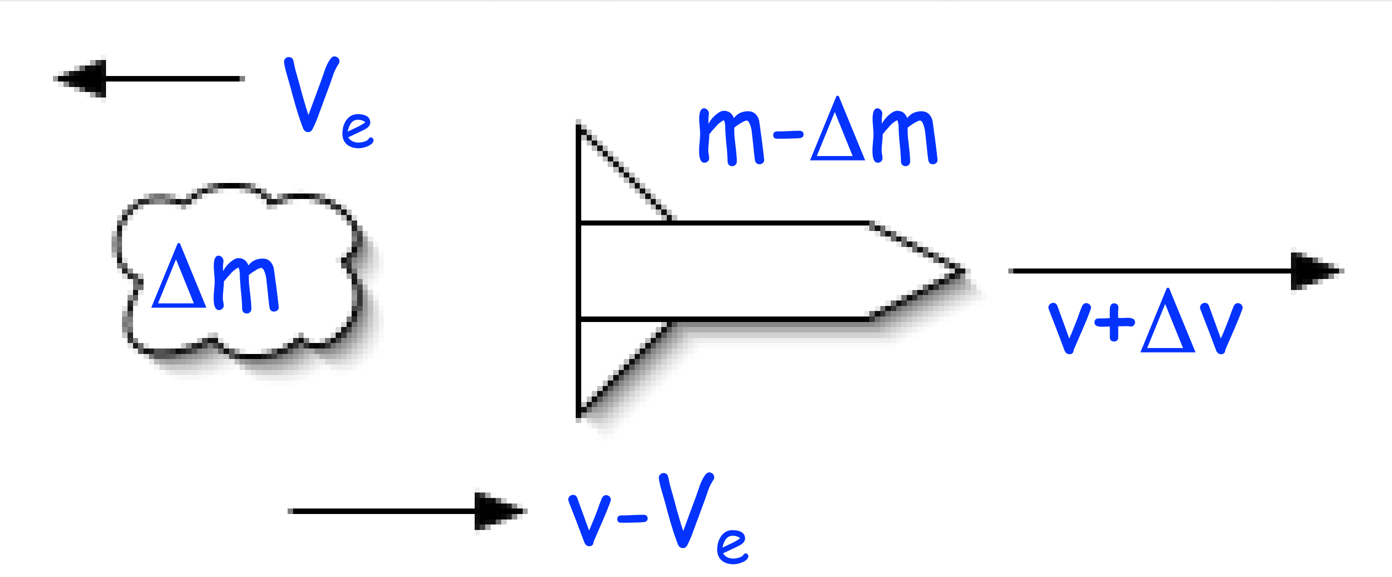
# Rocket Performance

- Lecture #02 – August 31, 2023
- The rocket equation
- Mass ratio and performance
- Structural and payload mass fractions
- Regression analysis
- Multistaging
- Optimal ΔV distribution between stages
- Trade-off ratios
- Parallel staging
- Modular staging

# Derivation of the Rocket Equation




- Momentum at time t:

$$M = mv$$

- Momentum at time t+Δt:

$$M = (m - \Delta m)(V + \Delta v) + \Delta m \left( v - V_e \right)$$

- Some algebraic manipulation gives:

$$m\Delta v = -\Delta m V_e$$

- Take to limits and integrate:

$$\int_{m_{initial}}^{m_{final}} \frac{dm}{m} = -\int_{V_{initial}}^{V_{final}} \frac{dv}{V_e}$$

# The Rocket Equation

- Alternate forms

$$r \equiv \frac{m_{final}}{m_{initial}} = e^{-\frac{\Delta V}{V_e}}$$

$$\Delta v = -V_e \ln\left(\frac{m_{final}}{m_{initial}}\right) = -V_e \ln r$$

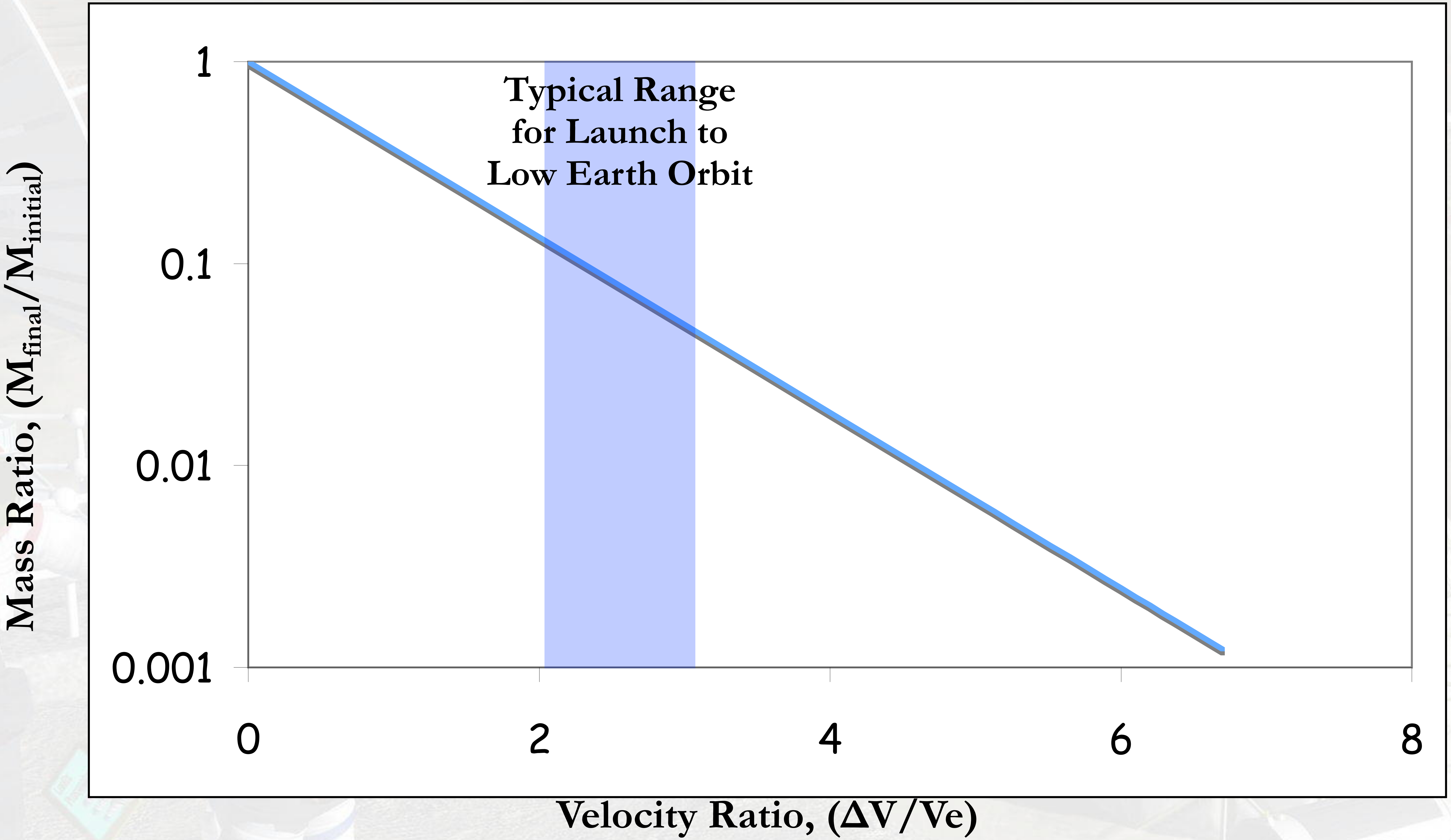
- Basic definitions/concepts
  - Mass ratio

$$r \equiv \frac{m_{final}}{m_{initial}} \text{ or } \mathfrak{R} \equiv \frac{m_{initial}}{m_{final}}$$

  - Nondimensional velocity change
    "Velocity ratio"

$$\nu \equiv \frac{\Delta V}{V_e}$$

# Rocket Equation (First Look)



# Sources and Categories of Vehicle Mass

Payload
Propellants
Structure
Propulsion
Avionics
Power
Mechanisms
Thermal
Etc.

# Sources and Categories of Vehicle Mass

- Payload
- Propellants
- Inert Mass
  - Structure
  - Propulsion
  - Avionics
  - Power
  - Mechanisms
  - Thermal
  - Etc.

