

Robotic Mobility – Above the Surface

- Free Space
- Relative Orbital Motion
- Airless Major Bodies (moons)
- Gaseous Environments (Mars, Venus, Titan)
 - Lighter-than-“air” (balloons, dirigibles)
 - Heavier-than-“air” (aircraft, helicopters)

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Robotic Mobility – Above the Surface
ENAE 788X - Planetary Surface Robotics

Propulsive Motion in Free Space

- Basic motion governed by Newton's Law
 $F = ma$ (actually, $\vec{F} = m \ddot{\vec{x}}$)
- Over a distance d and time t , assuming the motion is predominately coasting,

$$\Delta V = 2 \frac{d}{t}$$

(required to accelerate and decelerate)

- The rocket equation (relates propellant to ΔV)

$$\frac{m_{final}}{m_o} = e^{-\frac{\Delta V}{v_{exhaust}}}$$



Cost of Propulsive Maneuvering

- Assuming $\Delta V \ll V_{exhaust}$
- Use the Taylor's Series expansion of e

$$\frac{m_{final}}{m_o} \approx 1 - \frac{\Delta V}{V_{exhaust}}$$

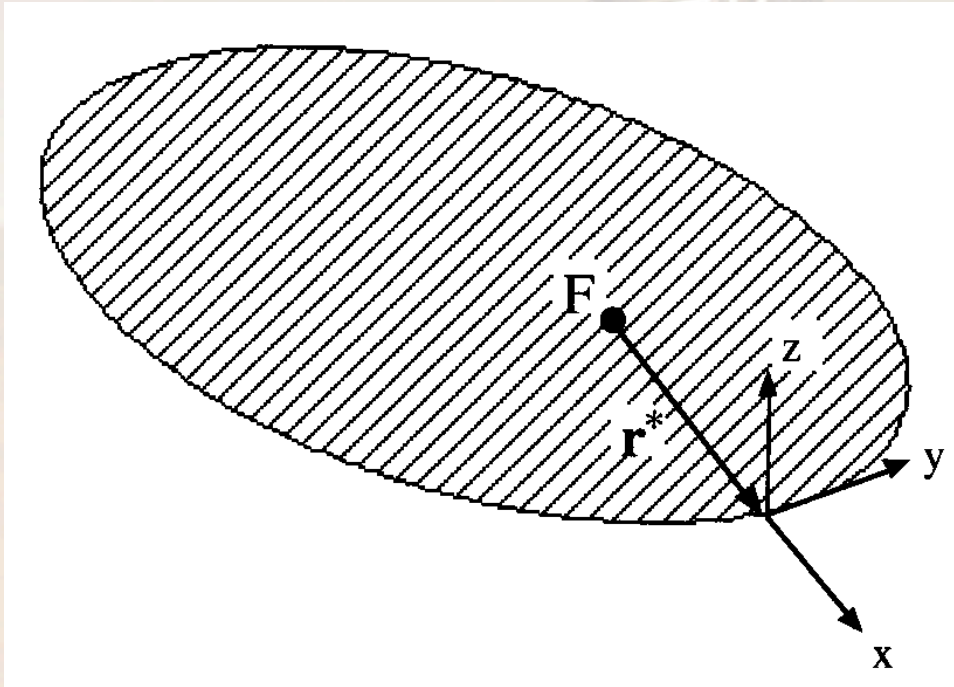
- Since $m_o = m_{initial} = m_{prop} + m_{final}$,

$$\frac{m_{prop}}{m_o} \approx \frac{2}{V_{exhaust}} \frac{d}{t} \quad \text{or} \quad \frac{m_{prop}}{m_o} \approx 2 \frac{V_{travel}}{V_{exhaust}}$$



Hill's Equations (Proximity Operations)

Linearized equations of motion relative to a target in circular orbit in a rotating Cartesian reference frame



$$\ddot{x} = 3n^2x + 2n\dot{y} + a_{dx}$$

$$\ddot{y} = -2n\dot{x} + a_{dy}$$

$$\ddot{z} = -n^2z + a_{dz}$$

$$n = \sqrt{\frac{\mu}{a^3}}$$

Ref: J. E. Prussing and B. A. Conway, *Orbital Mechanics*
Oxford University Press, 1993

a_{dx} , a_{dy} , a_{dz} are disturbing accelerations (e.g., thrust, solar pressure)



Clohessy-Wiltshire (“CW”) Equations

Force-free solutions to Hill’s Equations

$$x(t) = [4 - 3 \cos(nt)]x_o + \frac{\sin(nt)}{n}\dot{x}_o + \frac{2}{n}[1 - \cos(nt)]\dot{y}_o$$

$$y(t) = 6[\sin(nt) - nt]x_o + y_o - \frac{2}{n}[1 - \cos(nt)]\dot{x}_o + \frac{4 \sin(nt) - 3nt}{n}\dot{y}_o$$

$$\dot{x}(t) = 3n \sin(nt)x_o + \cos(nt)\dot{x}_o + 2 \sin(nt)\dot{y}_o$$

$$\dot{y}(t) = -6n [1 - \cos(nt)]x_o - 2 \sin(nt)\dot{x}_o + [4 \cos(nt) - 3]\dot{y}_o$$

$$z(t) = z_o \cos(nt) + \frac{\dot{z}_o}{n} \sin(nt)$$

$$\dot{z}(t) = -z_o n \sin(nt) + \dot{z}_o \cos(nt)$$

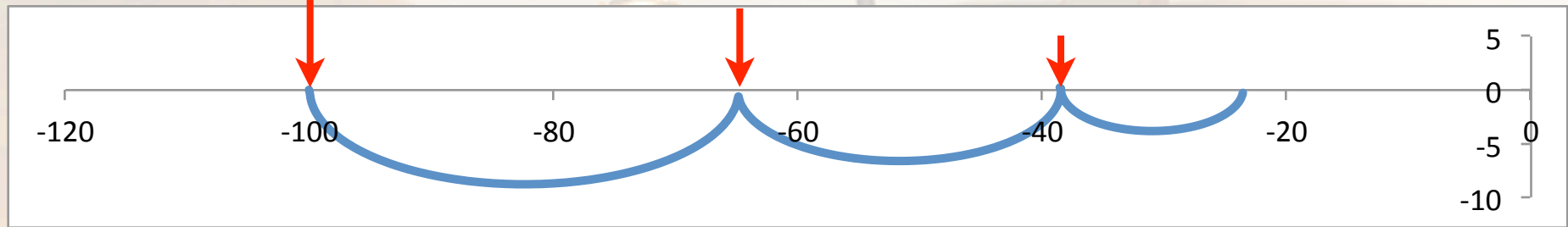


“V-Bar” Approach

0.01 m/sec

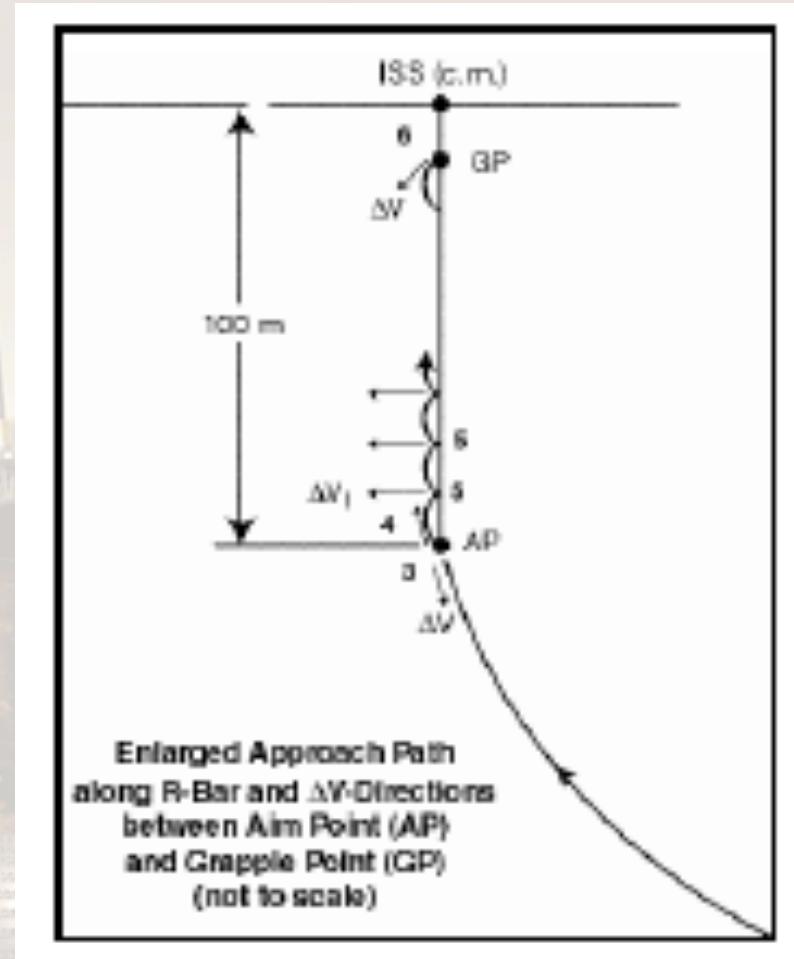
0.0075 m/sec

0.005 m/sec



“R-Bar” Approach

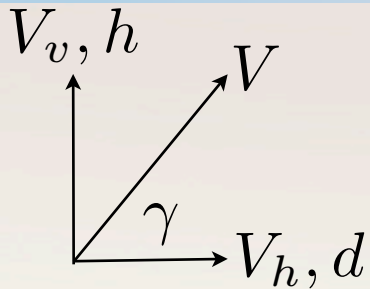
- Approach from along the radius vector (“R-bar”)
- Gravity gradients decelerate spacecraft approach velocity - low contamination approach
- Used for Mir, ISS docking approaches



