Terramechanics

- Origin and nature of lunar soil
- Soil mechanics
- Rigid wheel mechanics

Notes about Revised Course Schedule

- No class next week (9/11 and 9/13)
- Makeup lectures to be announced

Lunar Regolith

- Broken down from larger pieces over time
- Major constituents
  - Rock fragments
  - Mineral fragments
  - Glassy particles
- Local environment
  - 10^{-12} torr (= 1.22×10^{-10} Pa = 1.93×10^{-14} psi)
  - Meteorites at velocities >10^5 m/sec
  - Galactic cosmic rays, solar particles
  - Temperature range +250°F – -250°F

Regolith Creation Process

- Only "weathering" phenomenon on the moon is meteoritic impact!
- Weathering processes
  - Comminution: breaking rocks and minerals into smaller particles
  - Agglutination: welding fragments together with molten glass formed by impact energy
  - Solar wind spallation and implantation (miniscule)
  - Fire fountaining (dormant)

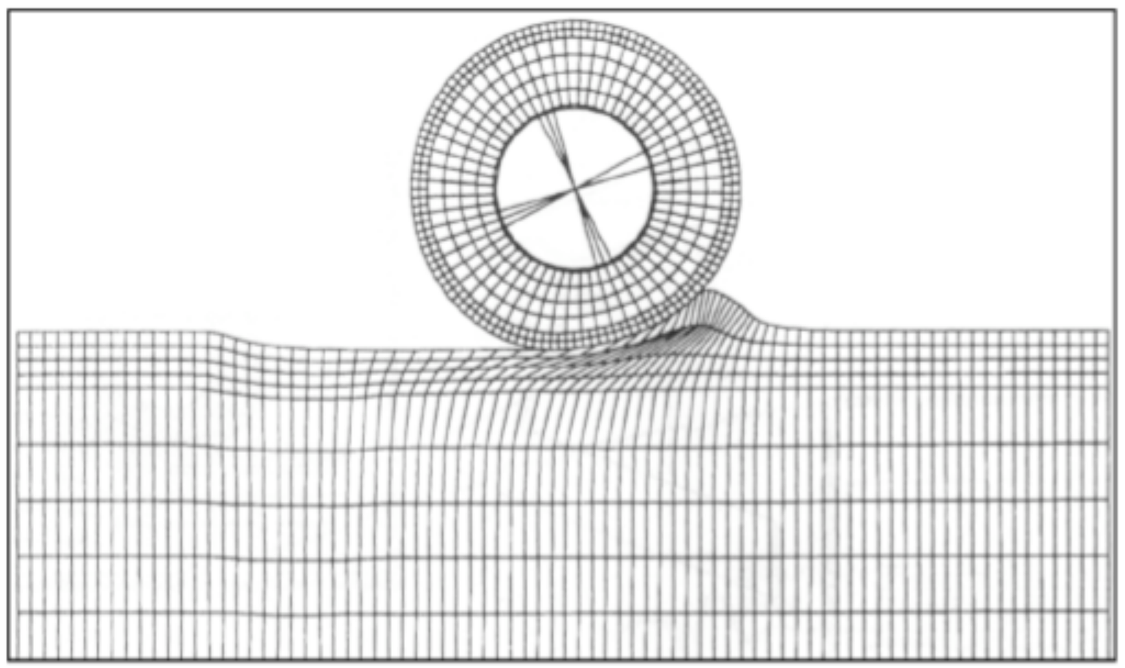
JSC-1 Simulant

- Ash vented from Merriam Crater in San Francisco volcano field near Flagstaff, AZ
- K-Ar dated at 150,000 years old ± 30,000
- Major constituents SiO_2, TiO_2, Al_2O_3, Fe_2O_3, FeO, MgO, CaO, Na_2O, other <1%
- Represents low-Ti regolith from lunar mare
- MLS-1 simulant (U.Minn.) preferred for simulation of highland material

Wheel-Soil Interaction

Wheel rolling over soil does work

- Compression
- “Bulldozing”



from Gibbesch and Schafer, “Advanced and Simulation Methods of Planetary Rover Mobility on Soft Terrain” *8th ESA Workshop on Advanced Space Technologies for Robotics and Automation*, Noordwijk, The Netherlands, November, 2004

