ENAE 483/788D LECTURE #07
(SPACE ENVIRONMENT) PROBLEMS – FALL, 2023

As preparation for your team projects this term and the large class project in ENAE 484 next term, consider a cylindrical habitat with a diameter of 8 meters, a cylindrical length of 10 m, and *only* for the purposes of keeping this problem set simpler, flat endcaps for problems (1) and (2).

(1) At an orbital altitude of 250 km, what is the drag force on the habitat with the cylindrical axis perpendicular to the velocity vector? *(Note: the graph in the lecture slides I wanted you to use is for 500 km orbital altitude, not 250. I hope you ignored this and used that graph anyway.)*

The planform area perpendicular to the flow would be length × diameter, or 80 m^2. From the notes (Lecture #06 pg. 19) the c_D for a cylinder in this orientation is 8/3. The orbital velocity is

$$v = \sqrt{\frac{\mu}{r}} = \sqrt{\frac{398604}{6378 + 250}} = 7.755\ km/sec$$

From page 12 of the lecture notes,

$$\rho = 3.875 \times 10^{-9} e^{-\frac{h}{59.06}} = 3.875 \times 10^{-9} e^{-\frac{250}{59.06}} = 5.622 \times 10^{-11}\ kg/m^3$$

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 5.622 \times 10^{-11} (7755)^2 80 (2.667) = \boxed{0.3606\text{ N}}$$

Note that the velocity had to be converted to m/sec to make the units work out correctly.

(2) In the same conditions, what is the drag force in the case of the cylindrical axis aligned with the velocity vector?

From page 17, c_D for a flat plate perpendicular to the flow is 4.

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} 8^2 = 50.27\ m^2$$

$$D = \frac{1}{2}\rho v^2 A c_D = \frac{1}{2} 5.622 \times 10^{-11} (7755)^2 50.27 (4) = \boxed{0.3399\text{ N}}$$

(3) The habitat could remain in Earth orbit for thirty years. Over that lifetime, what is the largest MMOD particle would you have to design for (on average)? For the purposes of this and the remaining questions, assume the habitat has a diameter of 8 m, a cylindrical length of 5 meters, and hemispherical endcaps.

$$A = 4\pi \left(\frac{d}{2}\right)^2 + \pi \ell d = \pi d \left(d + \ell\right) = 8\pi \left(8 + 5\right) = 326.7\ m^2$$

$$Flux = \frac{1\ hit/m^2 - yr}{(326.7\ m^2)(30\ yrs)} = 1.020 \times 10^{-4}\ hits/m^2 - yr$$

From the chart on page 33 of the lecture notes, find the flux on the vertical axis and read off the corresponding particle size on the horizontal axis. This gives the approximate size particle you would have to design to would be orbital debris of 4×10^{-1} cm or $\boxed{4\ mm}$. You would also expect to be hit by micrometeoroid particles of approximate 2.5 mm, but the design case for the shielding would be the 4 mm orbital debris particle.

(4) Over that same time, how many hits would you expect to have from particles with a diameter of 0.2 mm?

From the same chart, a particle size of 0.2 mm corresponds to a micrometeoroid flux of 1 hit/m^2-yr. It also would correspond to an orbital debris flux of 0.15 hit/m^2-yr, for a total flux of 1.15 hit/m^2-yr. For the spacecraft area and lifetime in low Earth orbit, this would be 1.15(326.7)(30)= $\boxed{11,270 \textit{ hits}}$ *by all particles of this size over the lifetime of the vehicle.*