**ENAE 483/788D LSM SPECIALTY PROBLEMS – FALL, 2023**

This is an extra-credit homework – if you turn it in prior to the solution set being posted on Dec. 7, it can add to your final grade in the class. Whether you do this homework or not, you *will be responsible for understanding and using this material on the final exam.*

A propellant tank for a reaction control system contains $N_2O_4$ ($\rho = 1450$ kg/m$^3$) and is pressurized to 2 MPa. The tank is spherical, 1 meter in diameter and 1 mm in thickness. It is made of Ti-6Al-4V, with a yield stress of 924 MPa and an ultimate stress of 1000 MPa and a density of 4430 kg/m$^3$. For this problem, the required factors of safety for a metallic pressurized propellant tank are 1.1 for yield and 1.4 for ultimate.

(1) If the only load source on the tank is due to the pressure, find the stress on the tank material.

$$For\ a\ sphere,\ \pi r^2 P = 2\pi r t\sigma \Rightarrow \sigma = \frac{Pr}{2t}$$
$$\sigma = \frac{Pd}{4t} = \frac{2\times 10^6(1)}{4(.001)} = \boxed{500\text{ MPa}}$$

(2) Find the margins of safety for yield and ultimate failure cases.

$$MoS = \frac{allowable\ stress}{FoS \times limit\ applied\ stress} - 1$$
$$MoS_{yield} = \frac{924}{1.1\times 500} - 1 = \boxed{0.6800}$$
$$MoS_{ult} = \frac{1000}{1.4\times 500} - 1 = \boxed{0.4286}$$

(3) In addition to pressure, the tank must also contain the inertial load of the propellants under acceleration. During launch, the tank will be fully loaded with propellant and can expect to experience 4.5 (Earth) gravities of acceleration. Find the total maximum pressure load on the propellant tank in this case.

$$P_{inertial} = (density)(depth)(acceleration) = 1450\ kg/m^3(1\ m)(4.5\times 9.8\ m/sec^2) = 63,950Pa$$
$$P_{tot} = P + P_{inertial} = 500,000,000 + 63,950 = \boxed{500.1\text{ MPa}}\ (to\ appropriate\ precision)$$

(4) Repeat (2) for this case.

$$MoS_{yield} = \frac{924}{1.1\times 500.1} - 1 = \boxed{0.6797}$$
$$MoS_{ult} = \frac{1000}{1.4\times 500.1} - 1 = \boxed{0.4283}$$