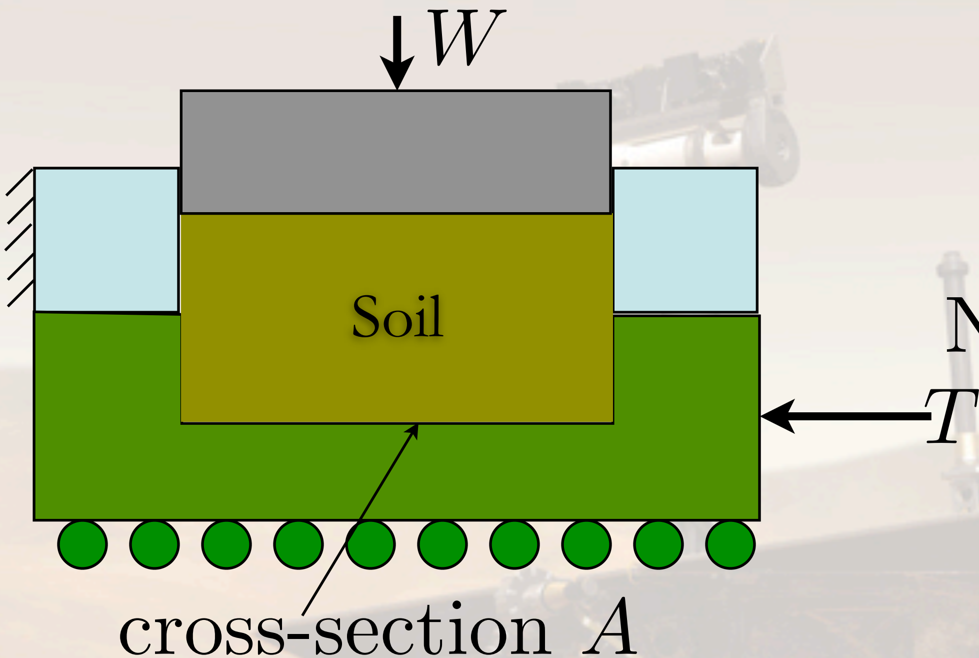
Soil Testing Apparatus

Bevameter (force vs. displacement)

Internal friction angle φ

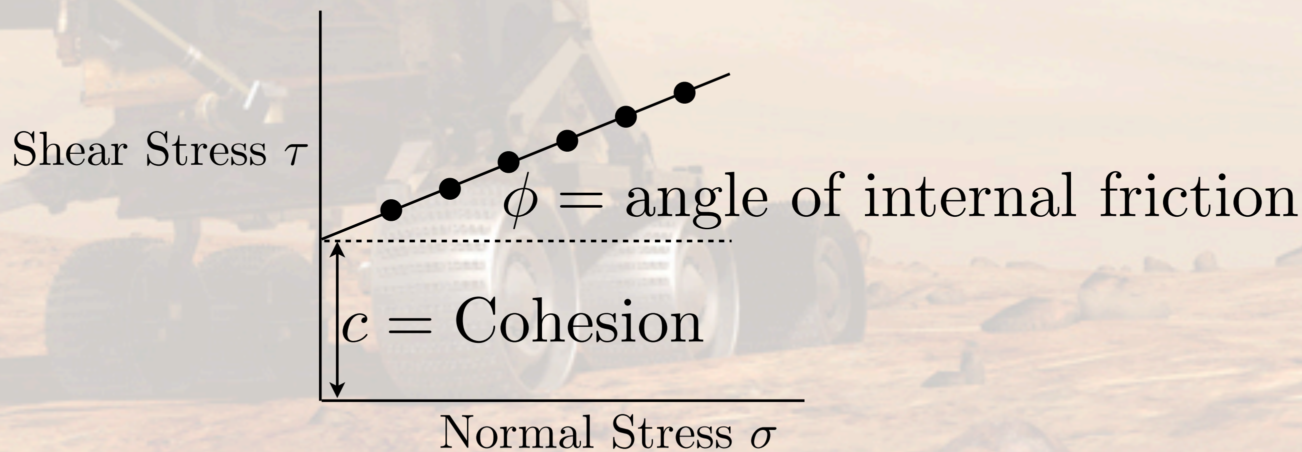
Shear deformation modulus K

Soil Characterization – Direct Shear



$$\text{Shear Stress } \tau = \frac{T}{A}$$

$$\text{Normal Stress } \sigma = \frac{W}{A}$$



Modeling Soil Reaction to a Wheel

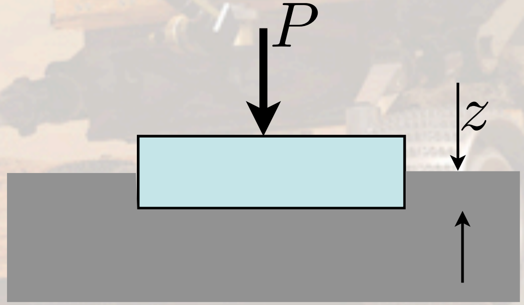
Assume soil reaction is like a (nonlinear) spring