# Basic Vehicle Parameters

- Basic mass summary

$$m_o = m_{pl} + m_{pr} + m_{in}$$

- Inert mass fraction

$$\delta \equiv \frac{m_{in}}{m_o} = \frac{m_{in}}{m_{pl} + m_{pr} + m_{in}}$$

- Payload fraction

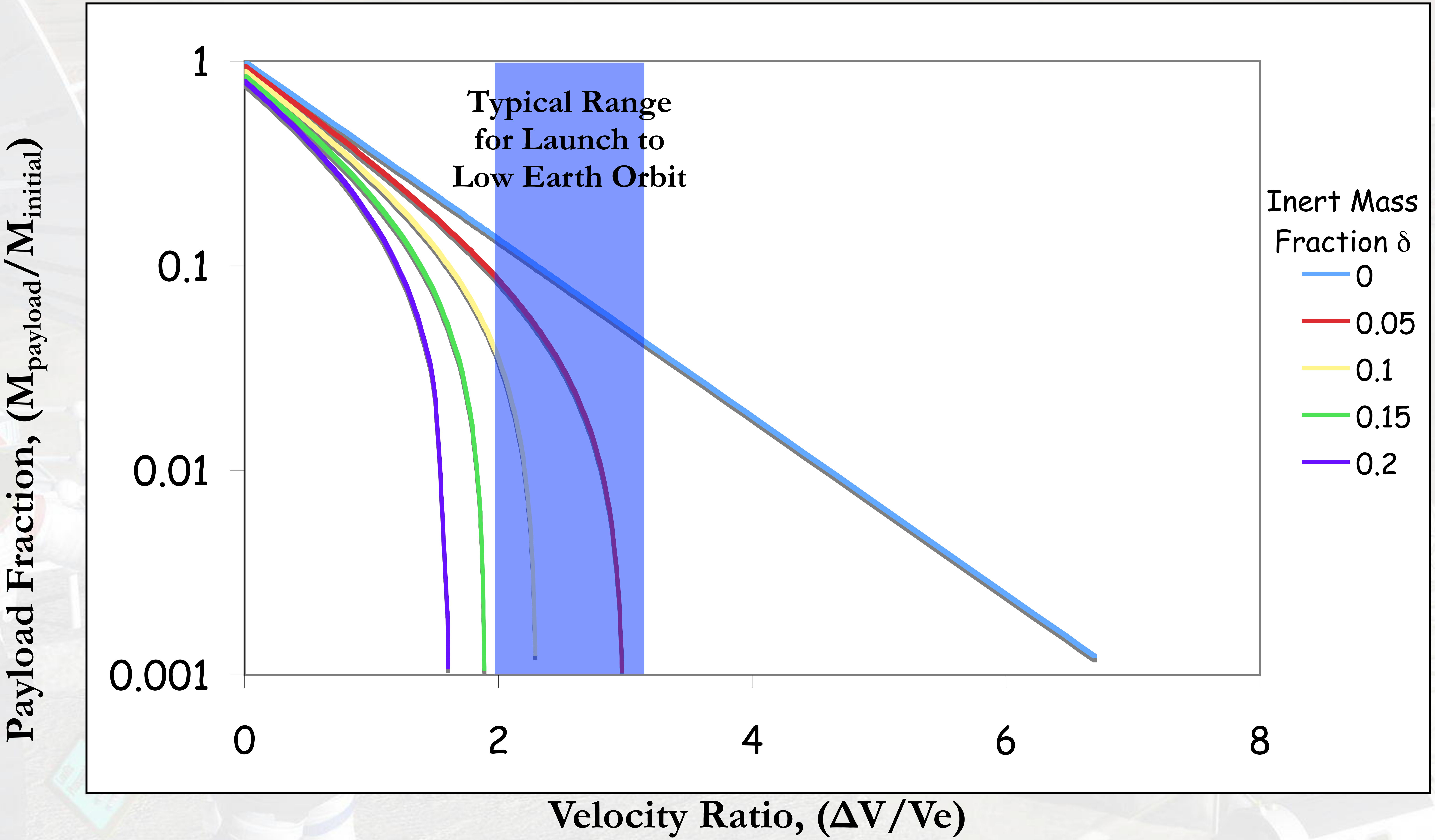
$$\lambda \equiv \frac{m_{pl}}{m_o} = \frac{m_{pl}}{m_{pl} + m_{pr} + m_{in}}$$

- Parametric mass ratio

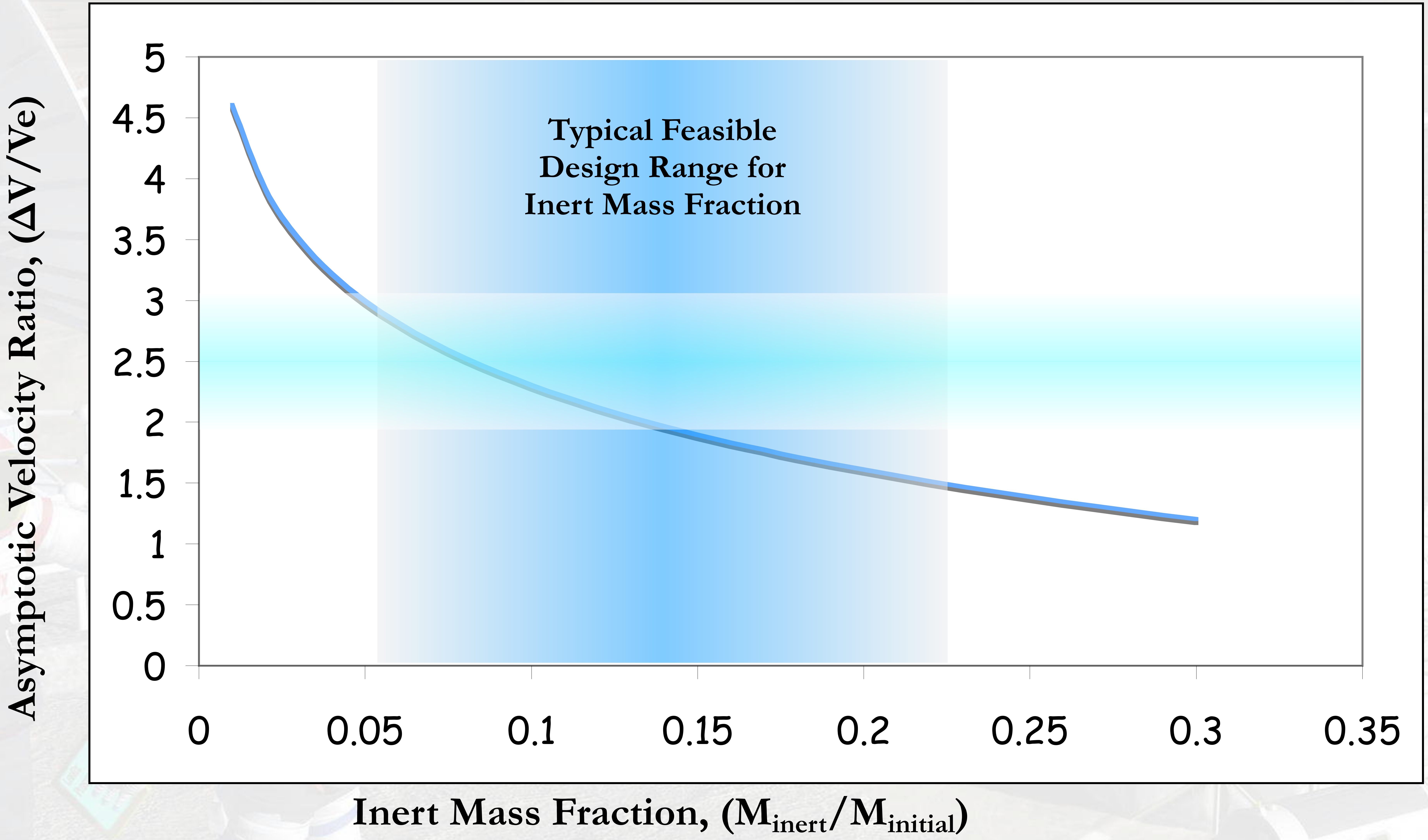
$$r = \lambda + \delta$$

$m_o \equiv$ initial mass
$m_{pl} \equiv$ payload mass
$m_{pr} \equiv$ propellant mass
$m_{in} \equiv$ inert mass