Ref: Collins, Meissinger, and Bell, Small Orbit Transfer Vehicle (OTV) for On-Orbit Satellite Servicing and Resupply, 15th USU Small Satellite Conference, 2001



Hopping (Airless Flat Planet)



Use $F=ma$ for vertical motion

$$\dot{V}_v = -g \quad h = V_v t - \frac{1}{2} g t^2$$

$$t_{flt} = 2V_v/g$$

Constant velocity in horizontal direction produces

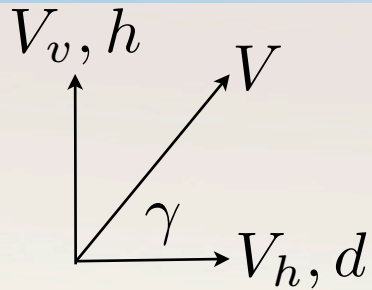
$$d = V_h t_{flt} = 2 \frac{V_h V_v}{g}$$

$$V_h = V \cos \gamma; V_v = V \sin \gamma$$

$$d = 2 \frac{V^2 \sin \gamma \cos \gamma}{g} = \frac{V^2}{g} \sin (2\gamma)$$



Hopping (Airless Flat Planet)



Horizontal distance is maximized when $\sin(2\gamma) = 1$

$$\gamma_{opt} = \frac{\pi}{2} = 45^\circ \quad d_{max} = \frac{V^2}{g}$$

$$V = \sqrt{gd} \quad \Delta V_{total} = 2V = \boxed{2\sqrt{gd}}$$

$$h_{max} = V_v \frac{V_v}{g} - \frac{1}{2}g \left(\frac{V_v}{g} \right)^2 \quad V_v = \frac{V}{\sqrt{2}}$$

$$h_{max} = \frac{V^2}{4g} = \frac{\sqrt{gd}^2}{4g} = \frac{d}{4}$$



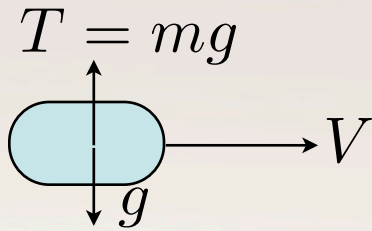
An Example of Propulsive Gliding



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Propulsive Gliding (Airless Flat Planet)



Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V$$

$$\Delta V_v = gt_{flt} = \frac{gd}{V}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V}$$



Propulsive Gliding (Airless Flat Planet)

Want to choose V to minimize

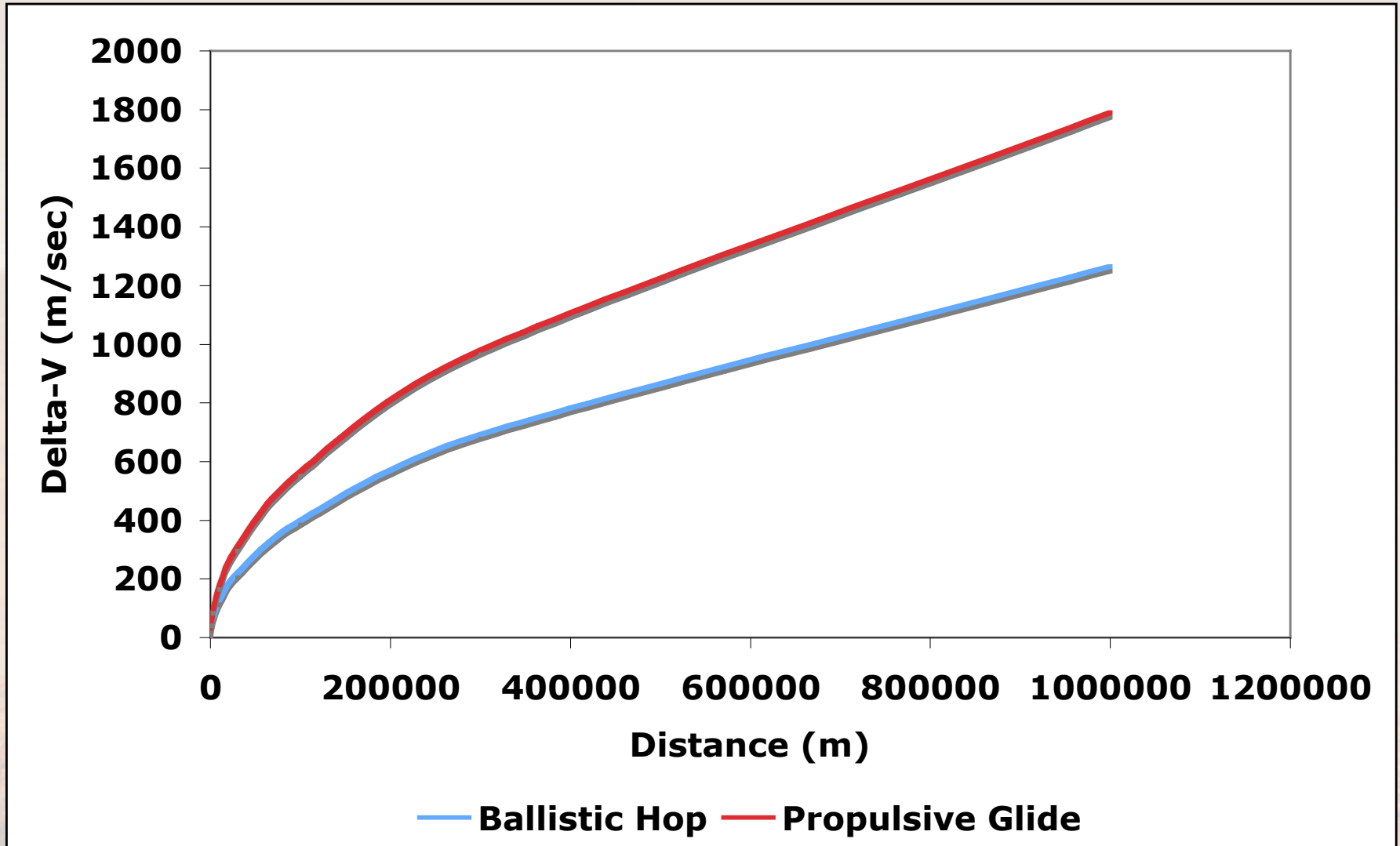
$$\frac{\partial}{\partial V} \left(2V + \frac{gd}{V} \right) = 0 \qquad 2 - \frac{gd}{V^2} = 0$$

$$V_{opt} = \sqrt{\frac{gd}{2}}$$

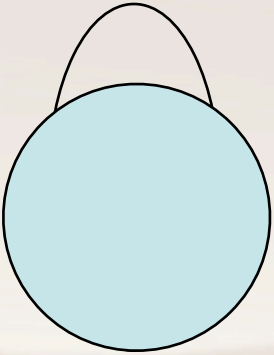
$$\Delta V_{total} = 2\sqrt{\frac{gd}{2}} + gd\sqrt{\frac{2}{gd}} = 2\sqrt{2}\sqrt{gd}$$



Delta-V for Hopping and Gliding



Hopping (Spherical Planet)



$$\Delta v = 2v_o$$

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)}$$

$$r = \frac{p}{1 + e \cos \nu} = \frac{a(1 - e^2)}{1 - e \cos \theta}$$

$$a = r \left(\frac{1 - e \cos \theta}{1 - e^2} \right) \quad v = \sqrt{\mu \left(\frac{2}{r} - \frac{1 - e^2}{r(1 - e \cos \theta)} \right)}$$

$$\frac{\partial v}{\partial e} = 0 \Rightarrow \frac{-r(1 - e \cos \theta)(-2e) + (1 - e^2)r(-\cos \theta)}{r^2(1 - e \cos \theta)^2} = 0$$



Hopping (Spherical Planet)

$$2er - 2e^2 r \cos \theta - r \cos \theta + re^2 \cos \theta = 0$$

$$\cos \theta e^2 - 2e + \cos \theta = 0$$

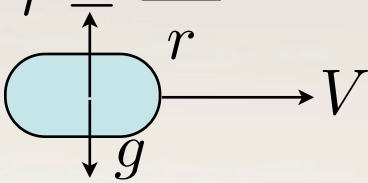
$$e_{opt} = \frac{2 \pm \sqrt{2^2 - 4 \cos^2 \theta}}{2 \cos \theta} = \frac{1 \pm \sin \theta}{\cos \theta}$$

+ produces $e > 1$ (hyperbolic orbit); - gives elliptical orbit

$$e_{opt} = \frac{1 - \sin \theta}{\cos \theta} \quad a_{opt} = r \left(\frac{1 - e_{opt} \cos \theta}{1 - e_{opt}^2} \right)$$



Propulsive Gliding (Airless Round Planet)

$$\omega^2 r = \frac{V^2}{r}$$


A diagram of a rounded planet with radius r . A horizontal arrow labeled V points to the right, representing the velocity. A vertical arrow labeled g points downwards from the center of the planet, representing gravity. The equation $\omega^2 r = \frac{V^2}{r}$ is written above the planet.

Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V \quad \Delta V_v = \left(g - \frac{V^2}{r} \right) t_{flt} = \frac{gd}{V} - \frac{dV}{r}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V} - \frac{dV}{r}$$



Propulsive Gliding (Airless Round Planet)

Want to choose V to minimize

$$\frac{\partial}{\partial V} \left(2V + \frac{gd}{V} - \frac{dV}{r} \right) = 0 \quad 2 - \frac{gd}{V^2} - \frac{d}{r} = 0$$

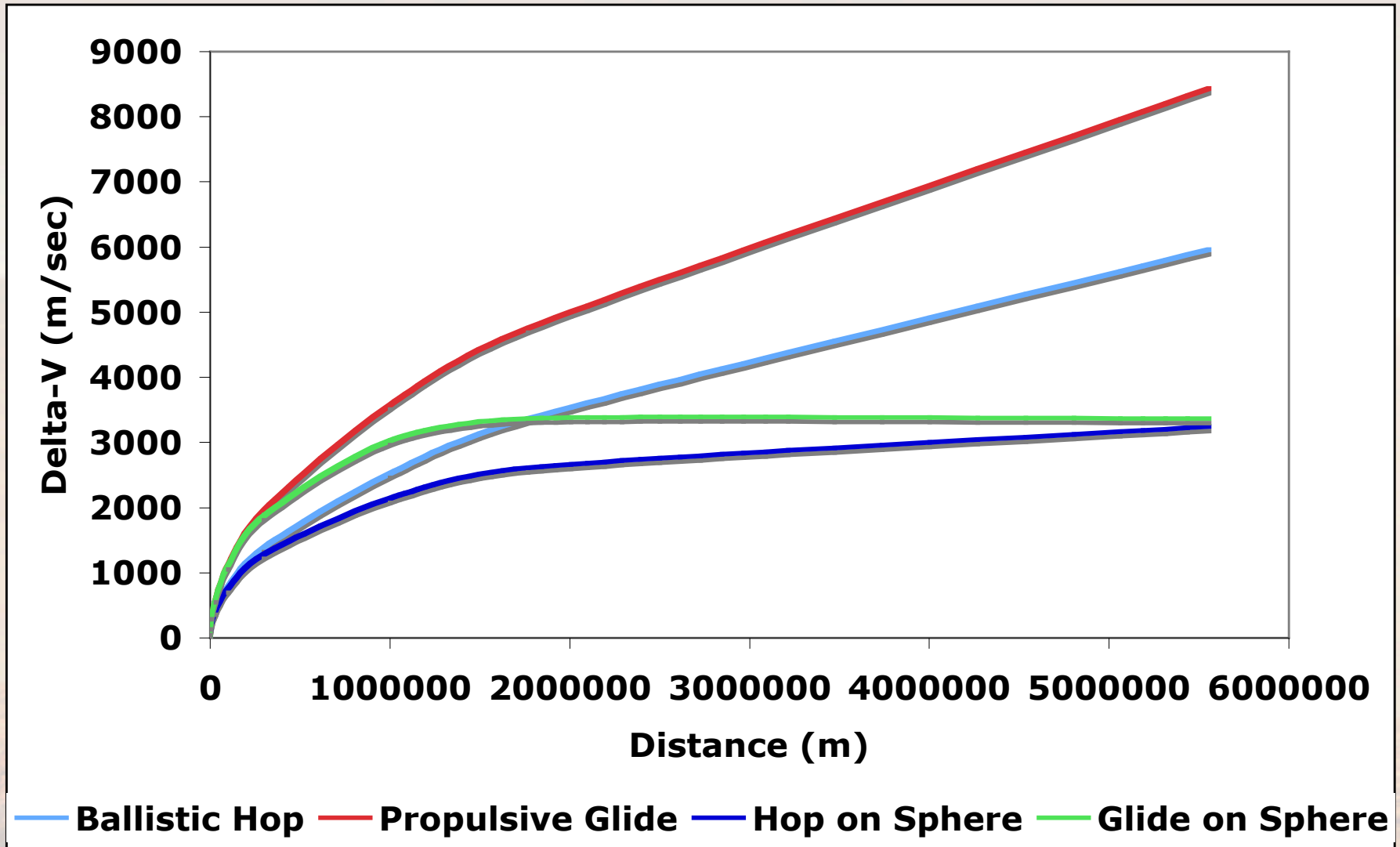
$$V_{opt} = \sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{\frac{gd}{2 - \frac{d}{r}}} + gd\sqrt{\frac{2 - \frac{d}{r}}{gd}} - \frac{d}{r}\sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{2 - \frac{d}{r}}\sqrt{gd}$$



Hopping on Flat and Round Bodies



Nondimensional Forms

Define $\nu \equiv \frac{V}{\sqrt{dg}}$ $\rho \equiv \frac{d}{r}$ $\eta \equiv \frac{h_{max}}{d}$

$$\nu_{flat\ glide} = 2\sqrt{2}$$

$$\nu_{flat\ hop} = 2 \quad \eta = \frac{1}{4}$$

$$\nu_{spherical\ glide} = 2\sqrt{2 - \rho} \quad (0 \leq \rho \leq 1)$$



Multiple Hops

- Assume n hops between origin and destination
- At each intermediate “touchdown”, v_v has to be reversed

$$\Delta V_{total} = 2V + 2(n - 1)V_v$$

$$t_{peak} = \frac{V_v}{g} \quad t_{total} = 2nt_{peak} = 2n\frac{V_v}{g}$$

$$d = V_h t_{total} = \frac{2n}{g} V_h V_v \quad V_v = \sqrt{2gh_{max}} \quad \nu_v = \sqrt{\frac{2\eta}{n}}$$

$$\nu \equiv \frac{V}{\sqrt{dg}} \quad \eta \equiv \frac{h_{max}}{d/n} \quad V_h = \frac{dg}{2nV_v} \quad \nu_h = \frac{1}{2} \sqrt{\frac{1}{2n\eta}}$$



Multiple Hop Analysis

$$\Delta\nu = 2\nu + 2(n-1)\nu_v$$
$$\Delta\nu = 2\sqrt{\nu_v^2 + \nu_h^2} + 2(n-1)\nu_v$$

$$\Delta\nu = 2\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}} + 2(n-1)\sqrt{\frac{2\eta}{n}}$$

$$\frac{\partial\Delta\nu}{\partial\eta} = \left[\frac{1}{\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}}} \left(\frac{2}{n} - \frac{1}{8n\eta^2} \right) \right] + (n-1)\sqrt{\frac{2}{n\eta}} = 0$$

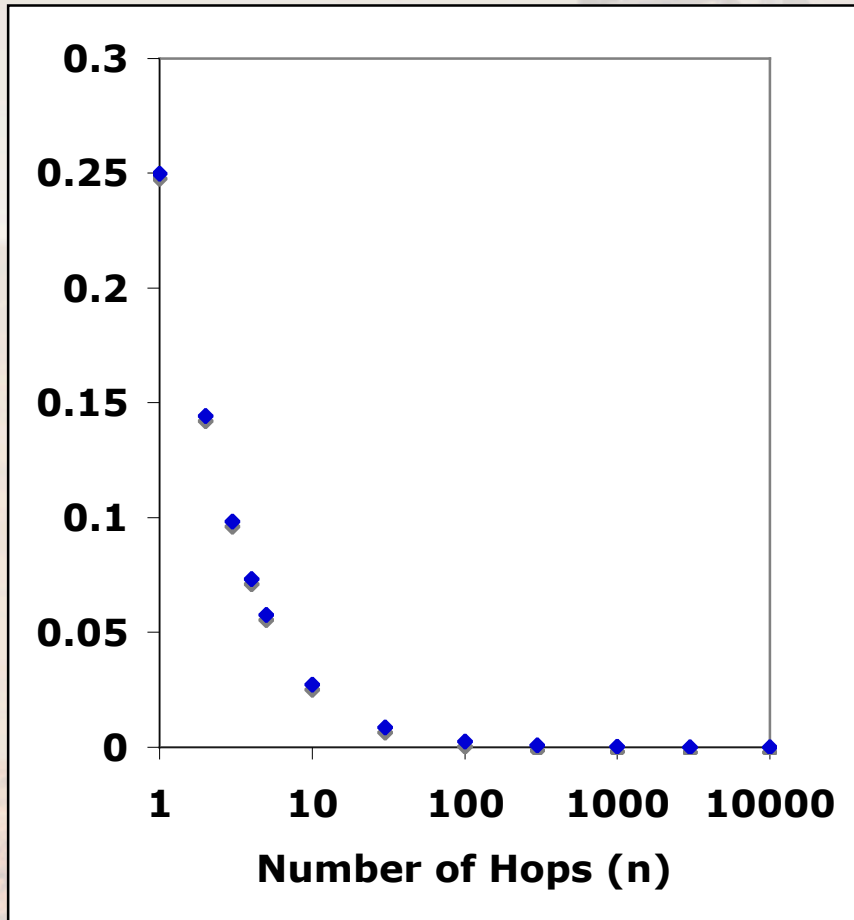
Analytically messy, but note that for $n = 1 \Rightarrow \eta_{opt} = \frac{1}{4}$

