$$

$$m_{in} = \delta m_o \Longrightarrow m_o = \frac{m_{in}}{\delta} = \frac{10}{0.1} = 100 \ MT$$

$$m_{pl} = \lambda m_o = 0.0353(100) = \boxed{3.53 \text{ MT}}$$

(b) What is the maximum payload you can achieve if you design a two-stage vehicle with the same v_e and δ as in (a), and a second-stage inert mass $m_{in,2}$=10 MT?

We know that the maximum payload given $v_{e1} = v_{e2}$ and $\delta_1 = \delta_2$ occurs when $\Delta v_1 = \Delta v_2 = \Delta v_{total}/2$=4600 m/sec

$$r_1 = r_2 = e^{-\Delta v_{1,2}/v_e} = e^{-4600/4600} = 0.3679 \Longrightarrow \lambda_2 = r_2 - \delta_2 = 0.3679 - 0.1 = 0.2679$$

$$m_{in2} = \delta_2 m_{o2} \Longrightarrow m_{o2} = \frac{m_{in2}}{\delta_2} = \frac{10}{0.1} = 100\ MT$$

$$m_{pl} = \lambda_2 m_{o2} = 0.2679(100) = \boxed{26.79 \text{ MT}}$$

You could go on and figure out the masses for the first stage, but I just asked for payload mass, and we've calculated that already.

(c) You could use two vehicles identical to the one you designed in (a) as a "siamese" launch vehicle: the first stage (same inert and propellant masses as in (a)) would carry the second stage, which in turn would carry the payload. What payload could this vehicle carry to orbit?

Okay, this one is a little tricky. We know from (a) that the upper stage has mass values of m_{o2} =100 MT, m_{in2}=10 MT, and we don't know m_{pl} (or, for that matter, m_{pr2}). For the first stage, m_{in1}=10 MT and m_{pr1} =86.47 MT (the original total mass was 100 MT, minus the 10 MT for inert mass and 3.53 MT of the original payload.) That makes m_o =196.47 MT, but we still don't know m_{pr2} or m_{pl}.

$$r_1 = \frac{m_{in1} + m_{o2}}{m_o} = \frac{10 + 100}{196.47} = 0.5599 \Longrightarrow \Delta v_1 = -v_e \ln r_1 = -4600 \ln 0.5599 = 2668\ m/sec$$

Does this make sense? The Δv for the first stage doesn't depend on the payload? Yes, because we've fixed the total mass of the second stage plus payload, and the first stage performance is only set by its own payload mass, m_{o2}. We now know the second stage has to provide the rest of the Δv, or 6532 m/sec.

$$r_2 = e^{-\Delta v_2/v_e} = e^{-6532/4600} = 0.2417 = \frac{m_{in2} + m_{pl}}{m_{o2}} = \frac{10 + m_{pl}}{100} \Longrightarrow m_{pl} = \boxed{14.17 \text{ MT}}$$

(4) The Falcon Heavy first stage has three Falcon 9 first-stage modules, with 9 Merlin-D engines in each for 27 engines overall.

(a) If you need the Falcon Heavy to be 99% reliable (defined here as all engines working), what is the required individual engine reliability?

$$P = R^n \Longrightarrow R = P^{1/n} = .99^{1/27} = \boxed{0.9996}$$

(b) If the Falcon Heavy launch will be successful with two engines failed and the reliability of each engine is 99%, what is the overall reliability of the first stage operation?

$$P = R^{27} + 27R^{26}(1-R) + \frac{27(26)}{2}R^{25}(1-R)^2 = 0.99^{27} + 27(0.99)^{26}(0.01) + \frac{27(26)}{2}0.99^{25}(0.01)^2 = \boxed{0.9976}$$

(c) How would your answer to (b) change with a 5% intercorrelated failure rate?

$$P = R^{27} + 27R^{26}(1-R)(1-f) + \frac{27(26)}{2}R^{25}(1-R)^2(1-f)^2$$

$$= 0.99^{27} + 27(0.99)^{26}(0.01)(0.95) + \frac{27(26)}{2}0.99^{25}(0.01)^2(0.95)^2 = \boxed{0.9845}$$

(d) The Falcon Heavy first stage can operate successfully following three engine failures, as long as only one failure occurs on each of the three modules. Again, if the individual engine reliability is 0.99, what is the overall success probability of the three-module first stage?