# Rocket Equation (including Inert Mass)



# Limiting Performance Based on Inert Mass



# Regression Analysis of Existing Vehicles

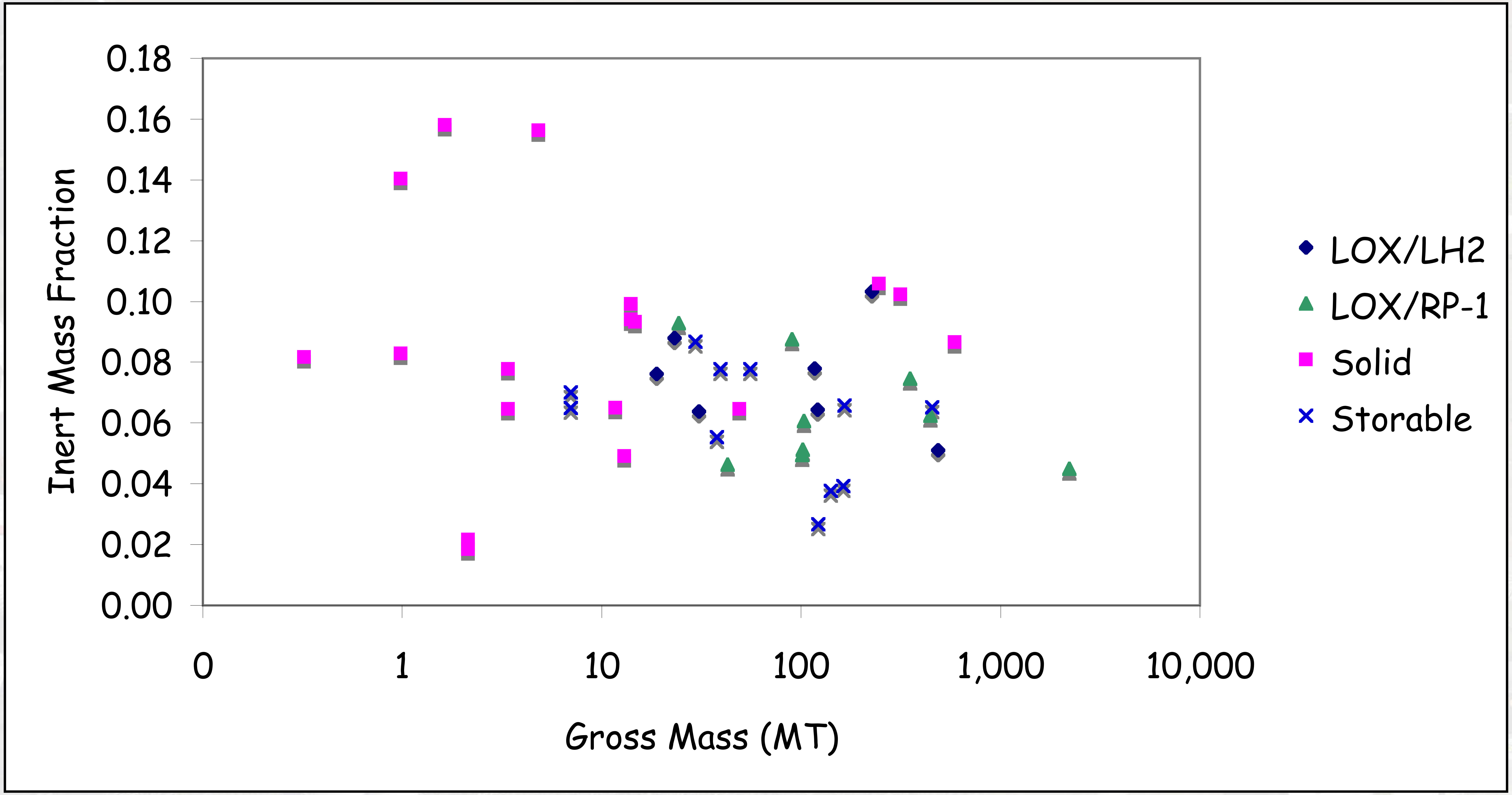
| Veh/Stage | prop mass (lbs) | gross mass (lbs) | Type | Propellants | Isp vac (sec) | isp sl (sec) | sigma | eps | delta |
|---|---|---|---|---|---|---|---|---|---|
| Delta 6925 Stage 2 | 13,367 | 15,394 | Storab | N2O4-A50 | 319.4 | | 0.152 | 0.132 | 0.070 |
| Delta 7925 Stage 2 | 13,367 | 15,394 | Storab | N2O4-A50 | 319.4 | | 0.152 | 0.132 | 0.065 |
| Titan II Stage 2 | 59,000 | 65,000 | Storab | N2O4-A50 | 316.0 | | 0.102 | 0.092 | 0.087 |
| Titan III Stage 2 | 77,200 | 83,600 | Storab | N2O4-A50 | 316.0 | | 0.083 | 0.077 | 0.055 |
| Titan IV Stage 2 | 77,200 | 87,000 | Storab | N2O4-A50 | 316.0 | | 0.127 | 0.113 | 0.078 |
| Proton Stage 3 | 110,000 | 123,000 | Storab | N2O4-A50 | 315.0 | | 0.118 | 0.106 | 0.078 |
| Titan II Stage 1 | 260,000 | 269,000 | Storab | N2O4-A50 | 296.0 | | 0.035 | 0.033 | 0.027 |
| Titan III Stage 1 | 294,000 | 310,000 | Storab | N2O4-A50 | 302.0 | | 0.054 | 0.052 | 0.038 |
| Titan IV Stage 1 | 340,000 | 359,000 | Storab | N2O4-A50 | 302.0 | | 0.056 | 0.053 | 0.039 |
| Proton Stage 2 | 330,000 | 365,000 | Storab | N2O4-A50 | 316.0 | | 0.106 | 0.096 | 0.066 |
| Proton Stage 1 | 904,000 | 1,004,000 | Storab | N2O4-A50 | 316.0 | 285.0 | 0.111 | 0.100 | 0.065 |
| average | | | | | 312.2 | 285.0 | 0.100 | 0.089 | 0.061 |
| standard deviation | | | | | 8.1 | | 0.039 | 0.033 | 0.019 |

# A Word About Specific Impulse

- Defined as "thrust/propellant used"
  - English units: lbs thrust/(lbs prop/sec)=sec
  - Metric units: N thrust/(kg prop/sec)=m/sec
- Two ways to regard discrepancy -
  - "lbs" is not mass in English units - should be slugs
  - Isp = "thrust/weight flow rate of propellant" - if $I_{sp}$ is in seconds, then $v_e = g_o I_{sp}$ where $g_o$ is for unit conversion (i.e., 9.8 m/sec everywhere!)
- If the real intent of specific impulse is

$$I_{sp} = \frac{T}{\dot{m}} \text{ and } T = \dot{m}V_e \text{ then } I_{sp} = V_e!!!$$

# Inert Mass Fractions for Existing LVs



# Regression Analysis

- Given a set of N data points ($x_i$,$y_i$)
- Linear curve fit: $y = Ax + B$
  - find $A$ and $B$ to minimize sum squared error

$$\text{error} = \sum_{i=1}^{N} (Ax_i + B - y_i)^2$$

  - Analytical solutions exist, or use Solver in Excel
- Power law fit: $y = Bx^A$

$$\text{error} = \sum_{i=1}^{N} \left[A \log(x_i) + B - \log(y_i)\right]^2$$

- Polynomial, exponential, many other fits possible

# Solution of Least-Squares Linear Regression

$$\frac{\partial(\text{error})}{\partial A} = 2\sum_{i=1}^{N}(Ax_i + B - y_i)x_i = 0$$

$$\frac{\partial(\text{error})}{\partial B} = 2\sum_{i=1}^{N}(Ax_i + B - y_i) = 0$$

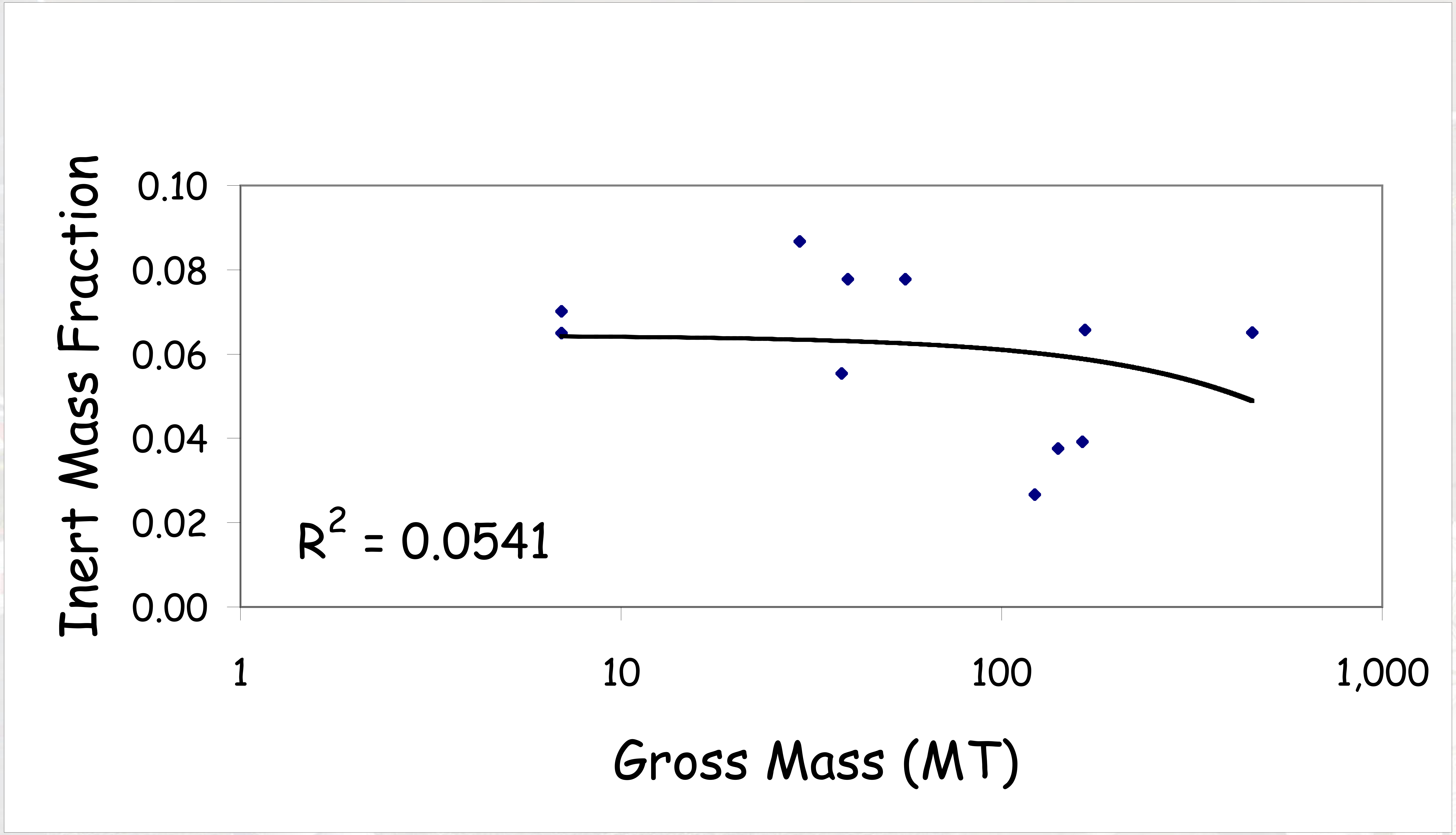
$$A\sum_{i=1}^{N} x_i^2 + B\sum_{i=1}^{N} x_i - \sum_{i=1}^{N} x_i y_i = 0$$