(In general, solve numerically)

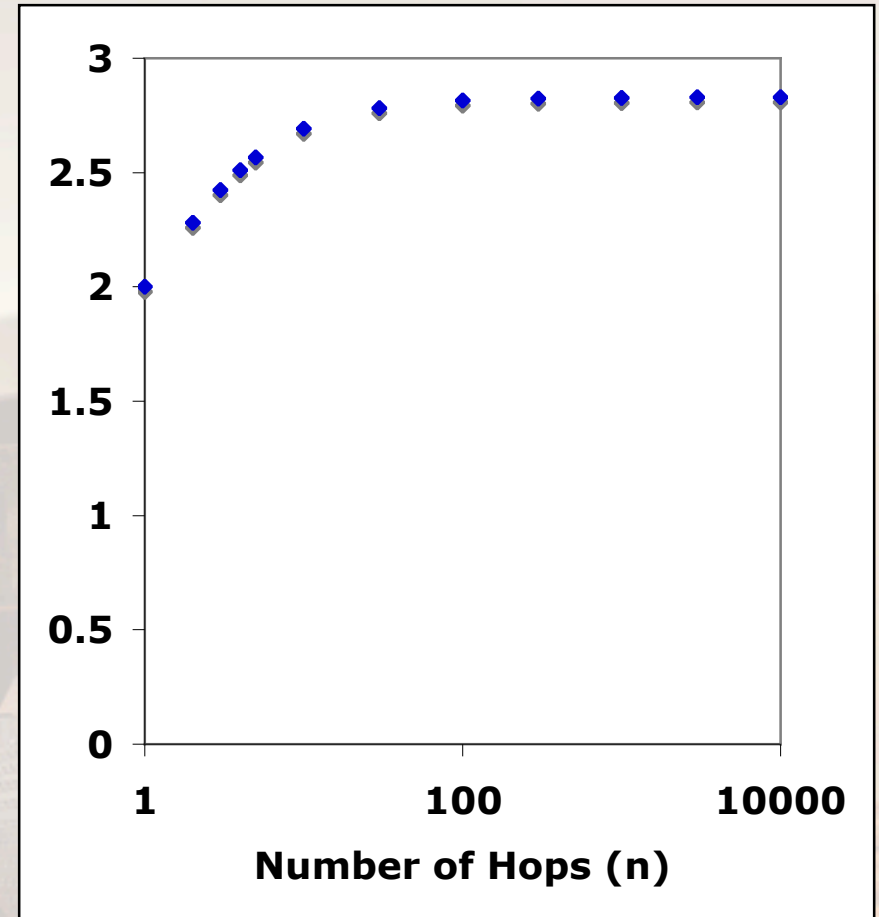


Optimal Solutions for Multiple Hops

η_{opt}



$\Delta\nu$



Hopping Between Different Altitudes

Relative to starting point, landing elevation $\equiv h_2$

$$v_1 = (v_h, v_{v1}) \quad v_2 = (v_h, v_{v2}) \quad v_{v1} \neq v_{v2}$$

$$h = v_{v1}t - \frac{1}{2}gt^2 \quad t_{peak} = \frac{v_{v1}}{g} \quad h_{peak} = \frac{1}{2} \frac{v_{v1}^2}{g}$$

$$v_{v1} = \sqrt{2gh_{peak}}$$

$$\text{From peak, } v_v = -gt_{fall}; \quad h = h_{peak} - \frac{1}{2}gt_{fall}^2$$

$$h_2 = h_{peak} - \frac{1}{2} \frac{v_{v2}^2}{g} \quad t_{fall} = \sqrt{\frac{2}{g}(h_{peak} - h_2)}$$

$$v_{v2} = \sqrt{2g(h_{peak} - h_2)}$$



Optimal Hop with Altitude Change

$$d = v_h(t_{peak} + t_{fall}) = v_h \left(\frac{v_{v1}}{g} + \sqrt{\frac{2}{g}(h_{peak} - h_2)} \right)$$

$$d = v_h \left(\sqrt{\frac{2h_{peak}}{g}} + \sqrt{\frac{2}{g}(h_{peak} - h_2)} \right)$$

$$d\sqrt{g} = v_h \left(\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)} \right)$$

$$v_h = \frac{d\sqrt{g}}{\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)}}$$

$$\Delta v = \left(\sqrt{v_h^2 + v_{v1}^2} + \sqrt{v_h^2 + v_{v2}^2} \right)$$



Nondimensional Form of Equations

Remember that $\nu \equiv \frac{v}{\sqrt{dg}}$; $\eta \equiv \frac{h_{peak}}{d}$; $\lambda \equiv \frac{h_2}{d}$

$$\Delta\nu = \left(\sqrt{\nu_h^2 + \nu_{v1}^2} + \sqrt{\nu_h^2 + \nu_{v2}^2} \right)$$

$$\nu_{v1} = \sqrt{2\eta} \qquad \nu_{v2} = \sqrt{2(\eta - \lambda)}$$

$$\nu_h = \frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}}$$

$$\Delta\nu = \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}} \right)^2 + 2\eta} + \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}} \right)^2 + 2(\eta - \lambda)}$$



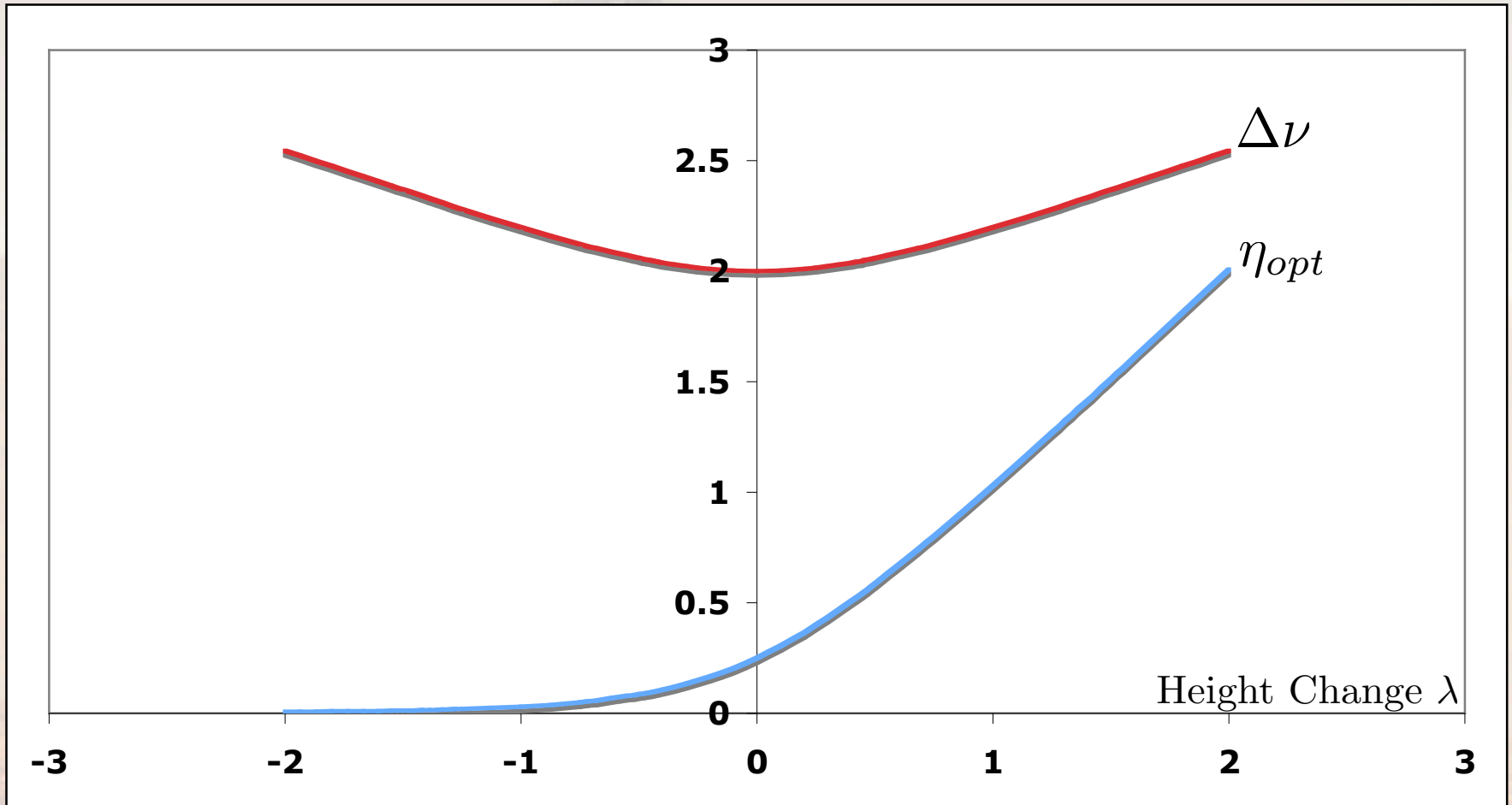
Optimization of Height-Changing Hop

- This is not going to be one where you can take the derivative and set equal to zero, so use the equation to find a numerical optimization
- Set $\lambda = 0$ to check for plain hop solution

