Terramechanics: Wheel-Soil Interaction

- Lecture #27 – December 5, 2023
- Origin and nature of lunar soil
- Soil mechanics
- Wheel-soil interactions
  - Soil compression
  - Bulldozing
  - Gravity effects
- Slip
- Grousers
- Drawbar pull

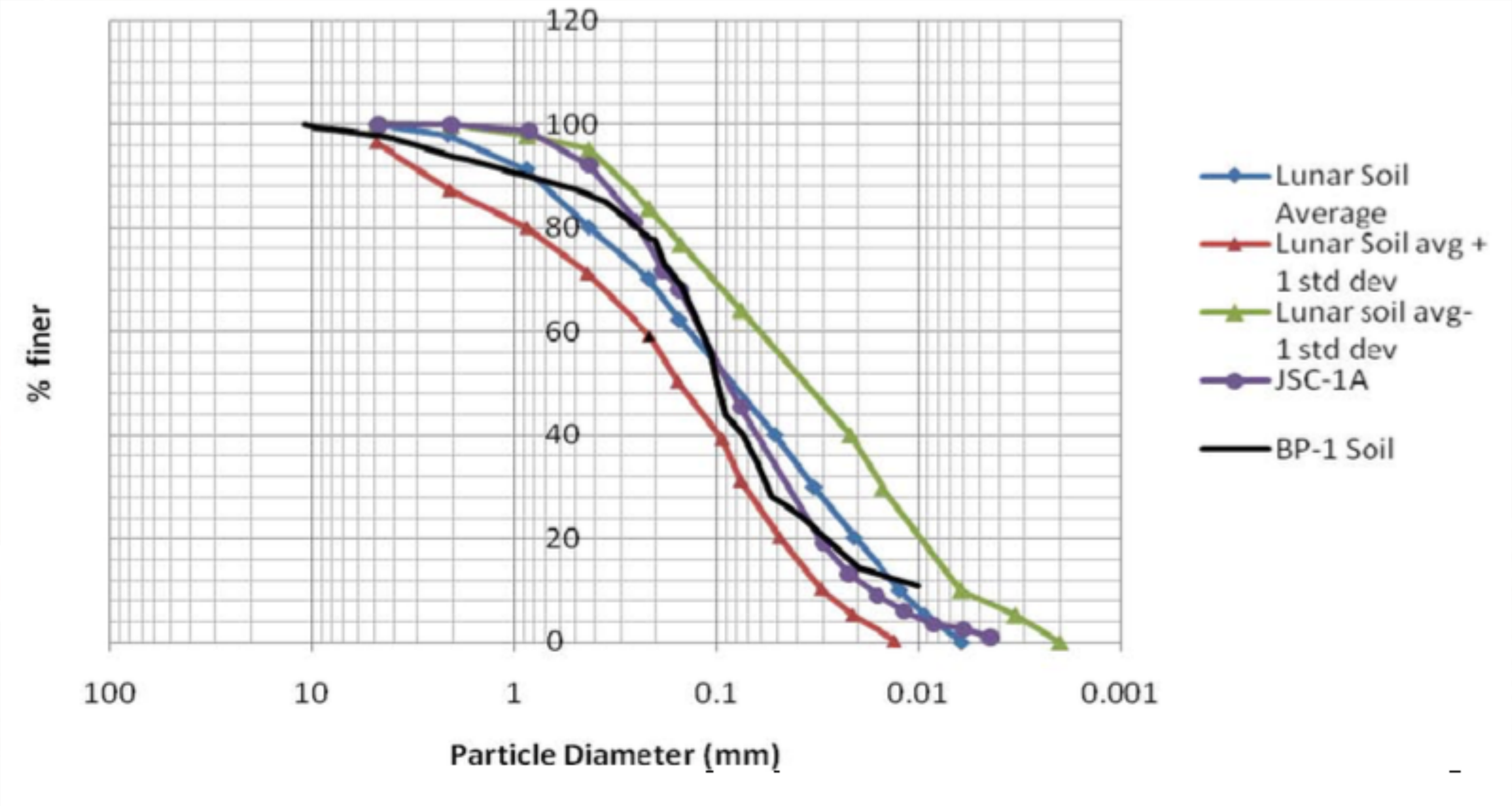
Full Disclosure

- Terramechanics is a highly complex field of research
- The material covered here today is approximately 25% of ENAE 788X, a graduate course in Planetary Surface Robotics
- I have deleted ALL derivations, and am presenting this as a "Quick Start" guide to rover wheel-soil interactions

Regolith Creation Process

- Only "weathering" phenomenon on the moon is meteoritic impact
- Weathering processes
  - Comminution: breaking rocks and minerals into smaller particles
  - Agglutination: welding fragments together with molten glass formed by impact energy
  - Solar wind spallation and implantation (minuscule)
  - Fire fountaining (dormant)
- Result is a fine powder composed of small dust-like pieces

Particle Size Distribution in Regolith



Rahmatian and Metzger, "Soil Test Apparatus for Lunar Surfaces" *Earth and Space 2010*, ASCE

Modeling Soil Reaction to a Wheel

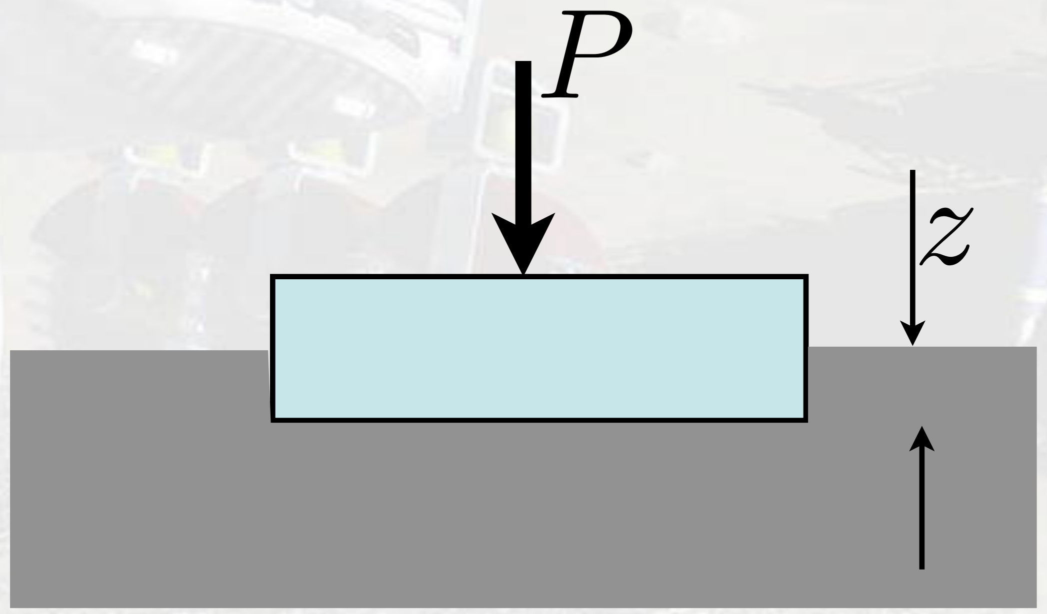
Assume soil reaction is like a (nonlinear) spring

$$P = kz^n$$

$$P = \text{applied pressure}$$

$$z = \text{compression depth}$$

$$k, n = \text{heuristic parameters}$$



More Detailed Soil Compression Equation

$$k = \frac{k_c}{b} + k_\phi$$

$$k_c = \text{modulus of cohesion of soil deformation}$$

$$k_c \text{ units} \Rightarrow < N/m^{(n+1)} >$$

$$k_\phi = \text{modulus of friction of soil deformation}$$

$$k_\phi \text{ units} \Rightarrow < N/m^{(n+2)} >$$

$$b = \text{wheel width}$$

$$\boxed{P = \left(\frac{k_c}{b} + k_\phi \right) z^n}$$

Lunar Soil Canonical Values

$$\text{Soil density: } \rho = 1600 \frac{kg}{m^3} \Longrightarrow \gamma = 2470 \frac{N}{m^3}$$

