ENAE 483/788D LECTURE #08
(COST ESTIMATING RELATIONS) PROBLEMS – FALL, 2023

We are going to use the NASA SVLCM costing model (in the lecture notes) to estimate the costs of the SpaceX Falcon 9 launch vehicle. Use the following parameters in the analysis: *Note: in this problem set, as throughout the course, "MT" refers to metric tons, or 1000 kg.*

Parameter	First Stage	Second Stage
Specific impulse (sec)	283	348
Propellant mass (MT)	418.7	111.5
Inert mass (MT)	27.6	4.5
Payload to LEO (MT)	–	22.8

(1) Calculate the nonrecurring costs for each of the two stages.
From page 13 of the slides, $c_{NR} = 12.73 m_{in}^{0.55}$ *for launch vehicles, where the units of* m_{in} *are kilograms. (Note that the values in the table are given in metric tons, or thousands of kilograms.) This gives values of* $c_{NR}(1st stage) = \boxed{\$3526M}$, *and* $c_{NR}(2nd stage) = \boxed{\$1300M}$, *for a total nonrecurring cost of \$4827M. It's worth noting that this would be the estimate for NASA developing this vehicle; SpaceX has said that the Falcon 9 development cost about \$500M.*

(2) Assume the development of the vehicle took six years. Find the year-by-year costs using a beta function with $c_f = 0.4$ and $P_k = 0.6$.

$$A = \frac{(1 - P_k)(c_f - 0.1875)}{0.625} = \frac{(1 - 0.6)(0.4 - 0.1875)}{0.625} = 0.1360$$
$$B = P_k \frac{c_f - 0.1875}{0.3125} = 0.6 \frac{0.4 - 0.1875}{0.3125} = 0.4080$$
$$C(\tau) = 10\tau^2(1 - \tau)^2(A + B\tau) + \tau^4(5 - 4\tau)$$

Year	τ	$C(\tau)$	Cumulative Cost, \$M	Yearly Cost, \$M
1	0.1667	0.0427	206.1	206.1
2	0.3333	0.1796	866.8	660.8
3	0.5	0.4000	1931	1064
4	0.6667	0.6624	3197	1266
5	0.8333	0.8956	4323	1126
6	1	1	4827	540.0

(3) Calculate the first unit production costs for each of the two stages.
From page 13 of the notes, $c_{R1} = 0.3024 m_{in}^{0.662}$ *for launch vehicles, where the units of* m_{in} *are kilograms. This gives values of* $c_{R1}(1st stage) = \boxed{\$263.3M}$, *and* $c_{R1}(2nd stage) = \boxed{\$79.25M}$, *for a total first unit production cost of \$342.6M for the first vehicle set.*

(4) For an 82.5% learning curve, calculate the production costs (unit by unit) for the first 5 vehicles.

$$p = \frac{\ln LC}{\ln 2} = \frac{\ln 0.825}{\ln 2} = -0.2775$$

Production #	LC fraction	Cost ($M)
1	1	342.6
2	0.8250	282.6
3	0.7372	252.5
4	0.6806	233.2
5	0.6398	219.2

(5) Assume for this problem that the Falcon 9 is expendable. For simple break-even, how much would you have to charge per flight to amortize both the nonrecurring and recurring costs over the first five vehicles?

The total recurring cost over the 5 vehicles is $1330M, so adding in the nonrecurring cost of $4827M the total cost is $6157M. Over 5 flights, that works out to **$1231M/flt** *or* **$54,010/kg of payload**. *(SpaceX charges $62M/flt, so you can see there's a huge cost advantage to flying more than five times!)*

(6) As we know, the Falcon 9 first stage is reusable. Assume the refurbishment fraction is f_R=0.05. What is the simple break-even cost over 25 missions assuming each first stage flies five times?

$$c_{refurb} = f_{refurb} c_{R,1} = 0.05(263.3) = \$13.17\ M/flt$$

(Note that we only refurbish the first stage, and that the refurbishment cost does not follow a learning curve.) The costs for the first six years are as per (2). With the 82.5% learning curve as per (4), we can calculate the production costs for the 5 first and 25 second stages as

Given these costs for the first and second stage production, 20 refurbishments of the first stages (if it's really a 25-mission program, you don't have to refurbish the stages after their fifth flights) which comes to $263.3M, and the nonrecurring costs from (1), the total program cost with reusability comes to $4827M, which equates to **$965.4M/flt** *or* **$42.34K/kg** *of payload.*

TABLE 1. default

Unit number	c_R Stage 1 ($M)	c_R Stage 2 ($M)
1	263.3	79.25
2	217.2	65.38
3	194.1	58.42
4	179.2	53.94
5	168.5	50.70
6		48.20
7		46.18
8		44.50
9		43.07
10		41.83
11		40.74
12		39.76
13		38.89
14		38.10
15		37.38
16		36.71
17		36.10
18		35.53
19		35.00
20		34.51
21		34.04
22		33.61
23		33.20
24		32.81
25		32.44
Totals	1330	1022

(7) Assume as per (2) the development of Falcon 9 took six years, that the production costs for the first stages are all paid in year 7, and that the cost for each second stage is paid in the year in which it flies. Refurbishment costs for the first stage are also paid in the year of flight. There are 5 flights/year in each of years 7-11 in which you pay production costs for the expendable second stages (with learning curve as per (5)) and refurbishment as necessary for the first stages (no learning effects on refurbishment). What is the total cost in each year of the program from 1-11?
Shown in table below

(8) At a 10% discount rate, what is the net present value in year 0 for each of the costs you calculated in (7)?
Shown in table below

Year	c_{NR} ($M)	c_R ($M)	c_{refurb} ($M)	c_{total} ($M)	NBV ($M)
1	206.1			206.1	187.3
2	660.8			660.8	546.1
3	1063.9			1063.9	799.3
4	1266.5			1266.5	865.0
5	1125.6			1125.6	698.9
6	504.0			504.0	284.5
7		1330.0		1330.0	682.5
8		223.8	65.8	289.6	135.1
9		194.9	65.8	260.7	110.6
10		177.9	65.8	243.7	94.0
11		166.1	65.8	231.9	81.3