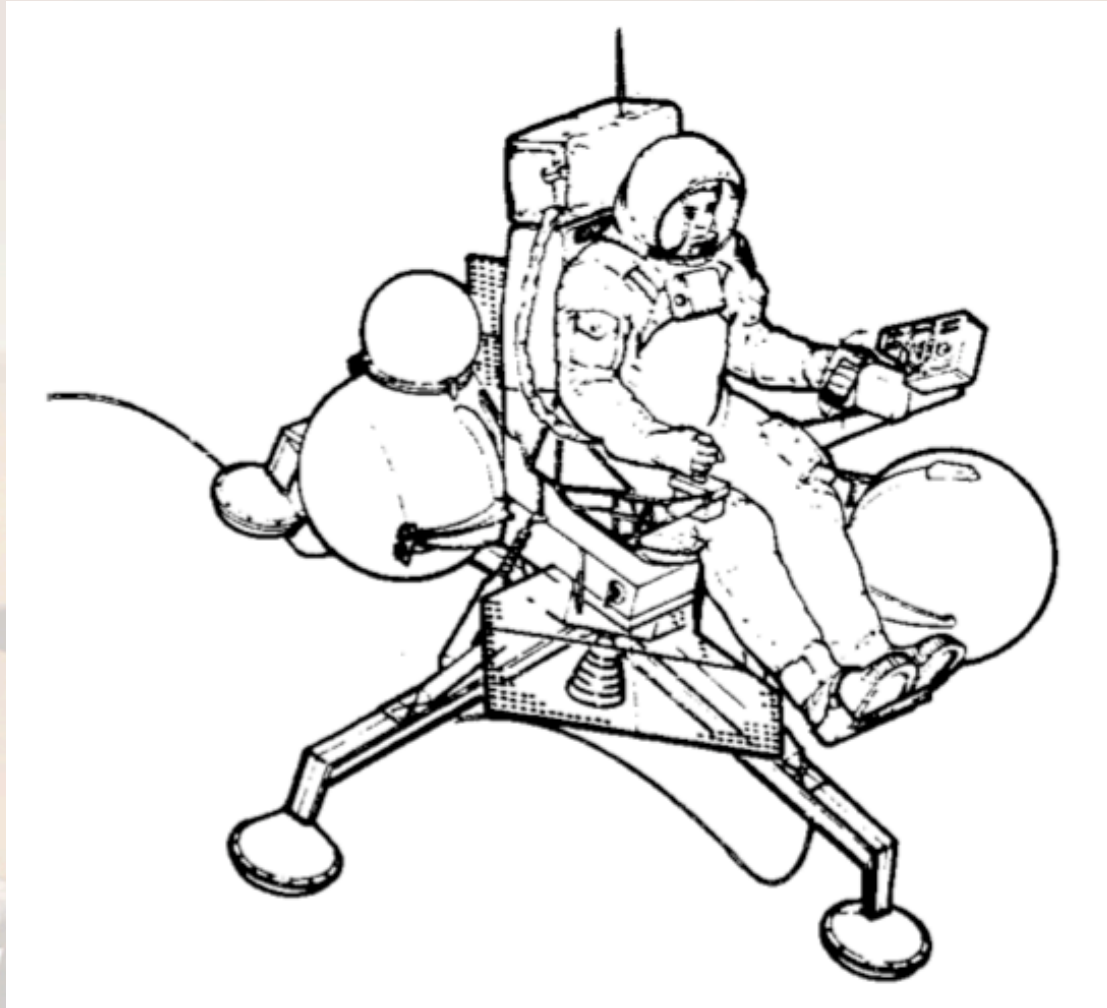
$$\Delta v = 2\sqrt{\frac{1}{8\eta} + 2\eta} \Rightarrow \eta_{opt} = \frac{1}{4}$$



Trajectory Design for Height Change



Apollo Concept of Lunar Flying Vehicle



from "Study of One-Man Lunar Flying Vehicle - Final Report Volume 1: Summary" North American Rockwell, NASA CR-101922, August 1969



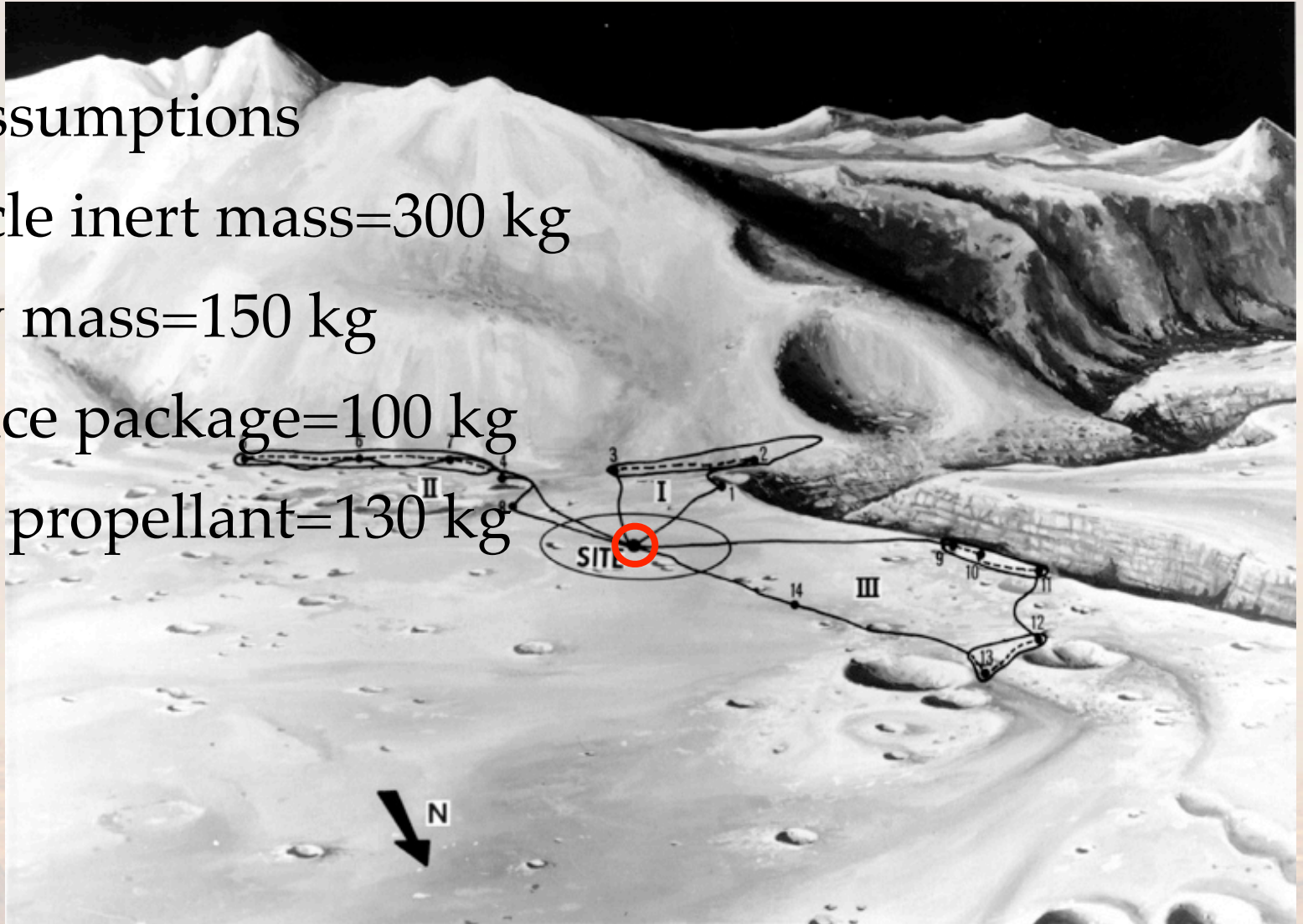
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Apollo 15 Revisited: LFV Sortie