$$n = 1$$

$$k_c = 1400\ N/m^2$$

$$k_\phi = 830{,}000\ N/m^3$$

$$\phi = 33^{O} = 0.576\ rad$$

$$\text{Cohesion } c = 170\ N/m^2$$

$$N_q = 32.23$$

$$N_c = 48.09$$

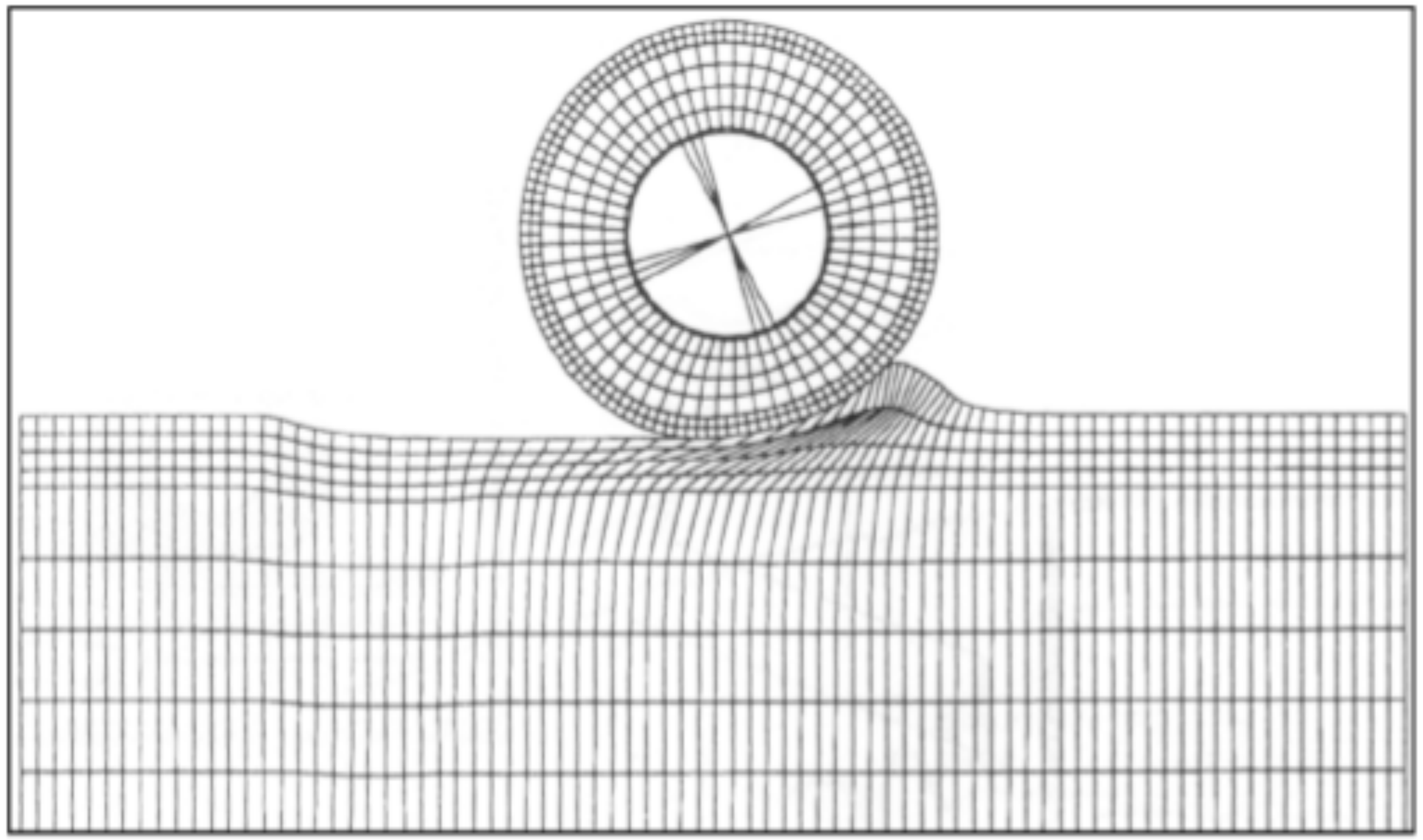
$$N_\gamma = 33.27$$

$$K_c = 33.37$$

$$K_\phi = 72.77$$

Sources of Wheel-Soil Interaction Resistance

- Compression of soil by wheel loads
- Internal friction in drive system
- Effects of gravity on slopes
- Bulldozing of soil ahead of whe



from Gibbesch and Schafer, "Advanced and Simulation Methods of Planetary Rover Mobility on Soft Terrain" *8th ESA Workshop on Advanced Space Technologies for Robotics and Automation*, Noordwijk, The Netherlands, November, 2004

Compression Resistance (Lunar Soil)