$$P = kz^n$$

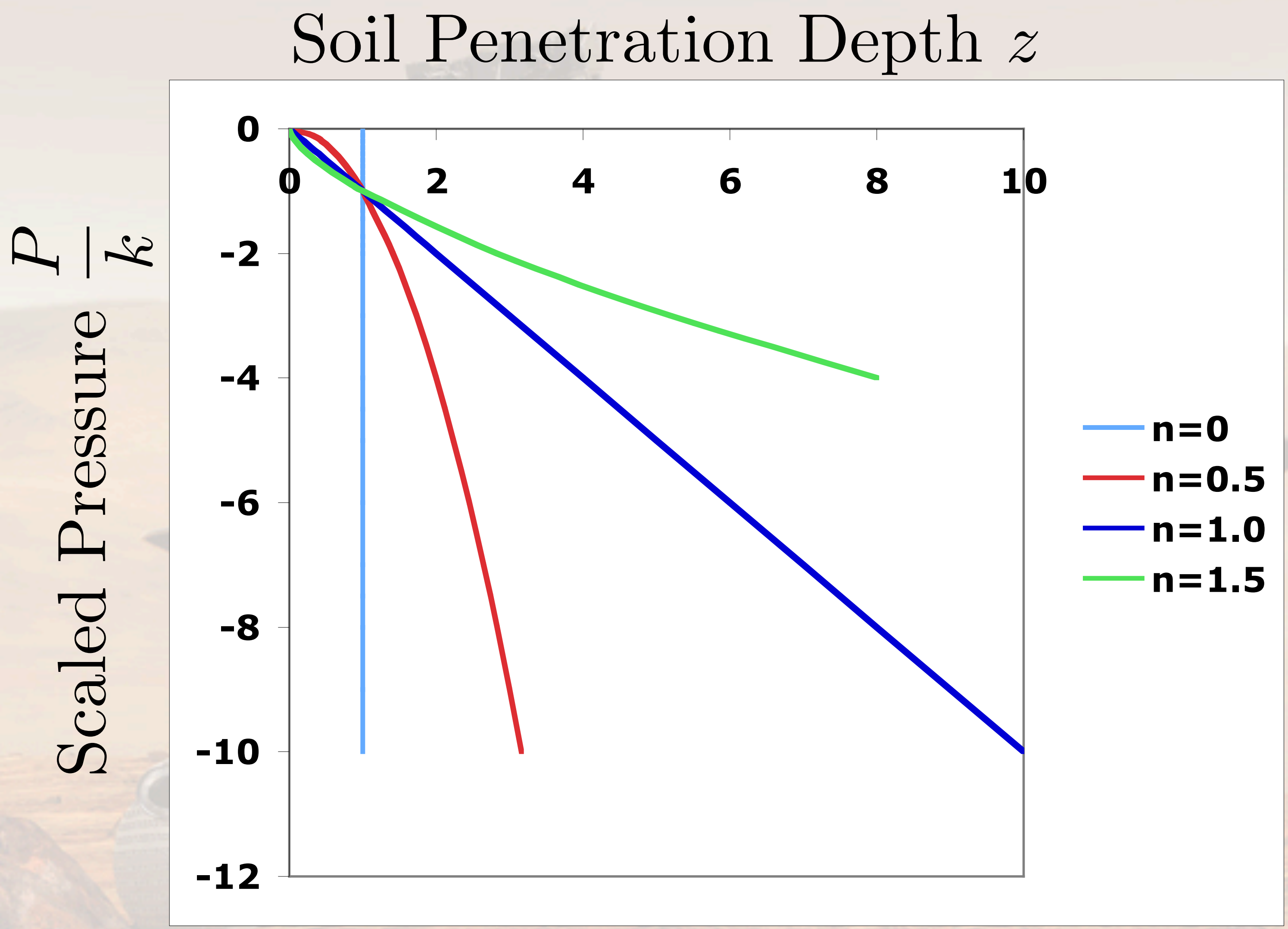
$$P = \text{applied pressure}$$

$$z = \text{compression depth}$$

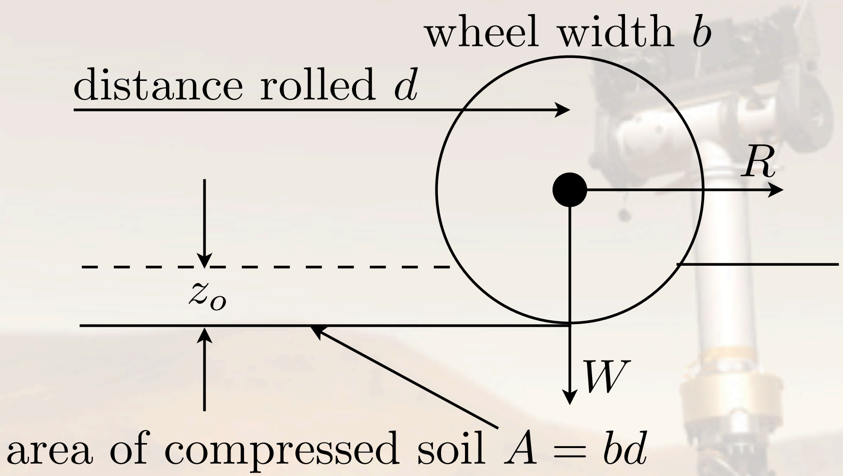
$$k, n = \text{heuristic parameters}$$



Effects of Soil Mechanics



Wheel-Soil Interactions



$$\text{Displacement Energy } \frac{E}{A} = \int \frac{F}{A} dz = \int P dz$$

$$\frac{E}{A} = \int_0^{z_o} P dz = \int_0^{z_o} k z^n dz = k \frac{z_o^{n+1}}{n+1}$$