Basic assumptions

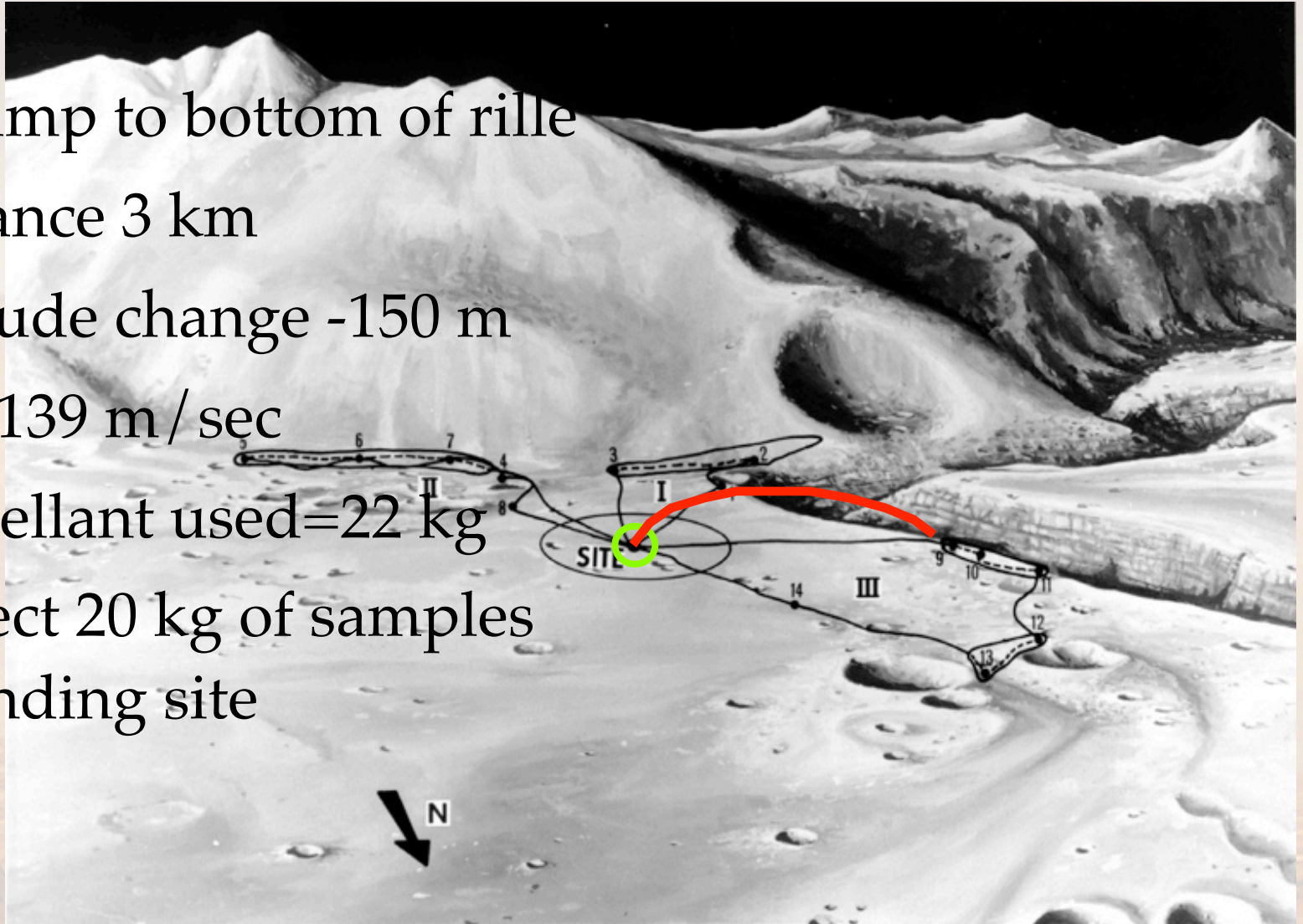
- Vehicle inert mass=300 kg
- Crew mass=150 kg
- Science package=100 kg
- Total propellant=130 kg



Apollo 15 Revisited: Leg 1

Base camp to bottom of rille

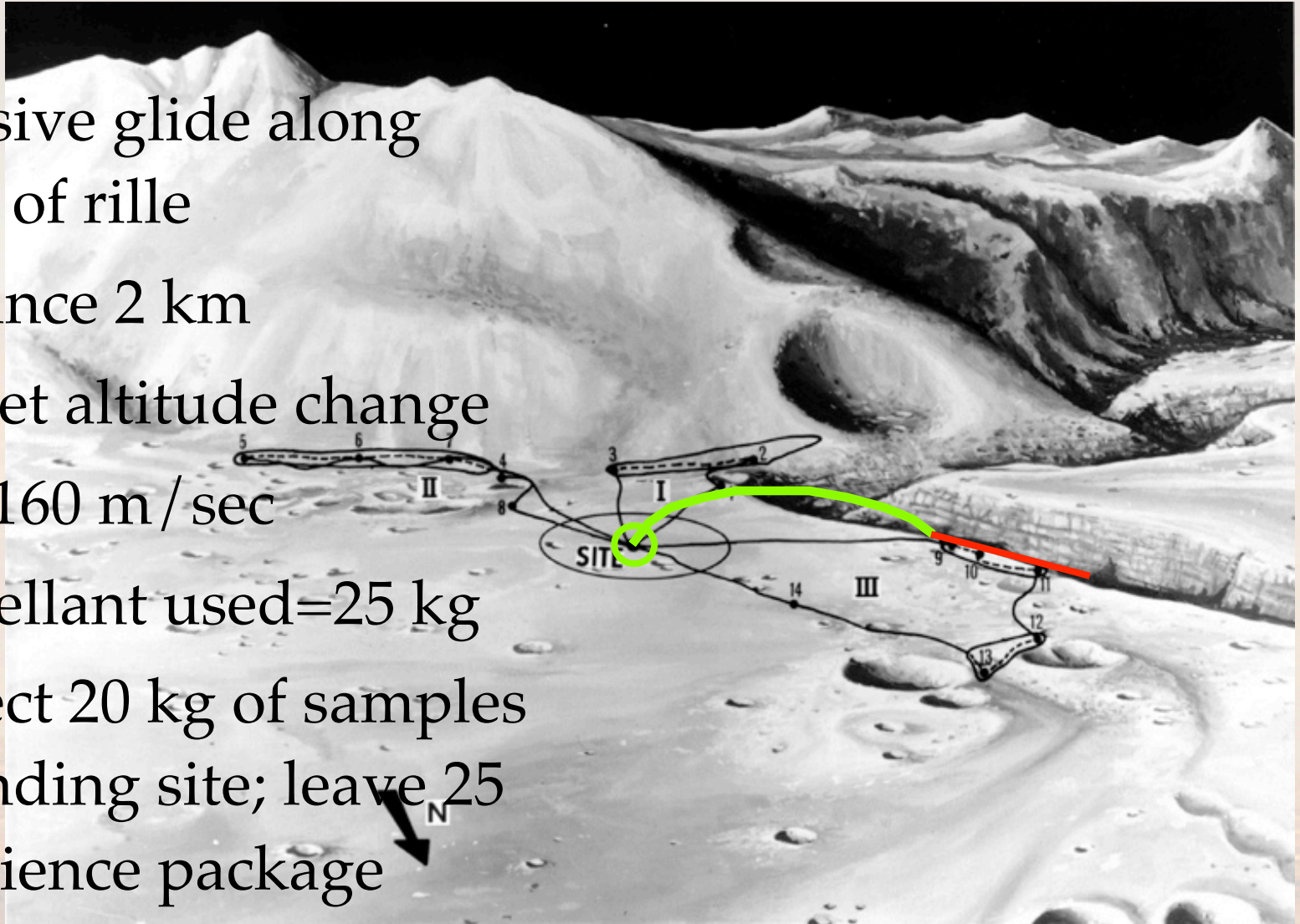
- Distance 3 km
- Altitude change -150 m
- $\Delta V = 139$ m/sec
- Propellant used = 22 kg
- Collect 20 kg of samples at landing site