Likelihood of a single module operating successfully (0 or 1 engine failures)

$$P_{mod} = R^9 + 9R^8(1-R) = 0.99^9 + 9(0.99)^8(0.01) = 0.9966$$

Likelihood of all three modules having a successful outcome

$$P_{stage} = P_{mod}^3 = 0.9966^3 = \boxed{0.9897}$$

(5) The fifth Falcon 9 cost \$50M to produce, and the 20th Falcon 9 cost \$32M to produce.

(a) Calculate the learning curve percentage

$$LC = \frac{C_{2n}}{C_n} = \frac{C_{20}}{C_{10}} = \frac{C_{10}}{C_5}$$

$$\frac{C_{20}}{C_5} = \frac{C_{20}}{C_{10}}\frac{C_{10}}{C_5} = (LC)^2 \Longrightarrow \frac{32}{50} = 0.64 = LC^2 \Longrightarrow \boxed{\text{LC=0.80=80\%}}$$

(b) Calculate the first unit production cost

$$p = \frac{\ln LC}{\ln 2} = \frac{\ln 0.80}{\ln 2} = 0.3219$$

$$C_5 = C_1 5^p \Longrightarrow C_1 = C_5 5^{-p} = 50(5)^{+0.3219} = \boxed{\$83.94\text{M}}$$

(6) A Borg ship has entered low Earth orbit at an altitude of 300 km. The ship is a cube 1 km on a side, and is flying such that one face of the cube is perpendicular to the velocity vector. The density of Earth's atmosphere at this altitude can be approximated by $\rho = 3.875 \times 10^{-9} e^{-h<km>/59.06}$ (in kg/m^3)

(a) What is the drag force on the Borg ship?

$$\rho = 3.875 \times 10^{-9} e^{-h<km>/59.06} = 3.875 \times 10^{-9} e^{-300/59.06} = 2.411 \times 10^{-11}\ kg/m^3$$

$$\textit{In circular orbit, } v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398604}{6378+300}} = 7726\ m/sec$$

As a flat plate perpendicular to the velocity vector, the drag coefficient c_D=*4. The area of the ship perpendicular to the velocity vector is (1 km)*^{2}*, or* $1 \times 10^6\ m^2$.

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 2.411 \times 10^{-11}(7726)^2(1 \times 10^6)4 = \boxed{2878\text{ N}}$$

(b) At this altitude, the flux for orbital debris impacts by particles of a centimeter in diameter is 10^{-5} hits/m^2/year. How long, on average, could the Borg cube stay in this orbit before it could expect to have an impact of this size?

$$(Flux)(Area) = 10^{-5}(6 \times 10^6) = 60\ hits/year \Longrightarrow 0.01667\ years/hit = \boxed{6.088 \text{ days}}$$

(7) A friend wants to borrow \$1000 from you today, with a promise of a lump sum payment of \$2000 in ten years. What annual interest rate does this correspond to?

$$C_1 = (1+r)^{-n} C_n \Longrightarrow r = \left(\frac{C_1}{C_n}\right)^{-1/n} - 1 = \left(\frac{1000}{2000}\right)^{-0.1} - 1 = \boxed{7.177\%}$$

(8) Extra credit

(a) You were told to watch two short video clips at the beginning of the term. In the clip entitled "The Deep Dive", what was the team designing?
A shopping cart

(b) In the excerpt from "Spider", what was the team designing?
A lunar module

(c) List the names (first and last) of all of the other members of your team from Team Project 1