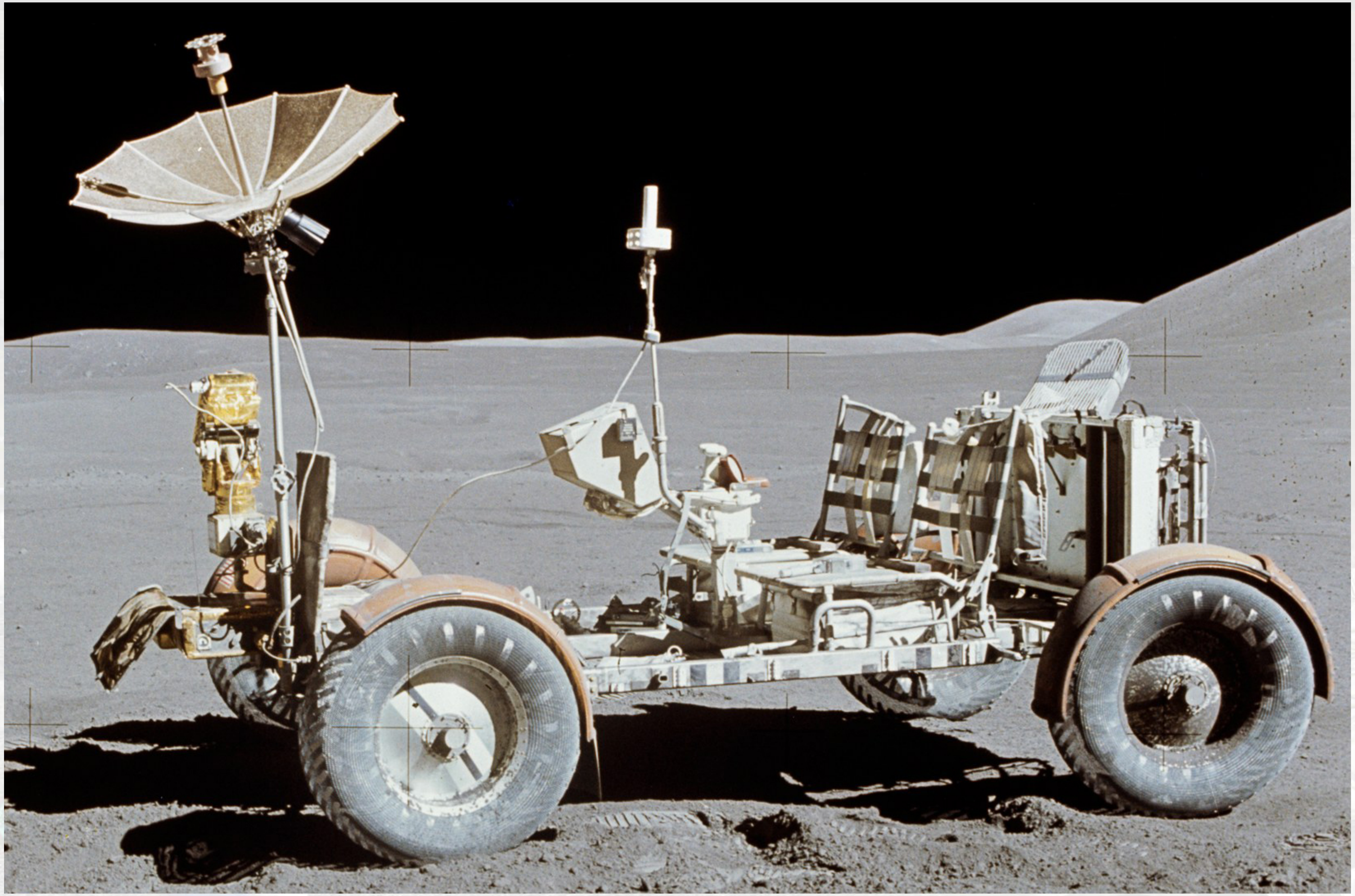
- Depth of compression

$$z = \left(\frac{3}{3-n} \frac{W_w}{(k_c + b k_\phi)\sqrt{D}} \right)^{\frac{2}{2n+1}}$$

- Compression resistance

$$R_c = \frac{1}{n+1} (k_c + b k_\phi)^{\frac{-1}{2n+1}} \left(\frac{3 W_w}{(3-n)\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}}$$

Apollo Lunar Roving Vehicle



Relevant LRV Parameters

$$W_w = 370\ lbs = 168\ kg = 259\ N \text{ (on Moon)}$$

$$D = 32\ in = 0.813\ m$$

$$b = 9\ in = 0.229\ m$$

$$z = \left(\frac{3}{2} \frac{259\ N}{\left(1400\ \frac{N}{m^2} + (0.229\ m)830000\ \frac{N}{m^3}\right)\sqrt{0.813\ m}} \right)^{\frac{2}{3}} = 0.0151\ m$$

Units check: $\left(\frac{N}{\left(\frac{N}{m^2} + \frac{Nm}{m^3}\right)\sqrt{m}} \right)^{\frac{2}{3}} = \left(m^{\frac{3}{2}}\right)^{\frac{2}{3}} = m$

Soil Bearing Limit

Safe weight on the soil

$$W_s = A\left(cN_c + \gamma z N_q + \frac{1}{2}\gamma b N_\gamma\right)$$

Safe soil pressure $P_s = \dfrac{W_s}{A}$

$c \equiv$ Soil cohesion (Pa)

$b \equiv$ Wheel width (m)

For our LRV example case, $P_s = 18{,}790\ Pa$

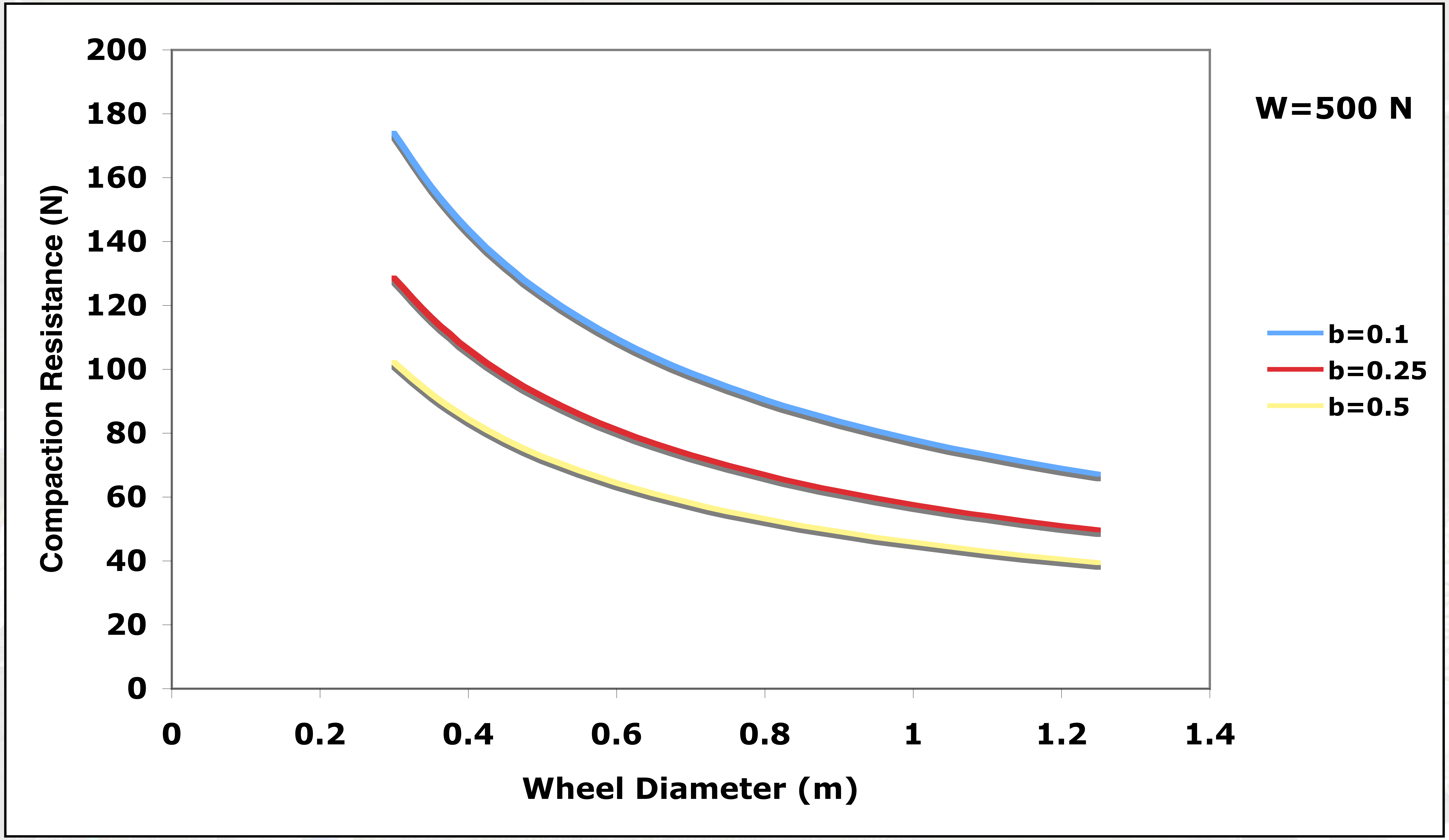
For $W_v = 1004\ N$, the minimum contact area is $534\ cm^2$ or $\ell > 5.8\ cm$

LRV Compression Resistance

$$R_c = \frac{1}{2}\left(1400\ \frac{N}{m^2} + (0.229\ m)820000\ \frac{N}{m^3}\right)^{\frac{-1}{3}}\left(\frac{3(259\ N)}{2\sqrt{0.813\ m}}\right)^{\frac{4}{3}} = 28.3\ N$$

Check units: $\left(\frac{N}{m^2} + \frac{Nm}{m^3}\right)^{\frac{-1}{3}}\left(\frac{N}{\sqrt{m}}\right)^{\frac{4}{3}} = \left(\frac{m^{2/3}}{N^{1/3}}\right)\left(\frac{N^{4/3}}{m^{2/3}}\right) = N$