Rolling Resistance

$$\text{Total Energy } \frac{E}{A}A = \frac{E}{A}bd = k\frac{z_o^{n+1}}{n+1}bd$$

Given a force resisting rolling $\equiv R$,

the energy required to roll a distance d is

$$E_{roll} = Rd$$

$$E_{roll} = E_{displacement} \Rightarrow Rd = \frac{E}{A}bd$$

Rolling Resistance

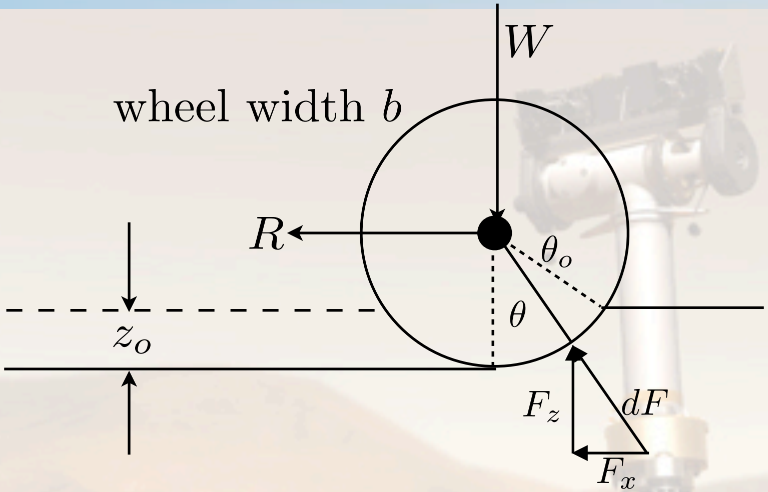
$$\text{For } n = 1 : P = kz; \frac{E}{A} = k\frac{z_o^2}{2}; R = \frac{1}{2}kbz_o^2$$

$$\text{For } n = \frac{1}{2} : P^2 = k^2 z; \frac{E}{A} = \frac{2}{3}kz_o^{\frac{3}{2}}; R = \frac{2}{3}kbz_o^{\frac{3}{2}}$$

$$\text{For } n = 0 : P = k; \frac{E}{A} = kz_o; R = kbz_o$$

$$\text{Generic case: } P = kz^n; \frac{E}{A} = k\frac{z_o^{n+1}}{n+1}; R = kb\frac{z_o^{n+1}}{n+1}$$

Soil Displacement Calculations



$$R - \int_0^{\theta_o} dF \sin\theta = 0$$

$$-W + \int_0^{\theta_o} dF \cos\theta = 0$$

$$dF = Pb \qquad dF\cos\theta = -Pb\ dx$$

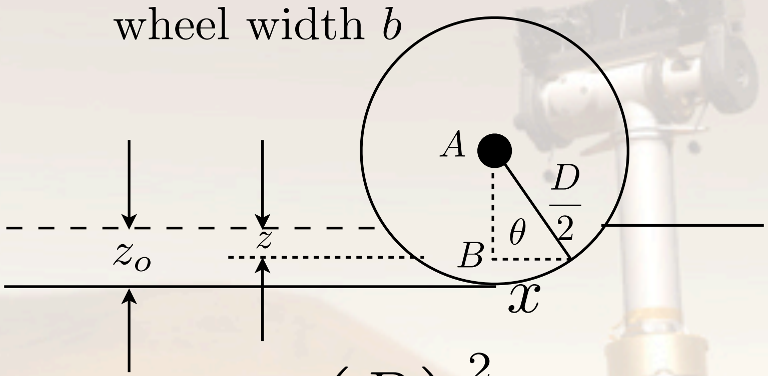
$$dF \sin\theta = Pb\ dz$$

$$R = \int_0^{\theta_o} Pb\ dz \qquad W = -\int_0^{\theta_o} Pb\ dx$$

In general, $P = kx^n$

$$W = -\int_0^{z_o} bkz^n dx$$

Soil Displacement Calculations



$$\bar{AB} = \frac{D}{2} - (z_o - z)$$

$$x^2 = \left(\frac{D}{2}\right)^2 - \bar{AB}^2 = \left(\frac{D}{2}\right)^2 - \left[\frac{D}{2} - (z_o - z)\right]^2$$

$$= \left(\frac{D}{2}\right)^2 - \left(\frac{D}{2}\right)^2 + 2\frac{D}{2}(z_o - z) - (z_o - z)^2$$

$$x^2 = [D - (z_o - z)](z_o - z)$$

Soil Compression Calculations

But $D \gg z_o - z$

$$x^2 \approx D(z_o - z) \Rightarrow 2x\ dx = -D\ dz$$

$$\text{so from } W = -\int_0^{z_o} bkz^n dx \text{ we get } W = -\int_0^{z_o} bkz^n \frac{-D}{2x} dz$$

$$W = -bk \int_0^{z_o} z^n \left(\frac{-D}{2\sqrt{D}\sqrt{z_o - z}} \right) dz$$

$$W = bk \int_0^{z_o} z^n \left(\frac{\sqrt{D} dz}{2\sqrt{z_o - z}} \right) dz$$

Soil Displacement Calculations

$$\text{Define } z_o - z \equiv t^2 \Rightarrow dz = -2t\,dt$$

$$W = bk\sqrt{D}\int_0^{\sqrt{z_o}} (z_o - t^2)^n\,dt$$

$$\text{Taylor Series expansion } (z_o - t^2)^n \cong z_o^n - n z_o^{n-1} t^2 + \cdots$$

$$W \approx \frac{bk\sqrt{Dz_o}}{3} z_o^n (3 - n)$$

$$\text{for } n = 1 \Rightarrow W = \frac{2}{3} bk z_o \sqrt{Dz_o}$$

$$\text{for } n = \frac{1}{2} \Rightarrow W = \frac{5}{6} bk z_o \sqrt{D}$$

$$\text{for } n = 0 \Rightarrow W = bk\sqrt{Dz_o}$$

Rolling Resistance as f(W)