Apollo 15 Revisited: Leg 2

Propulsive glide along bottom of rille

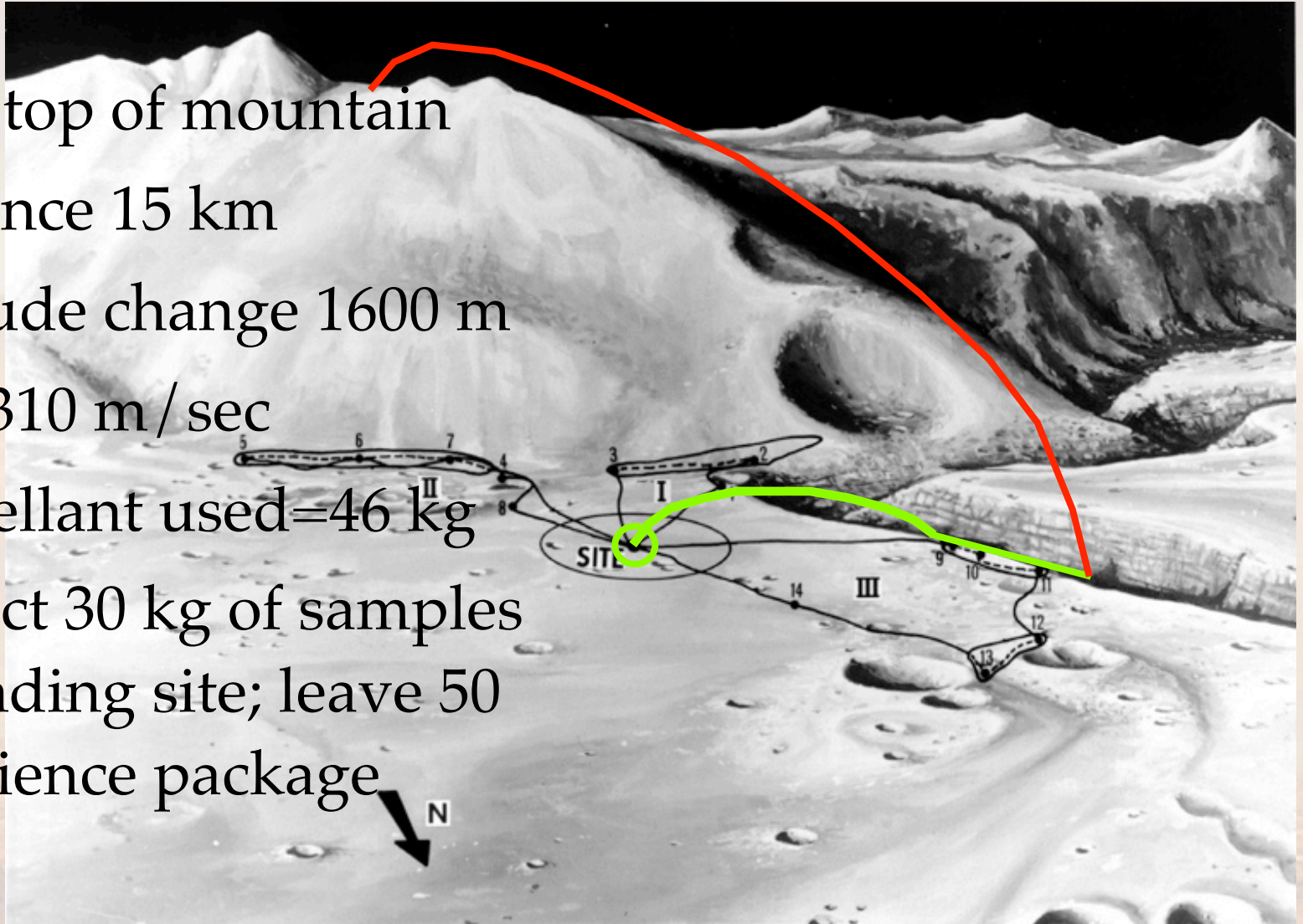
- Distance 2 km
- No net altitude change
- $\Delta V = 160$ m/sec
- Propellant used = 25 kg
- Collect 20 kg of samples at landing site; leave 25 kg science package



Apollo 15 Revisited: Leg 3

Hop to top of mountain

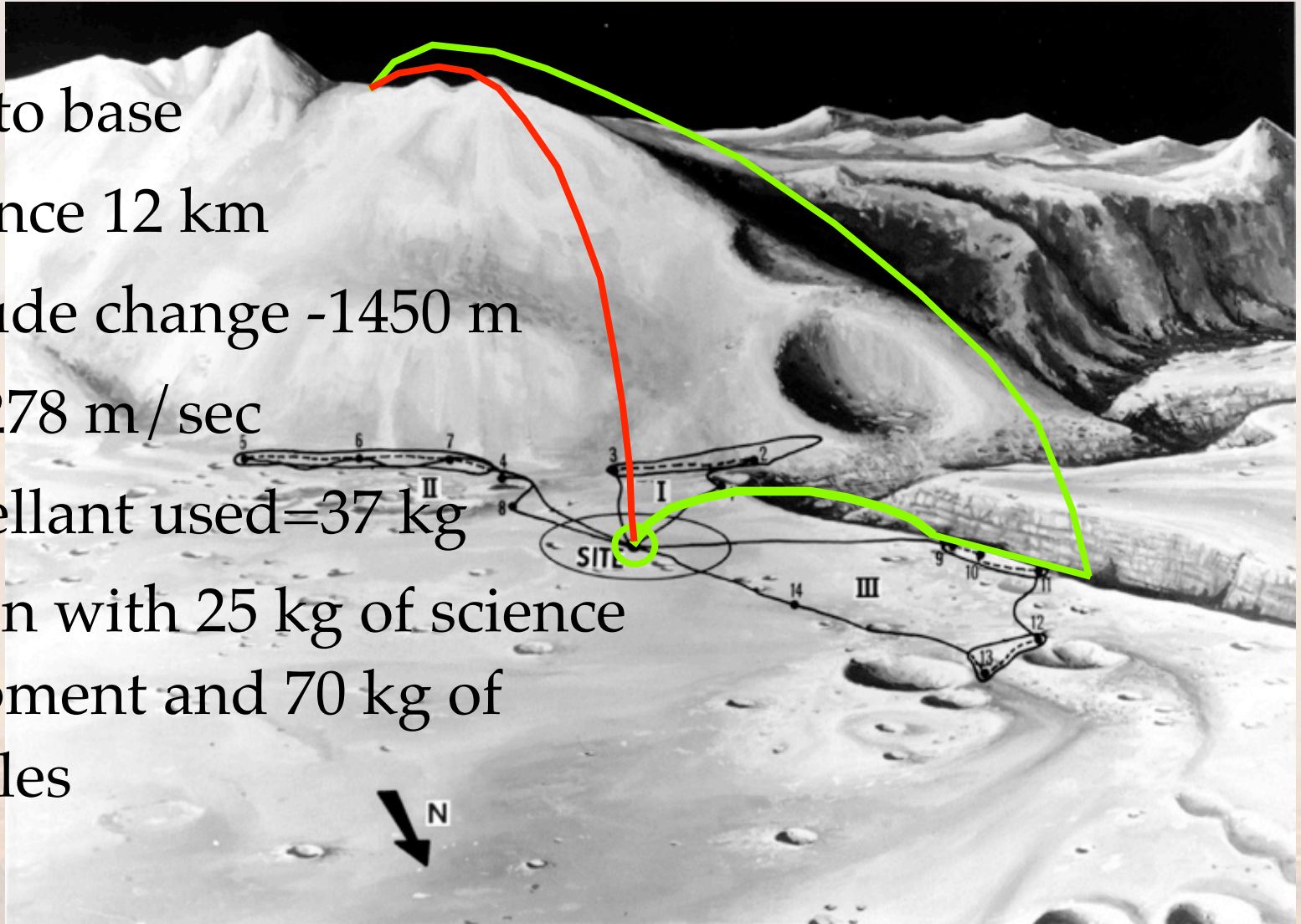
- Distance 15 km
- Altitude change 1600 m
- $\Delta V=310$ m/sec
- Propellant used=46 kg
- Collect 30 kg of samples at landing site; leave 50 kg science package



Apollo 15 Revisited: Leg 4

Return to base

- Distance 12 km
- Altitude change -1450 m
- $\Delta V=278$ m/sec
- Propellant used=37 kg
- Return with 25 kg of science equipment and 70 kg of samples



Apollo 15 Revisited: Discussion

- Current minimum estimates are for 400 kg of residual propellants in Altair at landing - would support three equivalent sorties
- Presence of water ice or ISRU propellant production at outpost would easily support moderate flier mission requirements
- Challenges in routine refueling of cryogenic propellants on the lunar surface, reliable flight and landing control system



Landing Impact Attenuation

- Cannot rely on achieving perfect zero velocity at touchdown
- Specifications for landing conditions
 - Vertical velocity ≤ 3 m/sec
 - Horizontal velocity ≤ 1 m/sec