Effects of Wheel Parameters



Rolling and Gravitation Resistance

- Rolling resistance (tires, bearings, etc.)

$$R_r = W_v c_f$$

$$W_v = \text{weight of vehicle}$$

$$c_f \equiv \text{coefficient of internal friction (typically 0.05)}$$

- Gravitational resistance

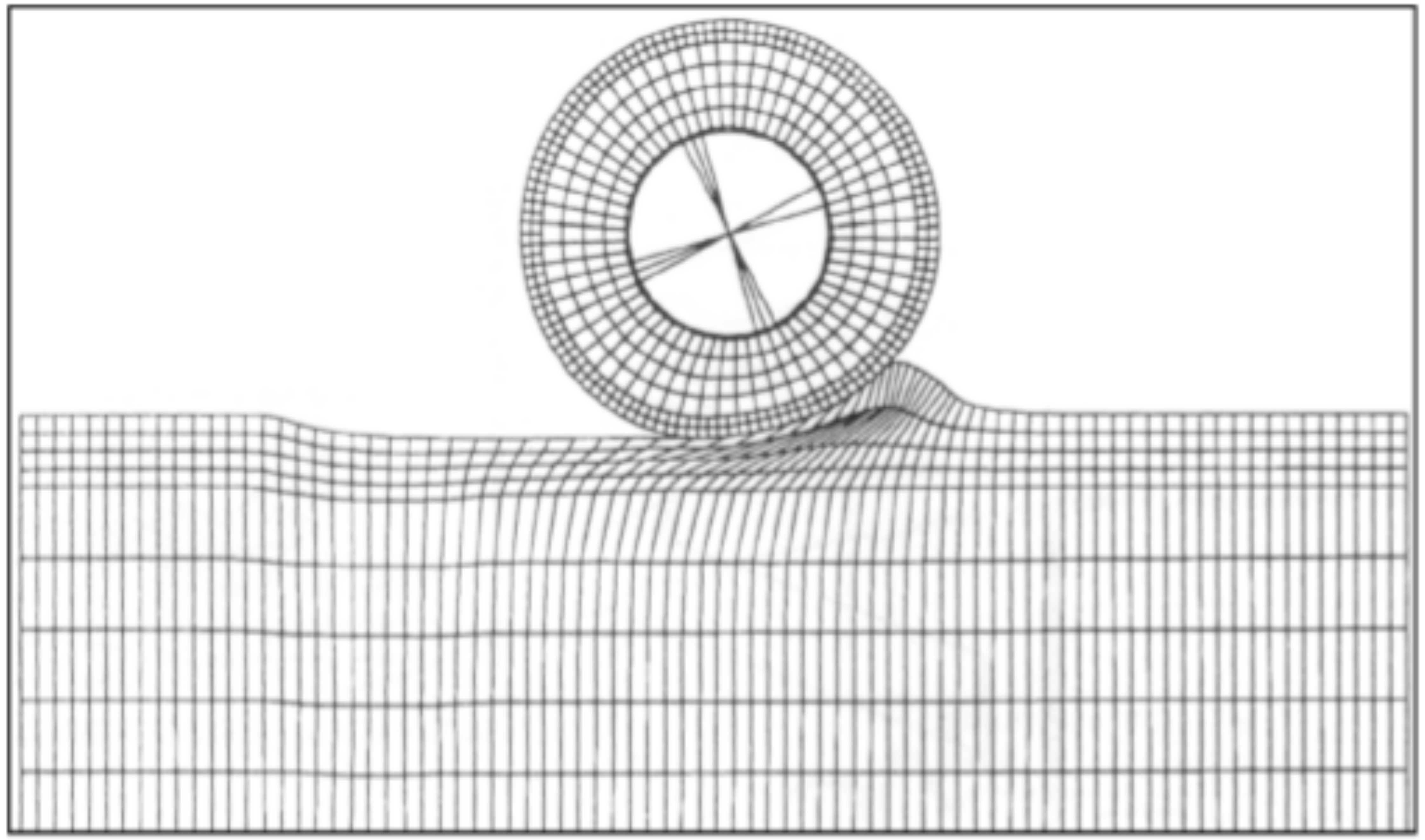
$$R_g = W_v \sin\theta_{slope}$$

- LRV examples (15° slope, W_v=1004 N)

$$R_r = 50\ N \qquad R_g = 260\ N$$

Bulldozing Resistance

- "Bulldozing" is the process of pushing soil up ahead of the wheel
- Ranges from a small factor to a huge one, depending on soil and wheel factors



from Gibbesch and Schafer, "Advanced and Simulation Methods of Planetary Rover Mobility on Soft Terrain" *8th ESA Workshop on Advanced Space Technologies for Robotics and Automation*, Noordwijk, The Netherlands, November, 2004

Bulldozing Resistance

General case:

All angles in radians!

$$R_b = \frac{b\sin(\alpha+\phi)}{2\sin\alpha\cos\phi}\left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\ell_o^3\gamma}{3}\left(\frac{\pi}{2}-\phi\right) + c\ell_o^2\left[1+\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right]$$

$$\alpha = \text{angle of attack of wheel in soil} \equiv \cos^{-1}\left(1-\frac{2z}{D}\right)$$

$$\gamma = \text{density of soil}\left\langle\frac{N}{m^3}\right\rangle \qquad \ell_o = z\tan^2\left(\frac{\pi}{4}-\frac{\phi}{2}\right)$$

For tracked vehicles, only the first term applies:

$$R_b = \frac{b\sin(\alpha+\phi)}{2\sin\alpha\cos\phi}\left(2zcK_c + \gamma z^2 K_\gamma\right)$$

LRV Bulldozing Example (1)

$$\phi = 33^o = 0.576\ rad$$

$$\alpha = \cos^{-1}\left(1 - \frac{2z}{D}\right) = \cos^{-1}\left(1 - \frac{2(1.812)}{81.2}\right) = 17.18° = 0.2999 \text{ rad}$$

$$\ell_o = z \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right) = 0.0051\ m$$

ℓ_o is length of soil ruptured by compression

Soil parameter values calculated on page 26

$$\ell = \frac{D}{2}\cos^{-1}\left(1 - \frac{2z}{D}\right) = 12.18 \text{ cm}$$

ℓ is the length of the wheel's circumference in contact with the soil

LRV Bulldozing Example (2)

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin\alpha \cos\phi} \left(2zcK_c + \gamma z^2 K_\gamma\right)$$

$$+ \frac{\ell_o^3 \gamma}{3}\left(\frac{\pi}{2} - \phi\right) + c\ell_o^2 \left[1 + \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right]$$

$$\langle R_b \rangle = cm \left(cm \ \frac{N}{cm^2} + \frac{N}{cm^3} cm^2 \right) + cm^3 \frac{N}{cm^3} + \frac{N}{cm^2} cm^2$$