$$\text{for } n = 0 \Rightarrow W = bk\sqrt{Dz_o} \Rightarrow z_o = \left(\frac{W}{bk}\right)^2 \frac{1}{D}$$

$$R = kbz_o \Rightarrow R = \frac{kb}{(kb)^2}\frac{W^2}{D} \Rightarrow R = \frac{W^2}{kbD}$$

$$\text{for } n = \frac{1}{2} \Rightarrow W = \frac{5}{6}bkz_o\sqrt{D} \Rightarrow z_o = \frac{6}{5}\frac{W}{bk\sqrt{D}}$$

$$R = \frac{2}{3}kbz_o^{\frac{3}{2}} \Rightarrow R = \frac{2}{3}kb\left(\frac{6}{5}\frac{W}{kb\sqrt{D}}\right)^{\frac{3}{2}} = \frac{2}{3}\left(\frac{6}{5}\right)^{\frac{3}{2}}\frac{W^{\frac{3}{2}}}{\sqrt{kb}D^{\frac{3}{4}}}$$

$$R = 0.876\frac{W^{\frac{3}{2}}}{\sqrt{kb}D^{\frac{3}{4}}}$$

Rolling Resistance as f(W)

$$\text{for } n = 1 \Rightarrow W = \frac{2}{3} b k z_o^{\frac{3}{2}} \sqrt{D} \Rightarrow z_o^2 = \left(\frac{3W}{2kb\sqrt{D}} \right)^{\frac{4}{3}}$$

$$R = \frac{1}{2} k b z_o^2 \Rightarrow R = \frac{1}{2} k b \left(\frac{3W}{2kb\sqrt{D}} \right)^{\frac{4}{3}} = \frac{1}{2} \left(\frac{3}{2} \right)^{\frac{4}{3}} \left(\frac{W^4}{kbD^2} \right)^{\frac{1}{3}}$$

$$R = 0.859 \left(\frac{W^4}{kbD^2} \right)^{\frac{1}{3}}$$

Rolling Resistance as f(W) (Generic)

$$W = \frac{bk\sqrt{Dz_o}}{3} z_o^n (3-n) = \frac{bk\sqrt{D}}{3} z_o^{n+\frac{1}{2}} (3-n)$$

$$z_o^{n+\frac{1}{2}} = \frac{3}{(3-n)} \frac{W}{bk\sqrt{D}}$$

$$z_o^{n+1} = \left(\frac{3}{3-n} \frac{W}{bk\sqrt{D}} \right)^{\frac{n+1}{n+\frac{1}{2}}} = \left(\frac{3}{3-n} \frac{W}{bk\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}}$$

$$R = \frac{bk}{n+1} z_o^{n+1} = \frac{bk}{n+1} \left(\frac{3}{3-n} \frac{W}{bk\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}}$$

$$R = \frac{1}{n+1} \left(\frac{3}{3-n} \frac{W}{\sqrt{D}} \right)^{\frac{2(n+1)}{2n+1}} \left(\frac{1}{bk} \right)^{\frac{1}{2n+1}}$$

More Detailed Soil Compression Equation

$$k = \frac{k_c}{b} + k_\phi$$

$$k_c = \text{modulus of cohesion of soil deformation}$$

$$k_c \text{ units} \Rightarrow < N/m^{(n+1)} >$$

$$k_\phi = \text{modulus of friction of soil deformation}$$

$$k_\phi \text{ units} \Rightarrow < N/m^{(n+2)} >$$

$$b = \text{wheel width}$$

$$P = \left(\frac{k_c}{b} + k_\phi \right) z^n$$

Soil Characteristics

soil type	n	$k_c \ \langle \frac{N}{m^{n+1}} \rangle$	$k_\phi \ \langle \frac{N}{m^{n+2}} \rangle$
Dry Sand	1.1	990	1,528,000
Lunar Regolith	1.0	1400	820,000
Sandy Loam	0.7	5270	1,515,000
Sandy Loam (MER-B)	1.0	28,000	7,600,000
Slope Soil (MER-B)	0.8	6800	210,000
Clay (Earth)	0.5	13,190	692,200

Equations for Compression Resistance

$$z = \left(\frac{3W_w}{(3-n)bk\sqrt{d}} \right)^{\frac{2}{2n+1}}$$

$$W_w = \text{weight on wheel}$$

$$d = \text{wheel diameter}$$

$$R_c = \left(\frac{bk}{n+1} \right) z^{n+1}$$

$$R_c = \text{compression resistance (per wheel)}$$

Soil Compression – Reece Formulation

$$P = \left(\frac{k_c}{b} + k_\phi \right) z^n$$

Problem is that k_c and k_ϕ have variable dimensions, based on n

$$k_c \text{ units} \Rightarrow < N/m^{(n+1)} >$$

$$k_\phi \text{ units} \Rightarrow < N/m^{(n+2)} >$$

Reece Formulation: nondimensionalize by b

Compression Resistance (Lunar Soil)

$$R_c = \frac{1}{n+1}\left(k_c + bk_\phi\right)^{\frac{-1}{2n+1}}\left(\frac{3W_w}{(3-n)\sqrt{d}}\right)^{\frac{2(n+1)}{2n+1}}$$

$$n = 1$$

$$k_c = 0.14\ N/cm^2$$

$$k_\phi = 0.827\ N/cm^3$$

$$R_c = \frac{1}{2}\left(k_c + bk_\phi\right)^{\frac{-1}{3}}\left(\frac{3W_w}{2\sqrt{d}}\right)^{\frac{4}{3}}$$