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{1}{2}m(v_h^2 + v_v^2)$$

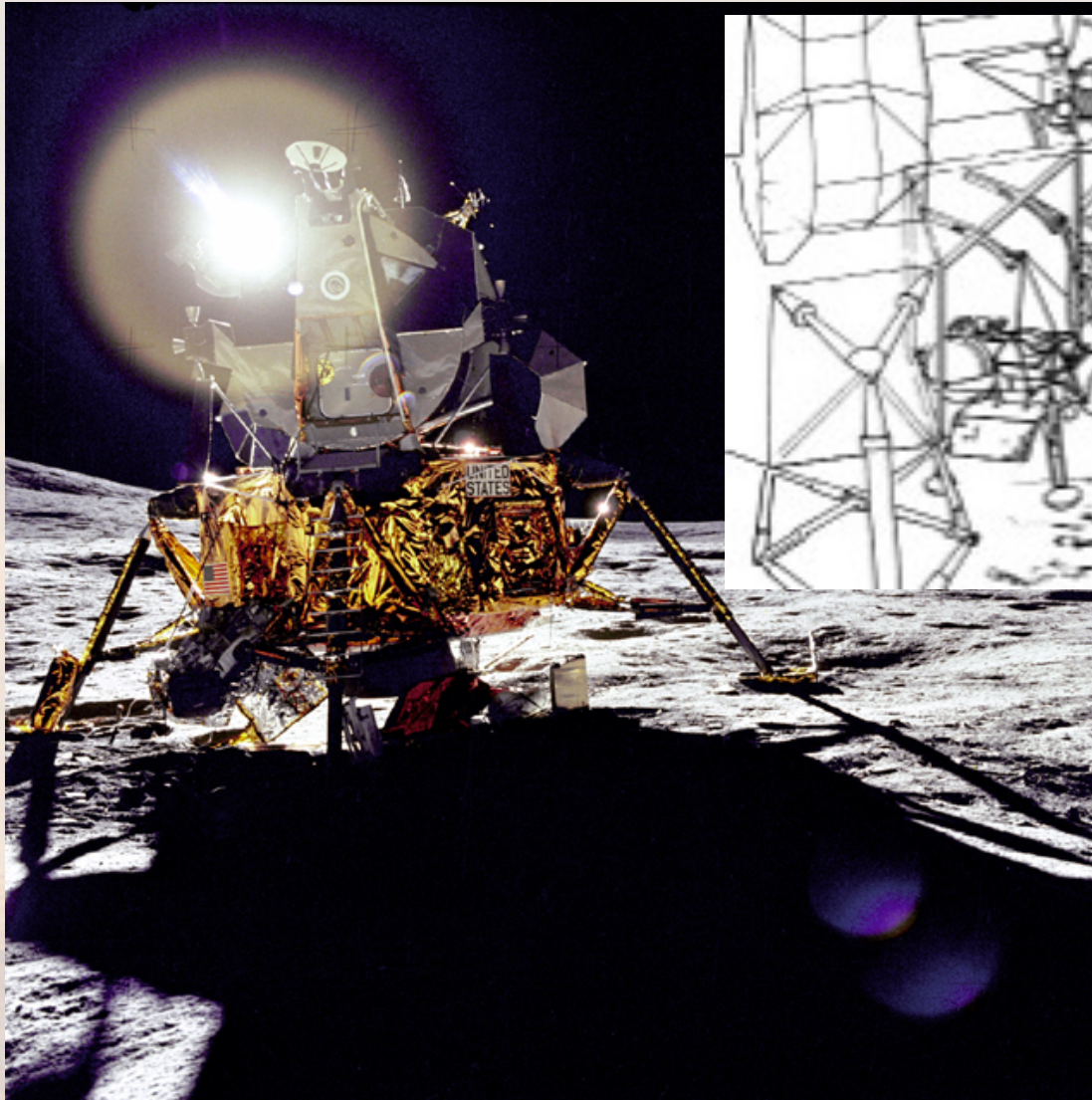
$$\text{Max case 500 kg vehicle} \implies E = 2500Nm$$



Mars Phoenix Lander



Apollo Lunar Module



Landing Deceleration

- Look at 3 m/sec vertical velocity
- Constant force deceleration

$$\frac{1}{2}mv^2 = Fd \quad \frac{1}{2}v^2 = \frac{F}{m}d = a_{desired}d \quad d = \frac{1}{2} \frac{v^2}{a_{desired}}$$

$$t_{decel} = \frac{v}{a_{desired}}$$

- Spring deceleration

$$F = kx \quad \int F dx = \frac{1}{2}mv^2$$

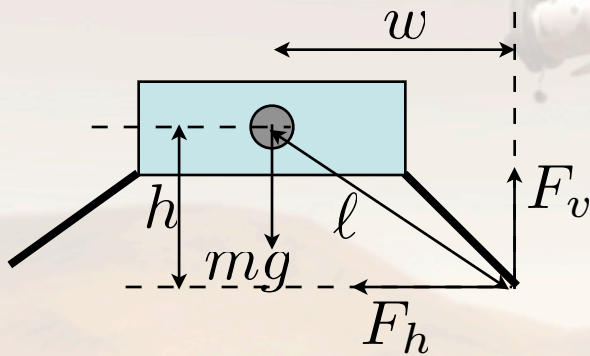
$$k = \frac{mv^2}{d^2} \quad a_{peak} = \frac{kd}{m}$$

$a_{desired}$	$d \langle cm \rangle$	$t_d \langle sec \rangle$
1/6 g	281	1.88
1/2 g	92	0.61
1 g	46	0.31
2 g	23	0.15
3 g	15	0.10



Effect of Lateral Velocity at Touchdown

- Resolve torques around landing gear footpad



$$\ddot{\theta} = \frac{\tau_{tot}}{I_{tot}}$$

$$\ddot{\theta} = \frac{F_h h - F_v w - mgw}{I_{cg} + m\ell^2}$$

- Worst cases - hit obstacle (high force), landing downhill
- Issue: rotational velocity induced is counteracted by vehicle weight
- Will vehicle rotation stop before overturn limit?

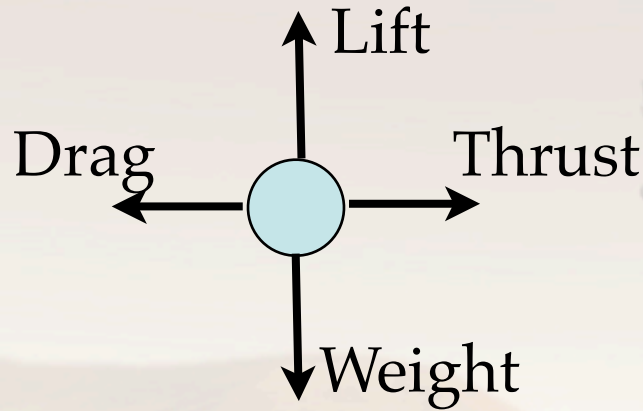


Atmospheric Neutral Buoyancy

- Given an enclosed volume V of gas with density ρ
- Lift force is $V(\rho_{\text{atm}} - \rho)$ - must be $\geq mg$
 - on Earth ~ 1 kg lift / cubic meter of He
 - on Mars ~ 10 gms lift / cubic meter of He
- Horizontal velocity at equilibrium is identical to wind speed
- Interior pressure generally identical to ambient (except for superpressure balloons)
- Can generate low density through choice of gas, heating



Dynamic Atmospheric Lift



$$D = \frac{1}{2} \rho v^2 S C_D$$

$$L = \frac{1}{2} \rho v^2 S C_L$$

For steady, level flight: $T = D$ $L = W$

$$W = L = D \frac{L}{D} = T \frac{L}{D}$$

$$L = \frac{1}{2} \rho v^2 S C_D \frac{L}{D}$$



Mars Atmosphere

$$\rho = 0.020 \frac{\text{kg}}{\text{m}^3}$$

$$T = 210 \text{ K}$$

$$g = 3.71 \frac{\text{m}}{\text{sec}^2}$$

$$R = 188.92 \frac{\text{J}}{\text{kg K}}$$

$$\gamma = 1.2941$$

$$\text{Speed of sound } a = \sqrt{\gamma RT} = 226.6 \frac{\text{m}}{\text{sec}}$$



Aircraft Flight Performance



- U-2 high-altitude spy plane
- Cruises at “70,000+ feet”
- $m=18,000$ kg
- $b=32$ m
- $S\sim 64$ m²

$$v_{stall} = \sqrt{\frac{mg}{S} \frac{2}{\rho C_{L(max)}}}$$

$$\text{U-2 } v_{stall(Mars)} = 228.4 \frac{m}{sec^2}$$



Looking for Equation for Range

$$\text{Efficiency} = \frac{\text{propulsive power}}{\text{fuel power}} = \frac{T v_e}{\dot{m}_f h}$$

$$\eta_{\text{overall}} = \frac{T v_e}{\dot{m}_f h} \quad \frac{dW}{dt} = -\dot{m}_f g = \frac{-W}{\frac{L}{D} \frac{T}{\dot{m}_f g}}$$

$$\frac{dW}{dt} = \frac{-W v_e}{\frac{h}{g} \frac{L}{D} \frac{T v_e}{\dot{m}_f h}} = \frac{-W v_e}{\frac{h}{g} \frac{L}{D} \eta_{\text{overall}}}$$



More Aerial Range

Rewrite and integrate

$$\frac{dW}{W} = \frac{-v_e dt}{\frac{h}{g} \frac{L}{D} \eta_{overall}} \implies \ln W = C - \frac{-v_e t}{\frac{h}{g} \frac{L}{D} \eta_{overall}}$$

Initial conditions - at $t = 0$ $W = W_{init} \rightarrow C = \ln W_{init}$

$$\text{Range} = \frac{h}{g} \frac{L}{D} \eta_{overall} \ln \frac{W_{init}}{W_{final}}$$

$$\text{Range} = \frac{V \frac{L}{D}}{g \text{ SFC}} \ln \frac{W_{init}}{W_{final}}$$

-->Breguet Range Equation