$$A\sum_{i=1}^{N} x_i + NB - \sum_{i=1}^{N} y_i = 0$$

$$A = \frac{N\sum x_i y_i - \sum x_i \sum y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2}$$

$$B = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N\sum x_i^2 - \left(\sum x_i\right)^2}$$

# Regression Analysis - Storables



# Regression Values for Design Parameters

| | Vacuum Ve (m/sec) | Inert Mass Fraction $\delta$ | Max $\Delta V$ (m/sec) |
|---|---|---|---|
| LOX/LH2 | 4273 | 0.075 | 11,070 |
| LOX/RP-1 | 3136 | 0.063 | 8664 |
| Storables | 3058 | 0.061 | 8575 |
| Solids | 2773 | 0.087 | 6783 |

# Revised Analysis With ε Instead of δ

$$\epsilon = \text{stage inert mass fraction}$$

$$r = \lambda + \delta \Longrightarrow \lambda = r - \delta$$

$$\epsilon \equiv \frac{m_{in}}{m_{in} + m_{pr}}$$

$$r = \frac{m_{pl} + m_{in}}{m_{pl} + m_{pr} + m_{in}}$$

$$r = \frac{\frac{m_{pl}}{m_{pr}+m_{in}} + \frac{m_{in}}{m_{pr}+m_{in}}}{\frac{m_{pl}}{m_{pr}+m_{in}} + \frac{m_{pr}+m_{in}}{m_{pr}+m_{in}}}$$

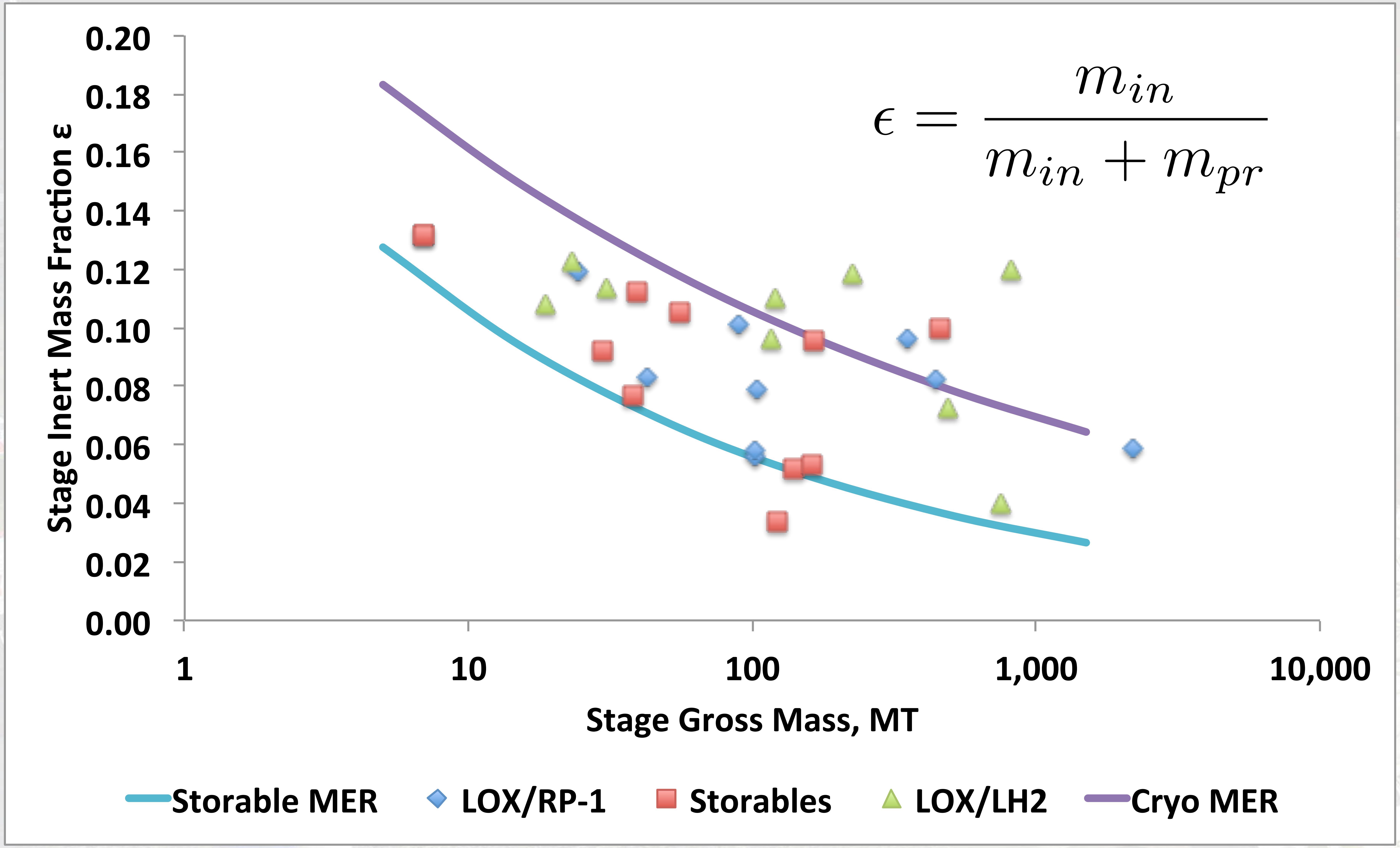
$$r = \frac{\rho + \epsilon}{\rho + 1} \text{ where } \rho \equiv \frac{m_{pl}}{m_{in} + m_{pr}}$$