Apollo Lunar Roving Vehicle Example

$$z = \left(\frac{3 * 253}{2(0.14 + 17.4 * 0.827)\sqrt{82}} \right)^{\frac{2}{3}} = 2.03\ cm$$

$$R_c = \frac{1}{2}\left(0.14 + 17.4 * 0.827\right)^{\frac{-1}{3}} \left(\frac{3 * 253}{2\sqrt{82}} \right)^{\frac{4}{3}} = 29.8\ N$$

check units -

$$\left(\frac{N^{-1/3}}{cm^{-2/3}} \right) \left(\frac{N^{4/3}}{cm^{2/3}} \right) = N$$

Rolling and Gravitation Resistance

- Rolling resistance (tires, bearings, etc.)

$$R_r = W_v c_f$$

$$W_v = \text{weight of vehicle}$$

$$c_f = \text{coefficient of friction (typ. 0.05)}$$

- Gravitational resistance

$$R_g = W_v \sin\theta_{slope}$$

- LRV examples (15° slope)

$$R_r = 51\ N \qquad R_g = 262\ N$$

Bulldozing Resistance

$$R_b = \frac{b\sin(\alpha+\phi)}{2\sin\alpha\cos\phi}\left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\pi \ell_o^3 \gamma (90-\phi)}{540} + \frac{c\pi\ell_o^2}{180}\tan\left(45+\frac{\phi}{2}\right)$$

$$\alpha = \text{angle of attack of wheel in soil} \equiv \cos^{-1}\left(1-\frac{2z}{D}\right)$$

$$\gamma = \text{density of soil}\left\langle \frac{\text{kg}}{\text{m}^3}\right\rangle$$

$$\ell_o = \text{length of soil rupture} \equiv z\tan^2\left(45-\frac{\phi}{2}\right)$$

Bulldozing Resistance

- “Bulldozing” is the process of pushing soil up ahead of the wheel
- Ranges from a small factor to a huge one, depending on soil and wheel factors
- Will be covered in detail in a later lecture

Tractive Force per Wheel (No Grousers)

$$H = [AC_b + W_w \tan\phi_b]\left[1 - \frac{K}{\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right]$$

$A =$ area of contact

$C_b =$ coefficient of soil/wheel cohesion

$\phi_b =$ wheel/soil friction angle

$s =$ wheel slip ratio

$K =$ coefficient of soil slip

$\ell =$ length of contact patch

Tractive Force per Wheel (With Grousers)

$$H = \left[b\ell C_b \left(1 + \frac{2h}{b}\right) N_g + W \tan\phi_b \left(1 + 0.64 \frac{h}{b} \arctan\frac{b}{h}\right)\right] \left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$