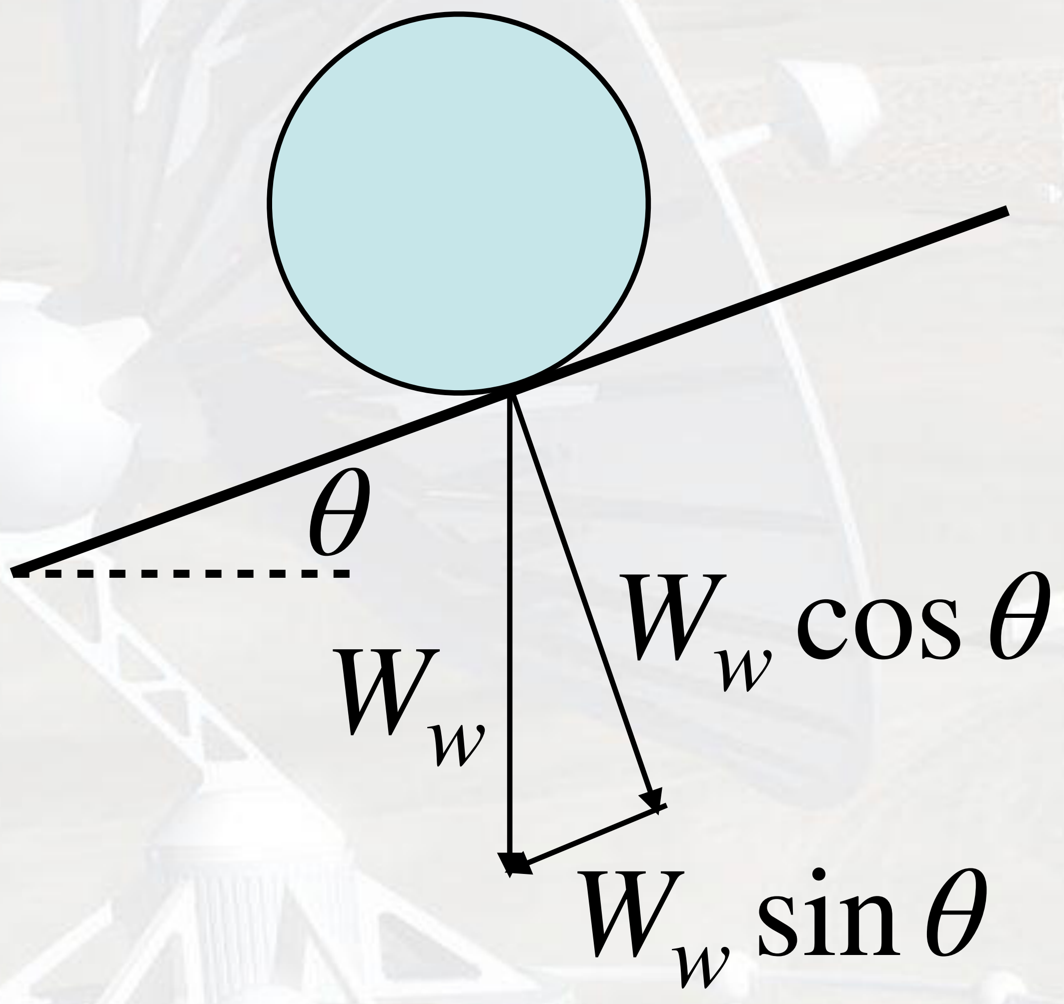
$$R_b = 94.98 + 0.000131 + 0.014 = 95.00 \ N \text{ per leading wheel}$$

$$R_{b,total} = 190.0 \ N$$

Total LRV Rolling Resistance Estimate

- Compression resistance R_c (4 wheels) 113 N
- Rolling resistance R_r 50 N
- Bulldozing resistance R_b (2 front wheels) 190 N
- *Total resistance, flat ground* *352 N*
- Gravitational resistance, 15° slope 260 N
- *Total resistance, 15° slope* *613 N*

Revisiting Rolling Resistance and Slopes



LRV example - $n = 1$

$$R_c = \frac{1}{2}(k_c + bk_\phi)^{\frac{-1}{3}}\left(\frac{3W_w\cos\theta}{2\sqrt{D}}\right)^{\frac{4}{3}}$$