$$\epsilon = 1 - \frac{m_{pr}}{m_{in} + m_{pr}} = 1 - PMF$$

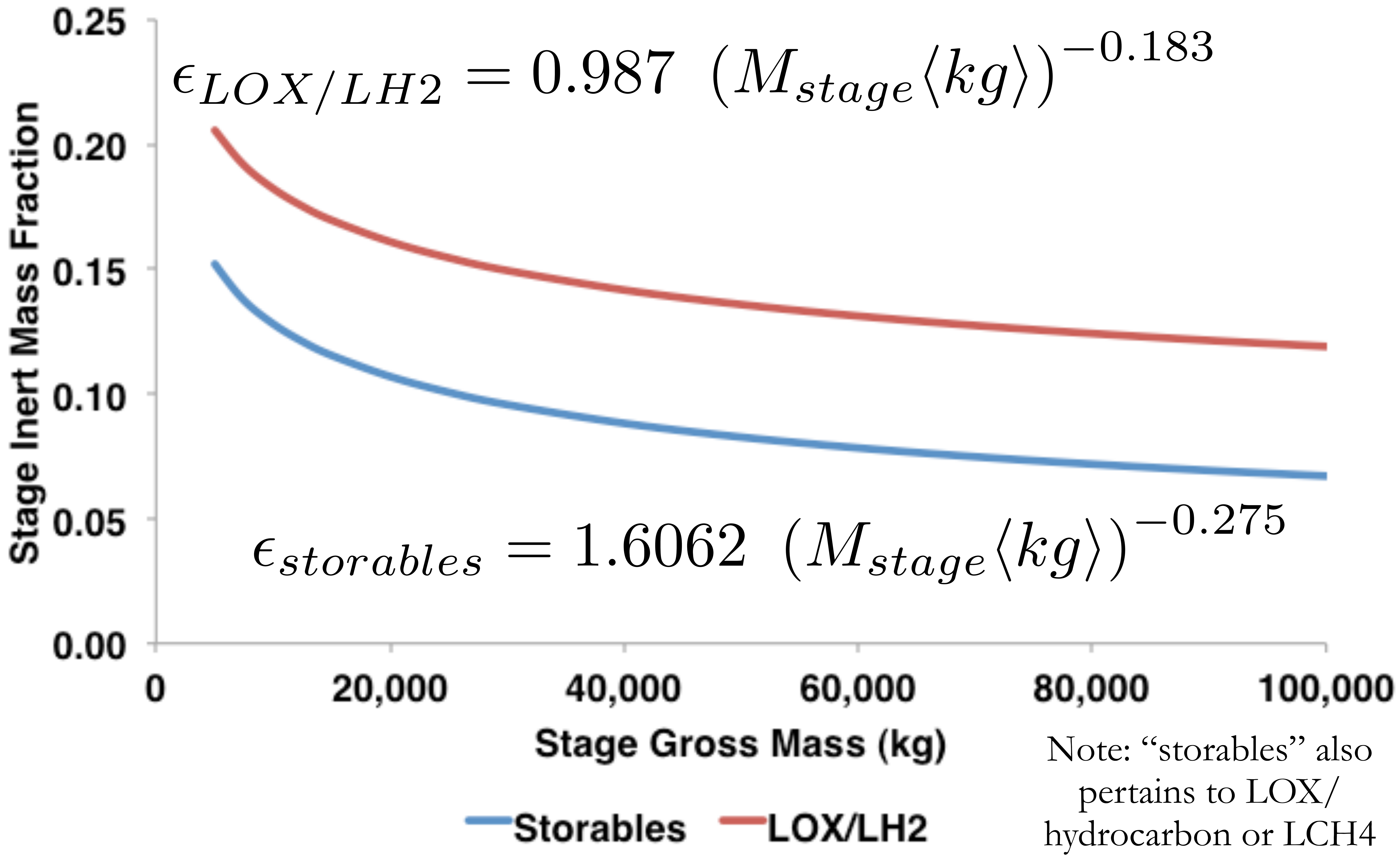
$PMF$ = Propellant Mass Fraction

$$\rho = \frac{r - \epsilon}{1 - r}$$

# Economy of Scale for Stage Size



# Stage Inert Mass Fraction Estimation




$$\epsilon_{LOX/LH2} = 0.987\ \left(M_{stage}\langle kg\rangle\right)^{-0.183}$$

$$\epsilon_{storables} = 1.6062\ \left(M_{stage}\langle kg\rangle\right)^{-0.275}$$

Note: “storables” also pertains to LOX/ hydrocarbon or LCH4

# Calculating $M_o$ from $M_{pl}$

Given $\Delta v$ and $I_{sp}$

$$r = e^{-\frac{\Delta v}{g I_{sp}}}$$

Given $\delta$

$$\lambda = r - \delta \implies m_o = \frac{m_{pl}}{\lambda}$$

Given $\epsilon$

$$m_{stage} = m_{in} + m_{pr}; \quad m_{in} = \epsilon m_{stage}; \quad m_o = m_{pl} + m_{stage}$$

$$r = \frac{m_{pl} + m_{in}}{m_{pl} + m_{in} + m_{pr}} = \frac{m_{pl} + \epsilon m_{stage}}{m_{pl} + m_{stage}}$$

$$m_{stage} = \left(\frac{1 - r}{r - \epsilon}\right) m_{pl} \implies m_o = \left(\frac{1 - \epsilon}{r - \epsilon}\right) m_{pl}$$

# The Rocket Equation for Multiple Stages

- Assume two stages

$$\Delta V_1 = -V_{e1} \ln\left(\frac{m_{final1}}{m_{initial1}}\right) = -V_{e1} \ln(r_1)$$

$$\Delta V_2 = -V_{e2} \ln\left(\frac{m_{final2}}{m_{initial2}}\right) = -V_{e2} \ln(r_2)$$

- Assume $V_{e1}=V_{e2}=V_e$

$$\Delta V_1 + \Delta V_2 = -V_e \ln(r_1) - V_e \ln(r_2) = -V_e \ln(r_1 r_2)$$

# Continued Look at Multistaging

- There's a historical tendency to define $r_0 = r_1 r_2$

$$\Delta V_1 + \Delta V_2 = -V_e \ln\left(r_1 r_2\right) = -V_e \ln\left(r_0\right)$$

- But it's important to remember that it's really

$$\Delta V_1 + \Delta V_2 = -V_e \ln\left(r_1 r_2\right) = -V_e \ln\left(\frac{m_{final1}}{m_{initial1}} \frac{m_{final2}}{m_{initial2}}\right)$$

- And that $r_0$ has **no** physical significance, since

$$m_{final1} \neq m_{initial2} \Rightarrow r_0 \neq \frac{m_{final2}}{m_{initial1}}$$

# Multistage Inert Mass Fraction

- Total inert mass fraction

$$\delta_0 = \frac{m_{in,1} + m_{in,2} + m_{in,3}}{m_0} = \frac{m_{in,1}}{m_0} + \frac{m_{in,2}}{m_0} + \frac{m_{in,3}}{m_0}$$

$$\delta_0 = \frac{m_{in,1}}{m_0} + \frac{m_{in,2}}{m_{0,2}} \frac{m_{0,2}}{m_0} + \frac{m_{in,3}}{m_{0,3}} \frac{m_{0,3}}{m_{0,2}} \frac{m_{0,2}}{m_0}$$

- Convert to dimensionless parameters

$$\delta_0 = \delta_1 + \delta_2 \lambda_1 + \delta_3 \lambda_2 \lambda_1$$

- General form of the equation

$$\delta_0 = \sum_{j=1}^{n \text{ stages}} \left( \delta_j \prod_{\ell=1}^{j-1} \lambda_\ell \right)$$

# Multistage Payload Fraction

- Total payload fraction (3 stage example)

$$\lambda_0 = \frac{m_{pl}}{m_0} = \frac{m_{pl}}{m_{0,3}} \frac{m_{0,3}}{m_{0,2}} \frac{m_{0,2}}{m_0}$$