$A = \text{area of contact} \cong b\ell$

$C_b = \text{soil/wheel cohesion} \;= 0.017\ N/cm^2$

$\phi_b = \text{wheel/soil friction angle} \;= 35°$

$s = \text{wheel slip ratio (typ. 0.02-0.05)}$

$K = \text{coefficient of soil slip} \;= 1.8 \text{ cm}$

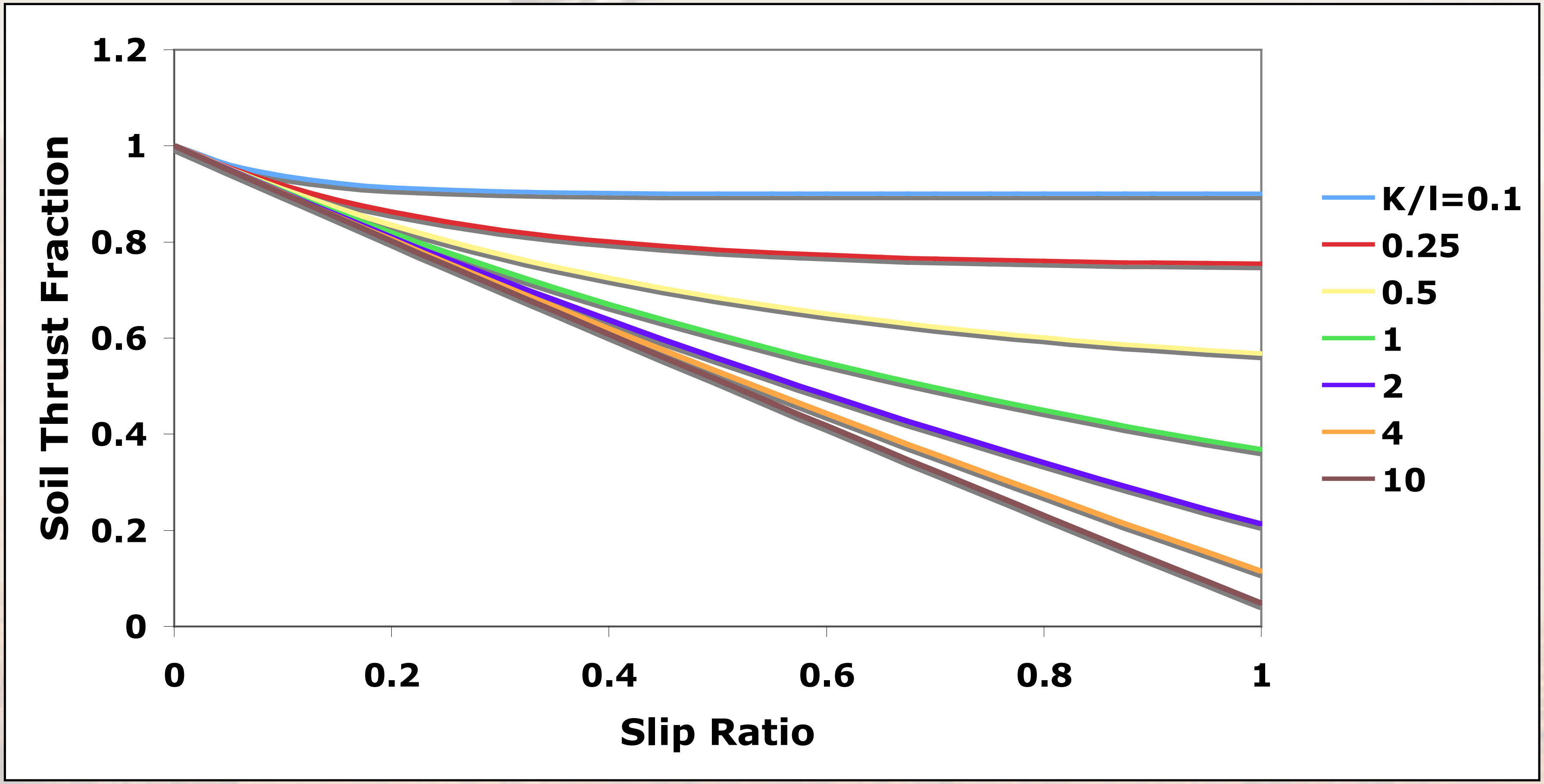
$\ell = \text{length of contact patch} = \frac{D}{2}\cos^{-1}\left(1 - \frac{2z}{D}\right)$

$h = \text{height of grouser}$

All values typical for lunar soil

Effect of Soil Thrust Fraction

$$\text{Soil Thrust Fraction} \quad \left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$