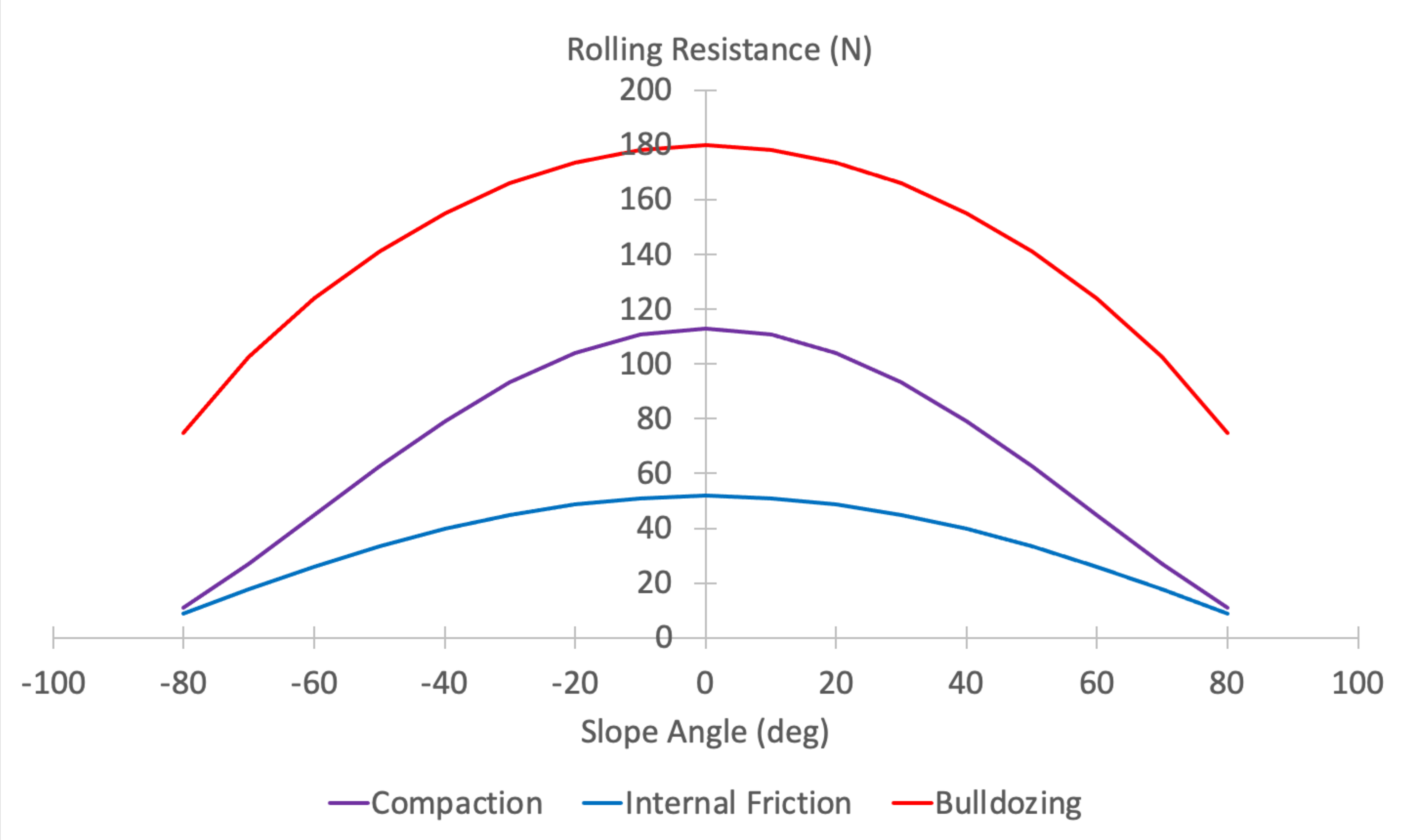
$$R_g = W_v \sin\theta$$

$$R_r = W_v \cos\theta \; c_f$$

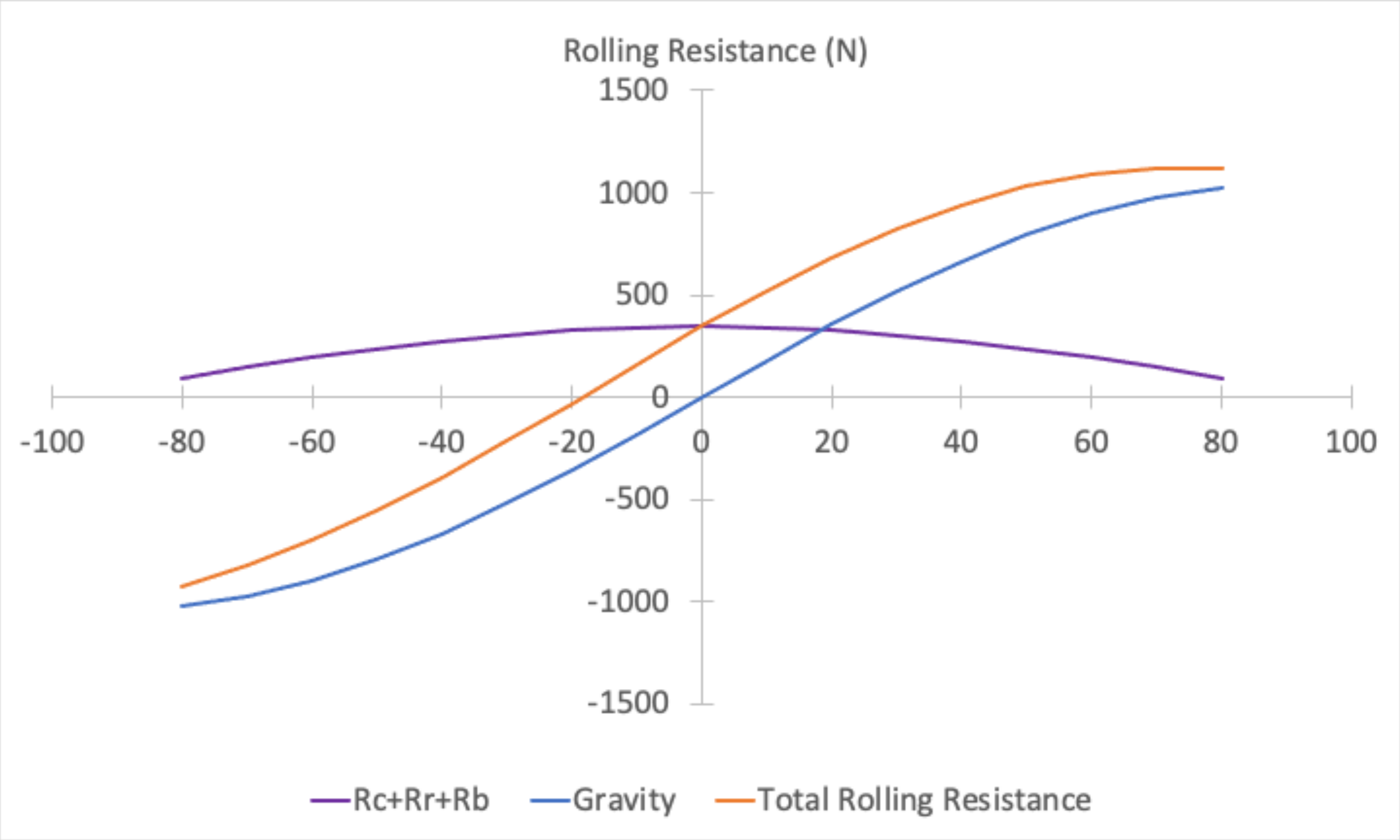
$$z = \left(\frac{3}{2}\frac{W_w\cos\theta}{(k_c + bk_\phi)\sqrt{D}}\right)^{\frac{2}{3}}$$

Bulldozing equation on pg. 43 is unchanged, but solution is altered by changing value of z

Slope Effects on LRV Rolling Resistance Factors



Slope Effects on Total LRV Rolling Resistance



Torque Transfer into Surface

- Calculation up until now focus on passive rolling resistance – how much force would be require to push the vehicle
- Need to calculate the ability of the wheel to convert torque of the drive actuator into soil thrust to propel the vehicle
- Strongly dependent on slip ratio

Wheel Slip Ratio

- Wheel circumferential speed ωr will never exactly match vehicle speed V
- The difference is slip ratio: $s \equiv 1 - \frac{V}{\omega r}$
- If $\omega r = V$ then $s = 0$
- If $\omega r > V$ then $0 < s < 1$
- If $\omega r \gg V$ (or $V = 0$) then $s = 1$
- If $\omega r < V$ (deceleration) then $-1 < s < 0$

Tractive Force per (Smooth) Wheel

$$H = \left(Ac + W\tan\phi\right)\left[1 - \frac{K}{s\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right]$$