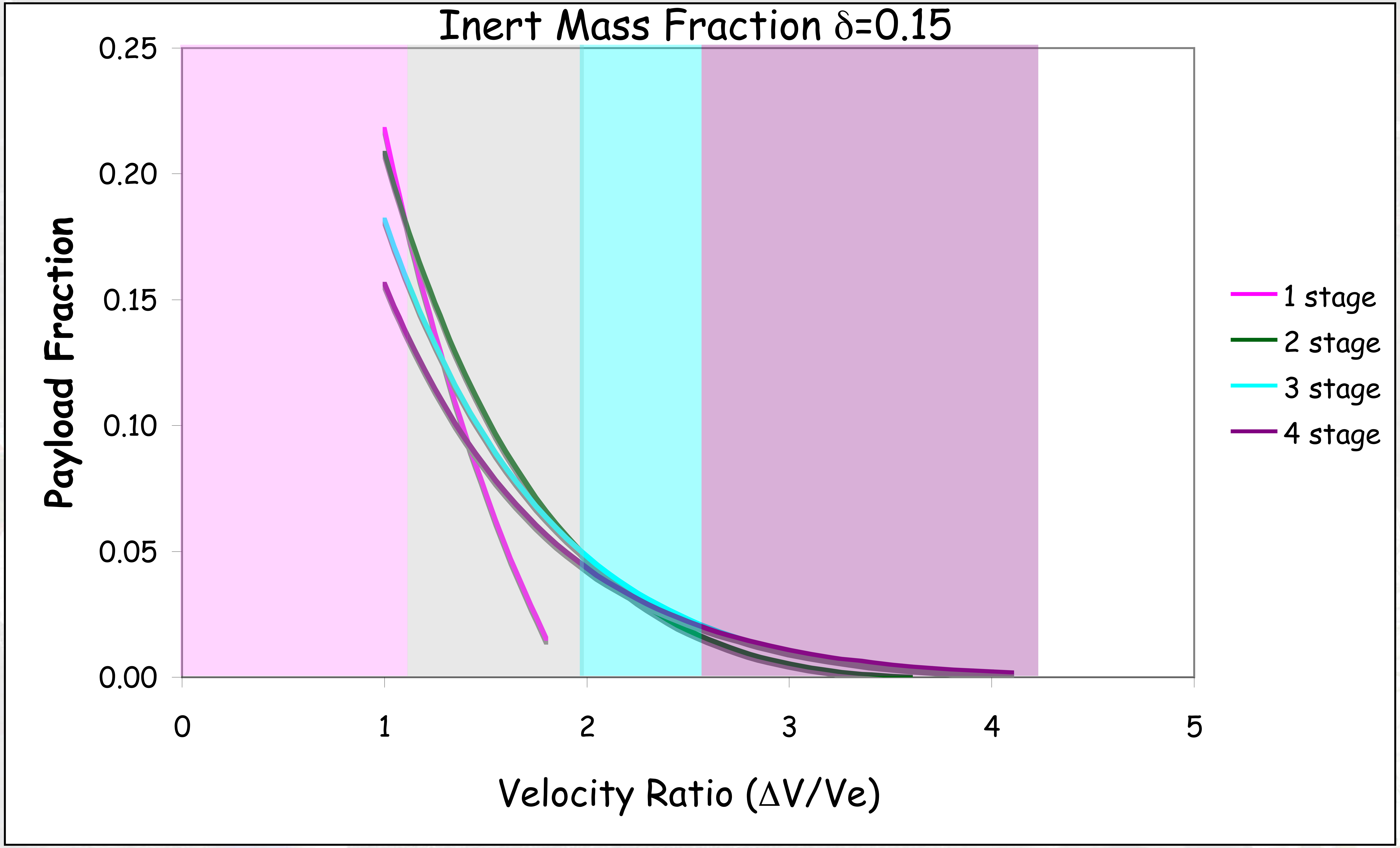
- Convert to dimensionless parameters

$$\lambda_0 = \lambda_3 \lambda_2 \lambda_1$$

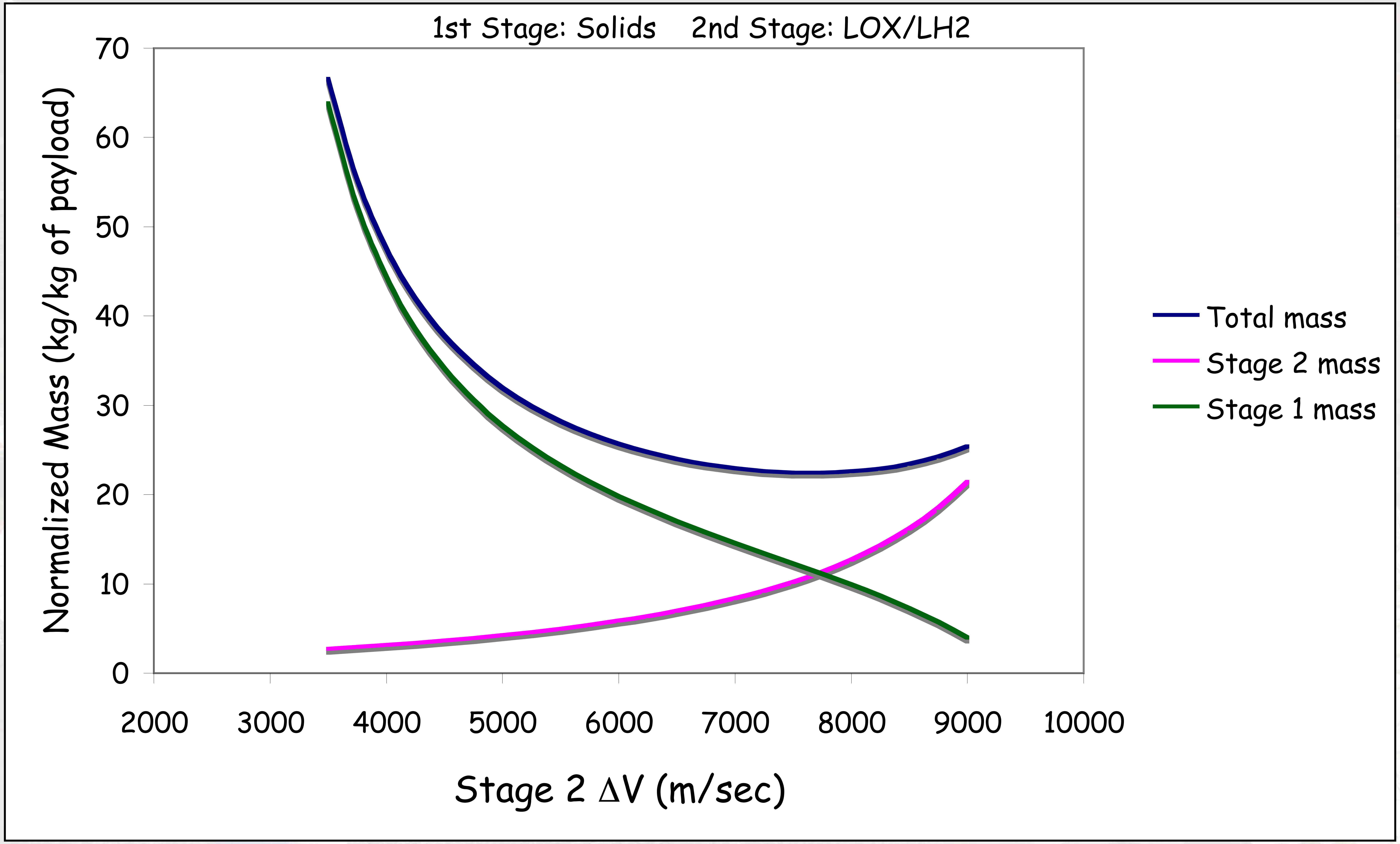
- Generic form of the equation

$$\lambda_0 = \prod_{j=1}^{n \text{ stages}} \lambda_j$$

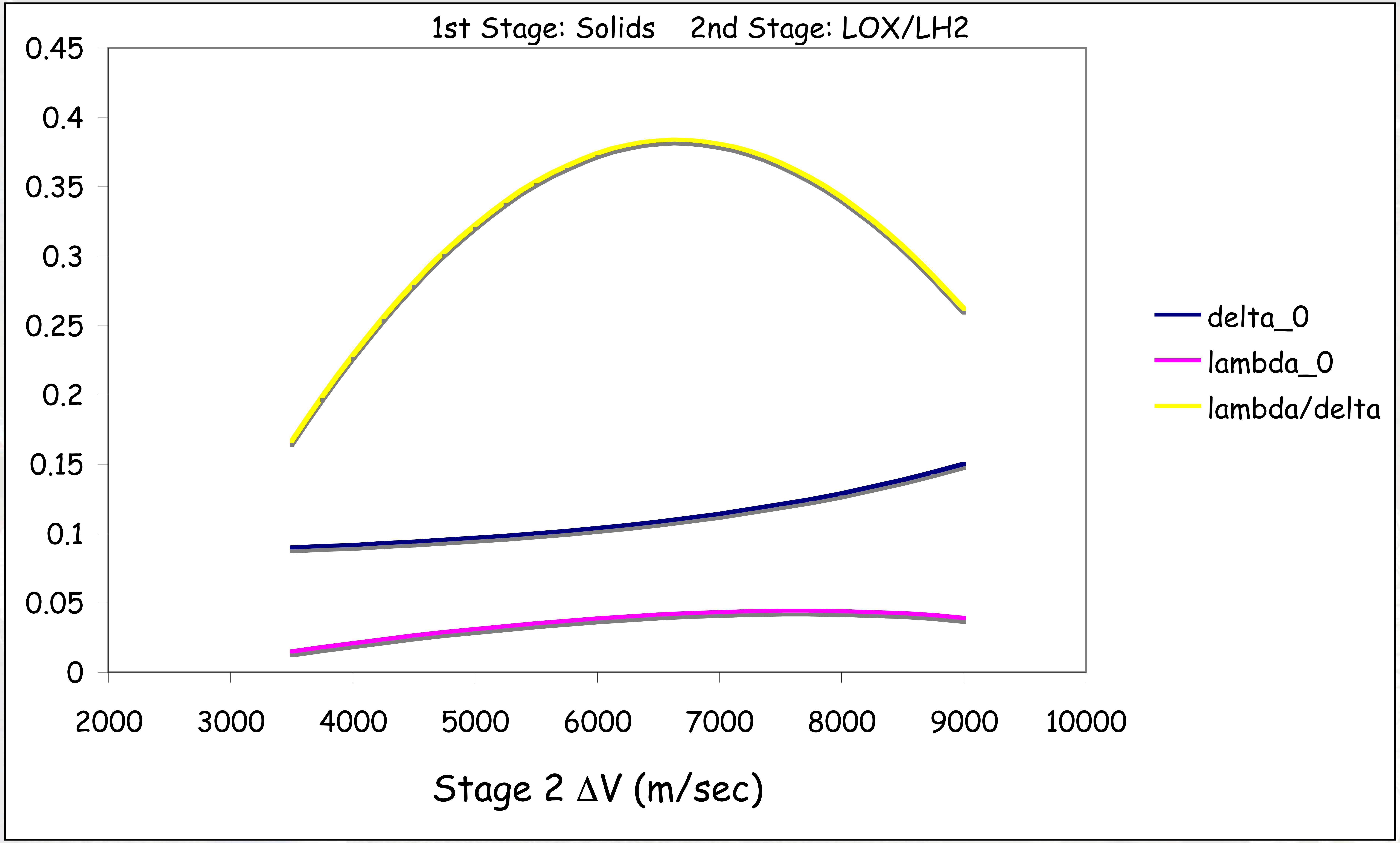
# Effect of Staging



# Effect of ΔV Distribution



# ΔV Distribution and Design Parameters



# Lagrange Multipliers

- Given an objective function

$$y = f(x)$$

subject to constraint function

$$z = g(x)$$

- Create a new objective function

$$y = f(x) + \lambda[g(x) - z]$$

- Solve simultaneous equations

$$\frac{\partial y}{\partial x} = 0 \qquad \frac{\partial y}{\partial \lambda} = 0$$

# Optimum ΔV Distribution Between Stages

- Maximize payload fraction (2 stage case)

$$\lambda_0 = \lambda_1 \lambda_2 = (r_1 - \delta_1)(r_2 - \delta_2)$$

subject to constraint function

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

- Create a new objective function

$$\lambda_o = \left( e^{\frac{-\Delta V_1}{V_{e,1}}} - \delta_1 \right) \left( e^{\frac{-\Delta V_2}{V_{e,2}}} - \delta_2 \right) + K \left[ \Delta V_1 + \Delta V_2 - \Delta V_{total} \right]$$

➥Very messy for partial derivatives!

# Optimum ΔV Distribution (continued)

- Use substitute objective function

$$max\left(\lambda_o\right) \Longleftrightarrow max\left[ln\left(\lambda_o\right)\right]$$

- Create a new constrained objective function

$$ln\left(\lambda_o\right) = ln\left(r_1 - \delta_1\right) + ln\left(r_2 - \delta_2\right) + K\left[\Delta V_1 + \Delta V_2 - \Delta V_{total}\right]$$

- Take partials and set equal to zero

$$\frac{\partial\left[ln\left(\lambda_o\right)\right]}{\partial r_1} = 0 \quad \frac{\partial\left[ln\left(\lambda_o\right)\right]}{\partial r_2} = 0 \quad \frac{\partial\left[ln\left(\lambda_o\right)\right]}{\partial K} = 0$$

# Optimum ΔV Special Cases

- "Generic" partial of objective function

$$\frac{\partial\left[ln\left(\lambda_o\right)\right]}{\partial r_i} = \frac{1}{r_i - \delta_i} + K\frac{V_{e,i}}{r_i} = 0$$

- Case 1: $\delta_1 = \delta_2$ $V_{e,1} = V_{e,2}$

$$r_1 = r_2 \Longrightarrow \Delta V_1 = \Delta V_2 = \frac{\Delta V_{total}}{2}$$

- Same principle holds for n stages

$$r_1 = r_2 = \cdots = r_n \Longrightarrow$$

$$\Delta V_1 = \Delta V_2 = \cdots = \Delta V_n = \frac{\Delta V_{total}}{n}$$

- For any other case, you'll have to solve it numerically…

# Sensitivity to Inert Mass

ΔV for multistaged rocket

$$\Delta V_{tot} = \sum_{k=1}^{n\ stages} \Delta V_k = \sum_{k=1}^{n} V_{e,k} \ln\left(\frac{m_{o,k}}{m_{f,k}}\right)$$

where

$$m_{o,k} = m_{pl} + m_{pr,k} + m_{in,k} + \sum_{j=k+1}^{n} \left(m_{pr,j} + m_{in,j}\right)$$

$$m_{f,k} = m_{pl} + m_{in,k} + \sum_{j=k+1}^{n} \left(m_{pr,j} + m_{in,j}\right)$$

# Finding Payload Sensitivity to Inert Mass

- Given the equation linking mass to ΔV, take

$$\frac{\partial(\Delta V_{tot})}{\partial m_{pl}} dm_{pl} + \frac{\partial(\Delta V_{tot})}{\partial m_{in,j}} dm_{in,j} = 0$$

and solve to find

$$\left.\frac{\partial m_{pl}}{\partial m_{in,k}}\right|_{\partial(\Delta V_{tot})=0} = \frac{-\sum_{j=1}^{k} V_{e,j}\left(\frac{1}{m_{o,j}} - \frac{1}{m_{f,j}}\right)}{\sum_{\ell=1}^{N} V_{e,\ell}\left(\frac{1}{m_{o,\ell}} - \frac{1}{m_{f,\ell}}\right)}$$

- This equation shows the "trade-off ratio" - Δpayload resulting from a change in inert mass for stage k (for a vehicle with N

# Trade-off Ratio Example: Gemini-Titan II

| | Stage 1 | Stage 2 |
|---|---|---|
| $m_o$ $(kg)$ | 150,500 | 32,630 |
| $m_f$ $(kg)$ | 39,370 | 6099 |
| $v_e$ $(m/sec)$ | 2900 | 3097 |
| $\frac{\partial m_{pl}}{\partial m_{in,k}}$ | -0.1164 | -1 |

# Payload Sensitivity to Propellant Mass

- In a similar manner, solve to find

$$\left.\frac{\partial m_{pl}}{\partial m_{pr,k}}\right|_{\partial(\Delta V_{tot})=0} = \frac{-\sum_{j=1}^{k} V_{e,j}\left(\frac{1}{m_{o,j}}\right)}{\sum_{\ell=1}^{N} V_{e,\ell}\left(\frac{1}{m_{o,\ell}} - \frac{1}{m_{f,\ell}}\right)}$$

- This equation gives the change in payload mass as a function of additional propellant mass (all other parameters held constant)

# Trade-off Ratio Example: Gemini-Titan II

| | Stage 1 | Stage 2 |
|---|---|---|
| $m_o\ (kg)$ | 150,500 | 32,630 |
| $m_f\ (kg)$ | 39,370 | 6099 |
| $v_e\ (m/sec)$ | 2900 | 3097 |
| $\frac{\partial m_{pl}}{\partial m_{in,k}}$ | -0.1164 | -1 |
| $\frac{\partial m_{pl}}{\partial m_{pr,k}}$ | 0.04124 | 0.2443 |

# Parallel Staging

- Multiple dissimilar engines burning simultaneously
- Frequently a result of upgrades to operational systems
- General case requires “brute force” numerical performance analysis

# Parallel-Staging Rocket Equation

- Momentum at time t:

$$M = mv$$

- Momentum at time t+Δt:
(subscript "b"=boosters; "c"=core vehicle)

$$M = (m - \Delta m_b - \Delta m_c)(v + \Delta v) + \Delta m_b (v - V_{e,b}) + \Delta m_c (v - V_{e,c})$$

- Assume thrust (and mass flow rates) constant

# Parallel-Staging Rocket Equation

- Rocket equation during booster burn

$$\Delta V = -\bar{V}_e \ln\left(\frac{m_{final}}{m_{initial}}\right) = -\bar{V}_e \ln\left(\frac{m_{in,b}+m_{in,c}+\chi m_{pr,c}+m_{0,2}}{m_{in,b}+m_{pr,b}+m_{in,c}+m_{pr,c}+m_{0,2}}\right)$$

where $\chi$= fraction of core propellant remaining after booster burnout, and where

$$\bar{V}_e = \frac{V_{e,b}\dot{m}_b+V_{e,c}\dot{m}_c}{\dot{m}_b+\dot{m}_c} = \frac{V_{e,b}m_{pr,b}+V_{e,c}(1-\chi)m_{pr,c}}{m_{pr,b}+(1-\chi)m_{pr,c}}$$

# Analyzing Parallel-Staging Performance

Parallel stages break down into pseudo-serial stages:

- Stage "0" (boosters and core)

$$\Delta V_0 = -\bar{V}_e \ln\left(\frac{m_{in,b}+m_{in,c}+\chi m_{pr,c}+m_{0,2}}{m_{in,b}+m_{pr,b}+m_{in,c}+m_{pr,c}+m_{0,2}}\right)$$

- Stage "1" (core alone)

$$\Delta V_1 = -V_{e,c} \ln\left(\frac{m_{in,c}+m_{0,2}}{m_{in,c}+\chi m_{pr,c}+m_{0,2}}\right)$$

- Subsequent stages are as before

# Parallel Staging Example: Space Shuttle

- 2 x solid rocket boosters (data below for single SRB)
  - Gross mass 589,670 kg
  - Empty mass 86,183 kg
  - Isp 269 sec
  - Burn time 124 sec
- External tank (space shuttle main engines)
  - Gross mass 750,975 kg
  - Empty mass 29,930 kg
  - Isp 455 sec
  - Burn time 480 sec