$A =$ area of contact $= b\ell$

$c =$ soil cohesion

$\phi =$ soil angle of internal friction

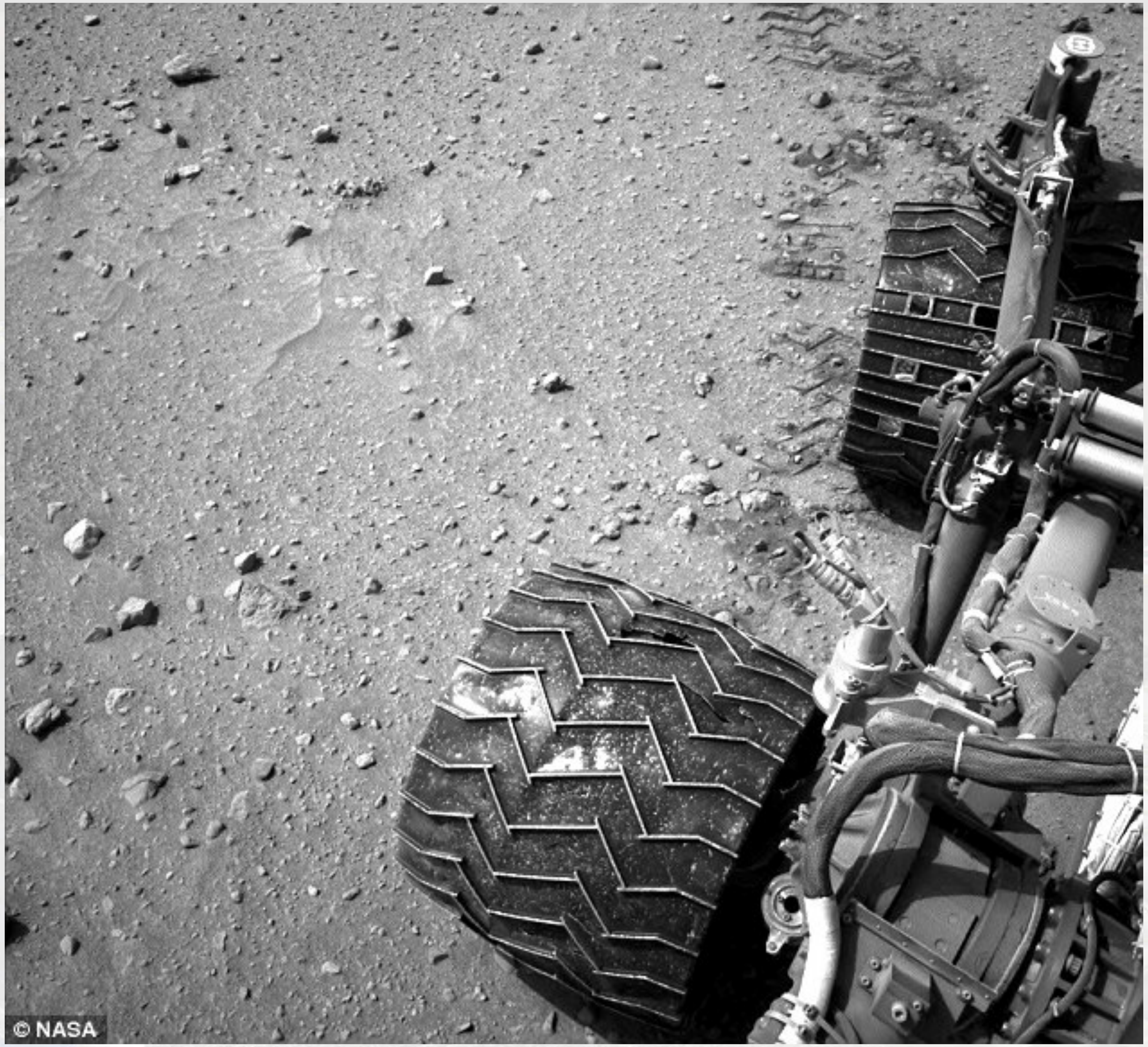
$s =$ wheel slip ratio

$K =$ shear deformation modulus

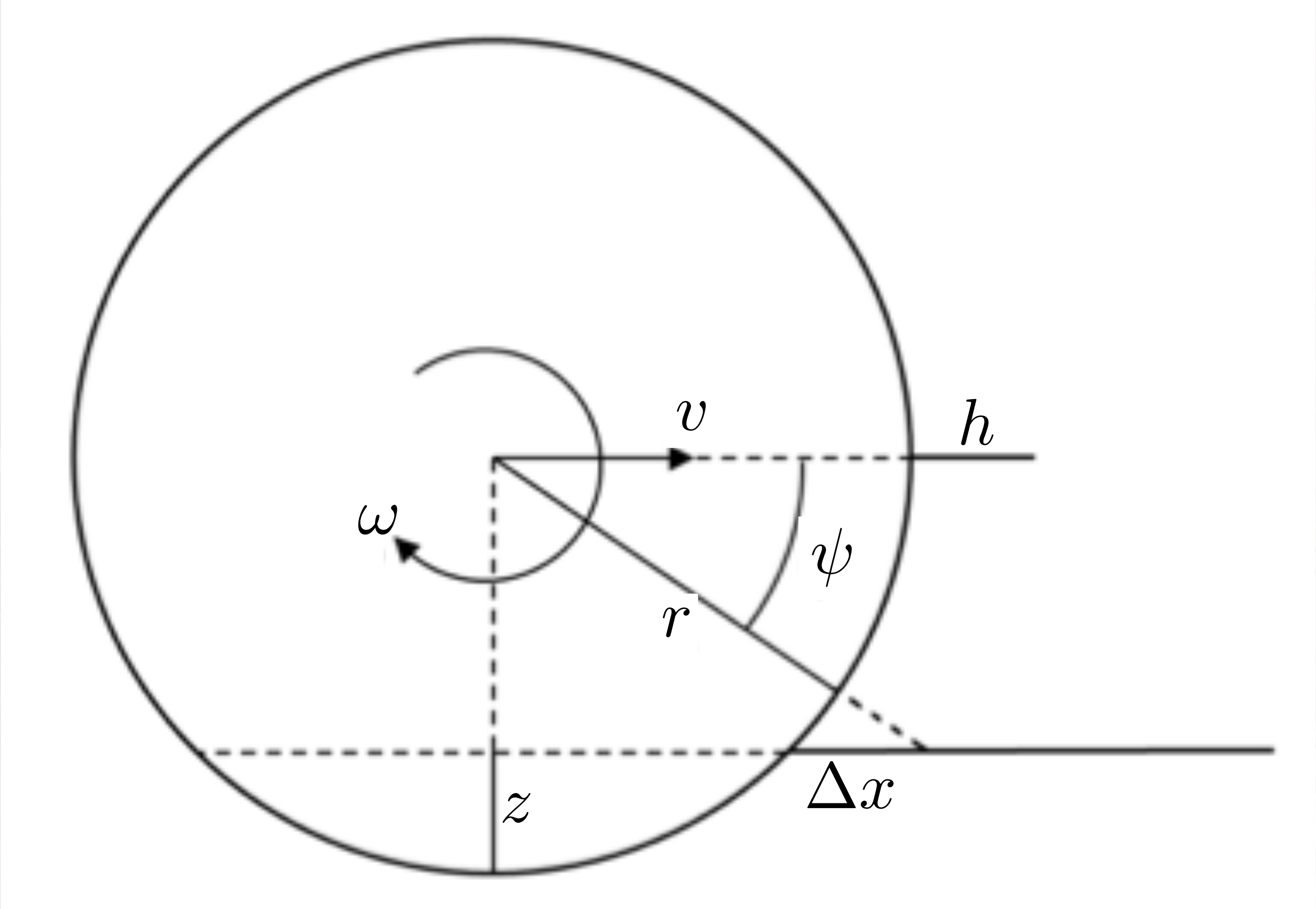
$\ell =$ length of contact patch $= \frac{D}{2}\cos^{-1}\left(1 - \frac{2z}{D}\right)$

Grousers (on Tracked Vehicle)

Grousers (MSL Wheels)



Minimum Grouser Condition



$$t_{rim\ contact} = \frac{\Delta x}{v}$$

$$t_{grouser\ contact} = \frac{\psi}{\omega}$$

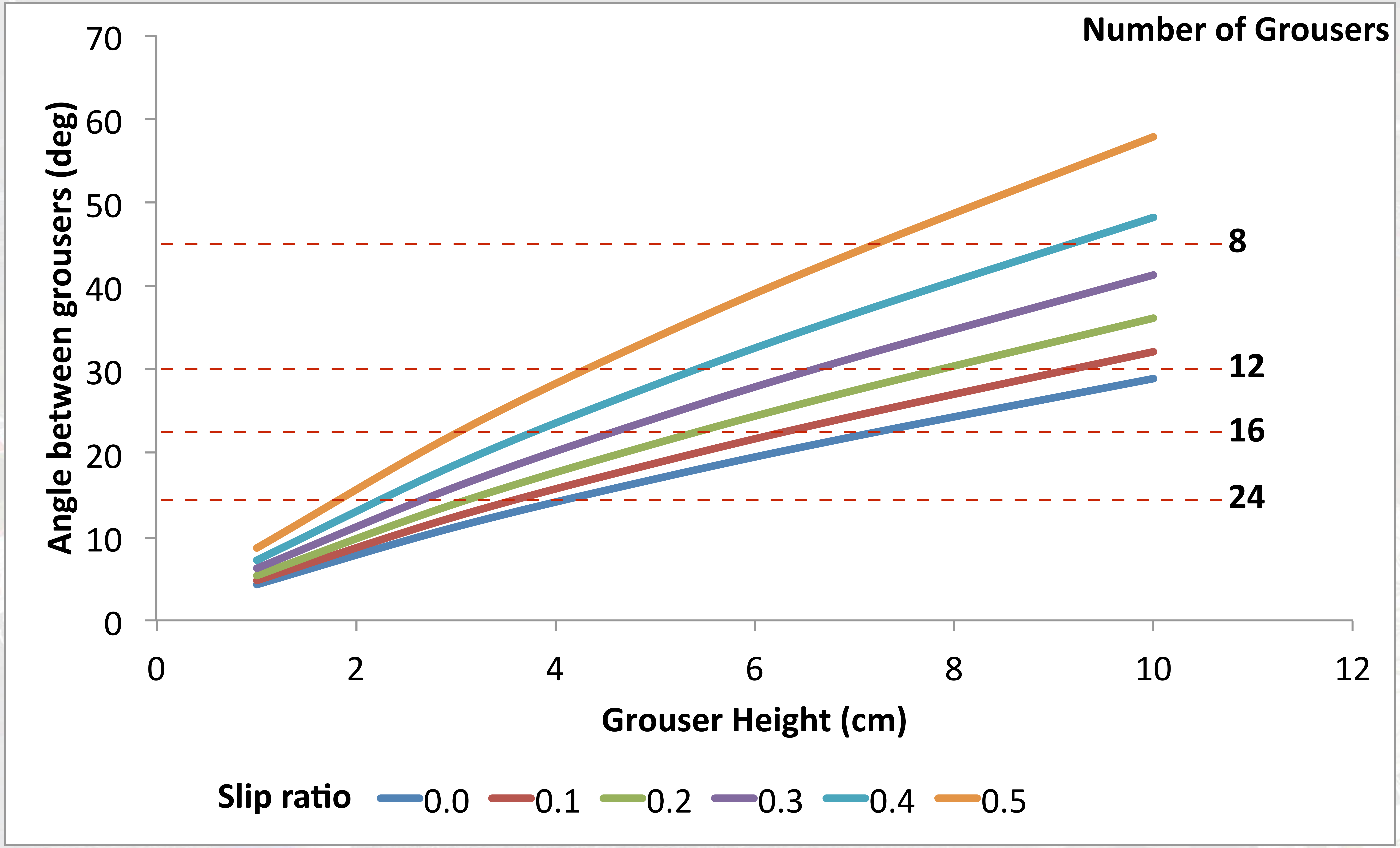
$$t_{grouser\ contact} \leq t_{rim\ contact}$$