# Shuttle Parallel Staging Example

$$V_{e,b} = gI_{sp,e} = (9.8)(269) = 2636\frac{m}{sec} \qquad V_{e,c} = 4459\frac{m}{sec}$$

$$\chi = \frac{480 - 124}{480} = 0.7417$$

$$\bar{V}_e = \frac{2636(1,007,000) + 4459(721,000)(1 - .7417)}{1,007,000 + 721,000(1 - .7417)} = 2921\frac{m}{sec}$$

$$\Delta V_0 = -2921 \ln \frac{862,000}{3,062,000} = 3702\frac{m}{sec}$$

$$\Delta V_1 = -4459 \ln \frac{154,900}{689,700} = 6659\frac{m}{sec}$$

$$\Delta V_{tot} = 10,360\frac{m}{sec}$$

# Modular Staging

- Use identical modules to form multiple stages
- Have to cluster modules on lower stages to make up for nonideal ΔV distributions
- Advantageous from production and development cost standpoints

# Module Analysis

- All modules have the same inert mass and propellant mass
- Because δ varies with payload mass, not all modules have the same payload mass or the same $\delta \implies$ use $\epsilon$ instead

# Rocket Equation for Modular Boosters

- Assuming n modules in stage 1,

$$r_1 = \frac{n(m_{in}) + m_{o2}}{n(m_{in} + m_{pr}) + m_{o2}} = \frac{n\varepsilon + \frac{m_{o2}}{m_{mod}}}{n + \frac{m_{o2}}{m_{mod}}}$$

- If all 3 stages use same modules, $n_j$ for stage j,

$$r_1 = \frac{n_1\varepsilon + n_2 + n_3 + \rho_{pl}}{n_1 + n_2 + n_3 + \rho_{pl}}$$

where

$$\rho_{pl} \equiv \frac{m_{pl}}{m_{mod}}; \; m_{mod} = m_{in} + m_{pr}$$

# Example: Conestoga 1620 (EER)

- Small launch vehicle (1 flight, 1 failure)
- Payload 900 kg
- Module gross mass 11,400 kg
- Module empty mass 1,400 kg
- Exhaust velocity 2754 m/sec
- Staging pattern
  – 1st stage - 4 modules
  – 2nd stage - 2 modules
  – 3rd stage - 1 module
  – 4th stage - Star 48V (gross mass 2200 kg, empty mass 140 kg, $V_e$ 2842 m/sec)

# Conestoga 1620 Performance

- 4th stage ΔV

$$\Delta V_4 = -V_{e4} \ln \frac{m_{f4}}{m_{o4}} = -2842 \ln \frac{900+140}{900+2200} = 3104 \ \frac{\text{m}}{\text{sec}}$$

- Treat like three-stage modular vehicle; $M_{pl}$=3100 kg

$$\epsilon = \frac{m_{in}}{m_{mod}} = \frac{1400}{11400} = 0.1228$$

$$\rho_{pl} = \frac{m_{pl}}{m_{mod}} = \frac{3100}{11400} = 0.2719$$

$$n_1 = 4; \ n_2 = 2; \ n_3 = 1$$

# Constellation 1620 Performance (cont.)

$$r_1 = \frac{n_1\epsilon + n_2 + n_3 + \rho_{pl}}{n_1 + n_2 + n_3 + \rho_{pl}} = \frac{4 \times 0.1228 + 2 + 1 + 0.2719}{4 + 2 + 1 + 0.2719} = 0.5175$$

$$r_2 = \frac{n_2\epsilon + n_3 + \rho_{pl}}{n_2 + n_3 + \rho_{pl}} = \frac{2 \times 0.1228 + 1 + 0.2719}{2 + 1 + 0.2719} = 0.4638$$

$$r_3 = \frac{n_3\epsilon + \rho_{pl}}{n_3 + \rho_{pl}} = \frac{1 \times 0.1228 + 0.2719}{1 + 0.2719} = 0.3103$$

$$V_1 = 1814\ \frac{\text{m}}{\text{sec}};\ V_2 = 2116\ \frac{\text{m}}{\text{sec}}$$

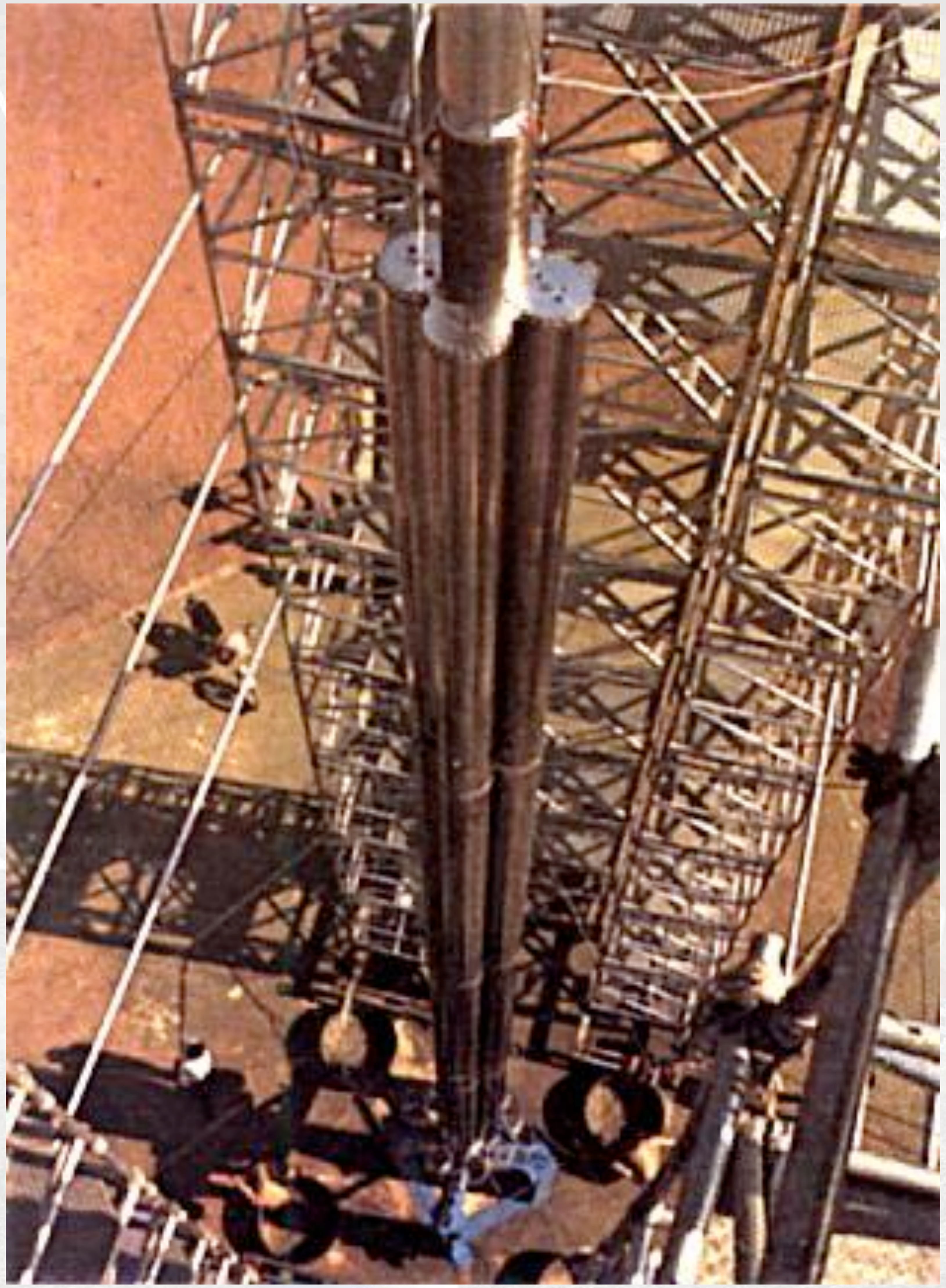
$$V_3 = 3223\ \frac{\text{m}}{\text{sec}};\ V_4 = 3104\ \frac{\text{m}}{\text{sec}}$$

$$V_{total} = 10,257\ \frac{\text{m}}{\text{sec}}$$

# Discussion about Modular Vehicles

- Modularity has several advantages
  - Saves money (smaller modules cost less to develop)
  - Saves money (larger production run = lower cost/module)
  - Allows resizing launch vehicles to match payloads
- Trick is to optimize number of stages, number of modules/stage to minimize total number of modules
- Generally close to optimum by doubling number of modules at each lower stage
- Have to worry about packing factors, complexity

# OTRAG - 1977-1983



# Today's Tools

- Mass ratios
- Estimation of vehicle masses from $\Delta v$ and $v_e$ and inert mass fractions (both $\delta$ and $\epsilon$)
- Regression analysis
- Staging calculations
- Optimization of $\Delta v$ distribution between stages
- Trade-off ratios
- Parallel staging calculations
- Modular vehicle calculations