$$\psi \leq \frac{\omega}{v}\Delta x$$

$$s = 1 - \frac{v}{\omega r} \Longrightarrow v = \omega r(1-s)$$

$$\psi \leq \frac{1}{r(1-s)}\Delta x$$

Required Grouser Spacing



Number of Grousers in Ground Contact

$$z = 1.812\ cm \qquad\qquad r = 40.6\ cm$$

$$\text{Number of grousers} = N$$

$$\text{Angle between grousers} \equiv \psi = \frac{2\pi}{N}$$

$$\text{Distance between grousers} \equiv \ell_g = \psi r = \frac{2\pi r}{N}$$

$$\text{Number of grousers in ground contact} \equiv N_g = \frac{2\pi r}{N\ell}$$

$$\text{LRV example: choose } N = 16$$

$$N_g = 1.3 \approx 1$$

Tractive Force per Wheel (With Grousers)

$$H = \left[b\ell c \left(1 + \frac{2h}{b}\right) N_g + W \tan\phi \left(1 + 0.64 \frac{h}{b} \arctan \frac{b}{h}\right)\right] \left[1 - \frac{K}{s\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$

$A =$ area of contact $\cong b\ell$

$c =$ soil cohesion $= 0.017$ N/cm^2

$\phi =$ soil angle of internal friction $= 35^o$

$s =$ wheel slip ratio (typ. 0.02-0.05)

$K =$ shear deformation modulus $= 1.8\ cm$

$\ell =$ length of contact patch $= \frac{D}{2} \cos^{-1}\left(1 - \frac{2z}{D}\right)$

$h =$ height of grouser

$N_g =$ number of grousers in contact with ground

All values typical for lunar soil

Basic Equation of Vehicle Propulsion

DP: Drawbar pull (residual drive force)

H: Maximum tractive force of wheels

R_c: Compaction resistance

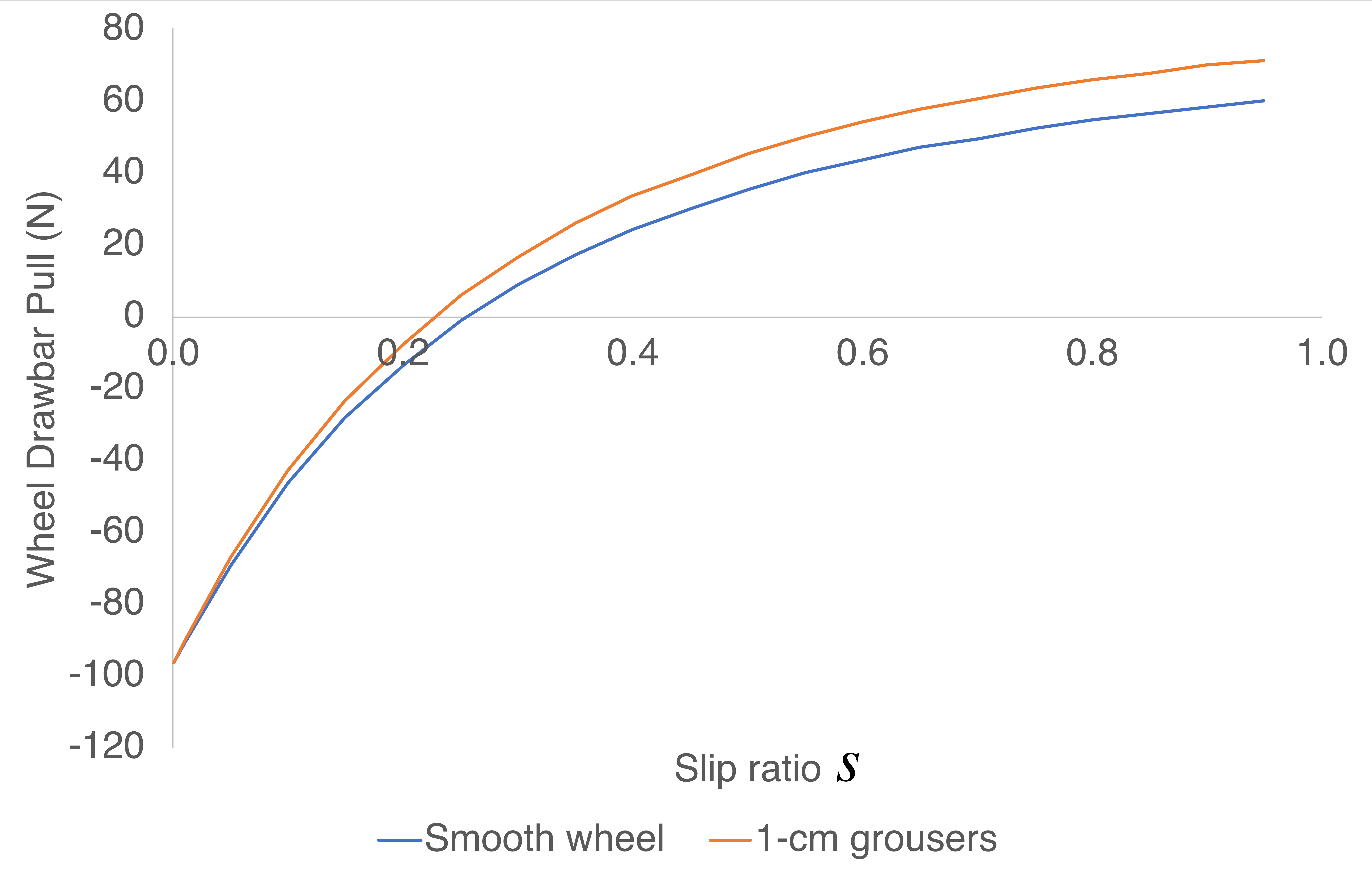
R_b: Bulldozing resistance

R_g: Gravitational resistance

R_r: Rolling resistance (internal)

$$DP = H - (R_c + R_b + R_g + R_r)$$

Drawbar Pull vs. Slip (per wheel)



Some Notes on Terramechanics

- This is the simplest approach to calculating wheel-soil interactions
- Real-world issues not modeled include
  - Non-homogeneities
  - Soil layering
  - Soil transport under wheel
- This technique is conservative in estimating drawbar pull
- Braking is more complicated than sticking $s < 0$ into the equation for H, but beyond the scope for this lecture