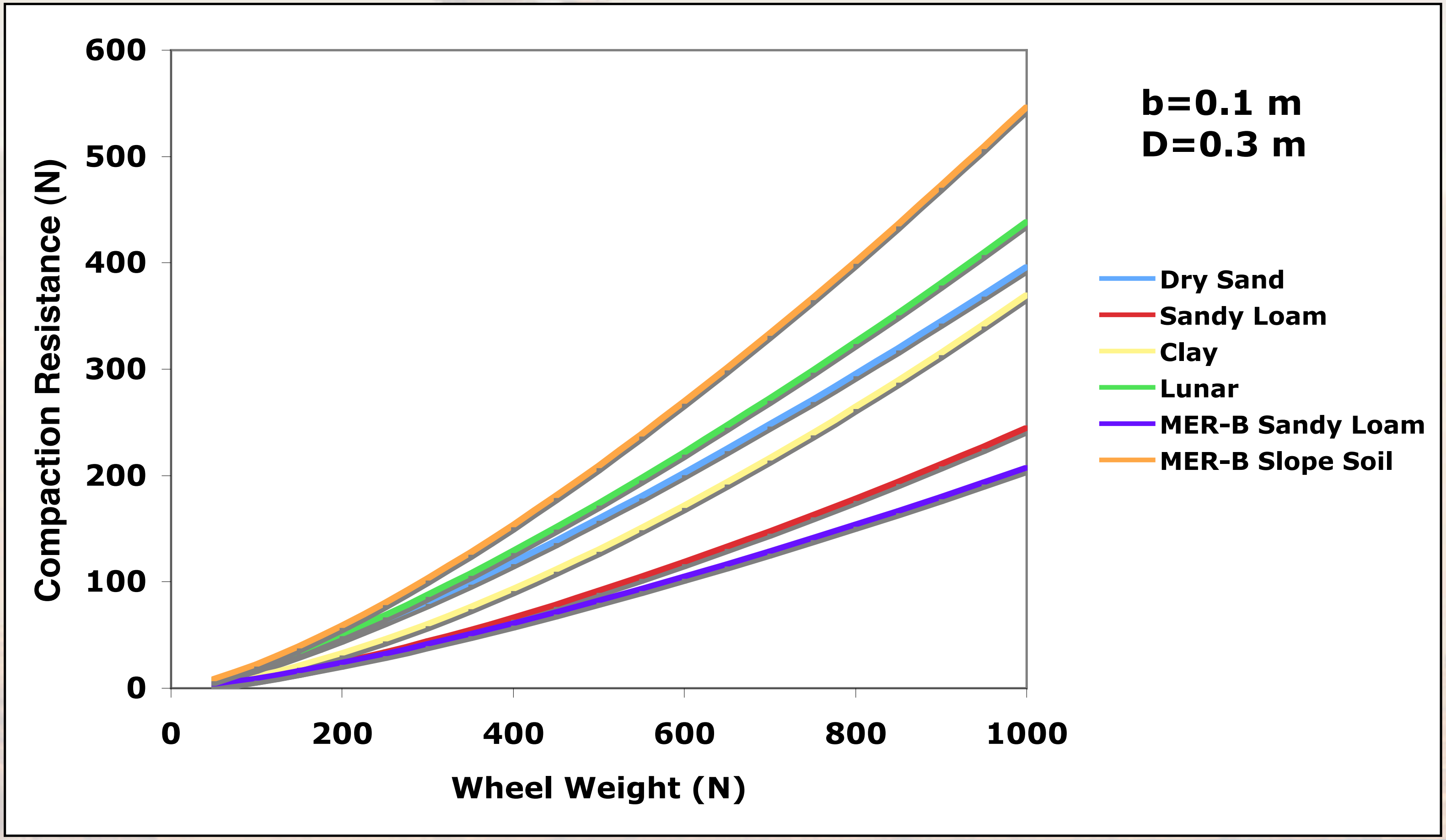
Basic Equation of Vehicle Propulsion

$$DP = H - (R_c + R_b + R_g + R_r)$$

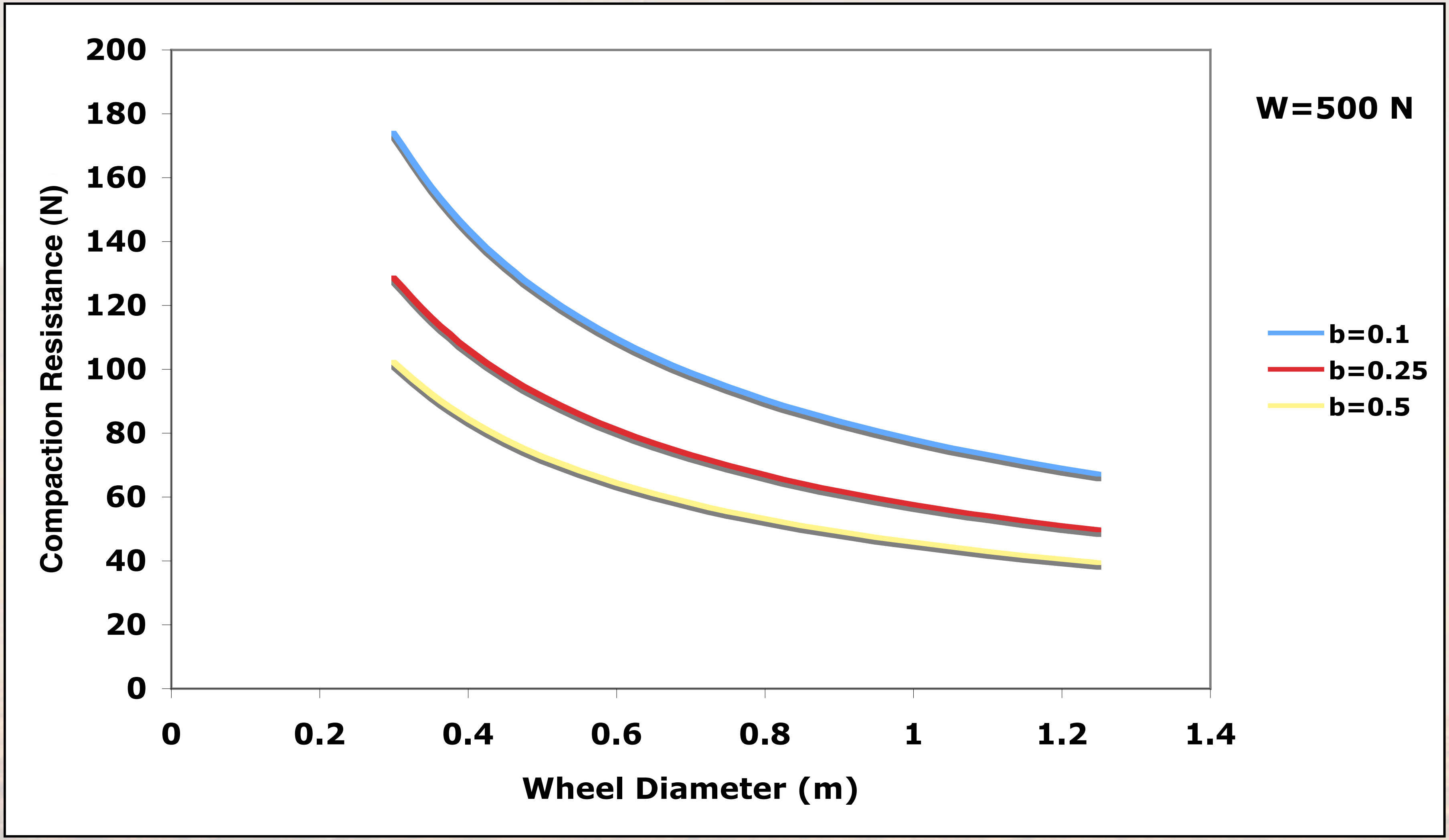
- DP: Drawbar pull (residual drive force)
- H: Maximum tractive force of wheels
- R_c: Compaction resistance
- R_b: Bulldozing resistance
- R_g: Gravitational resistance
- R_r: Rolling resistance (internal)

Example: Wheelbarrow (Single) Wheel

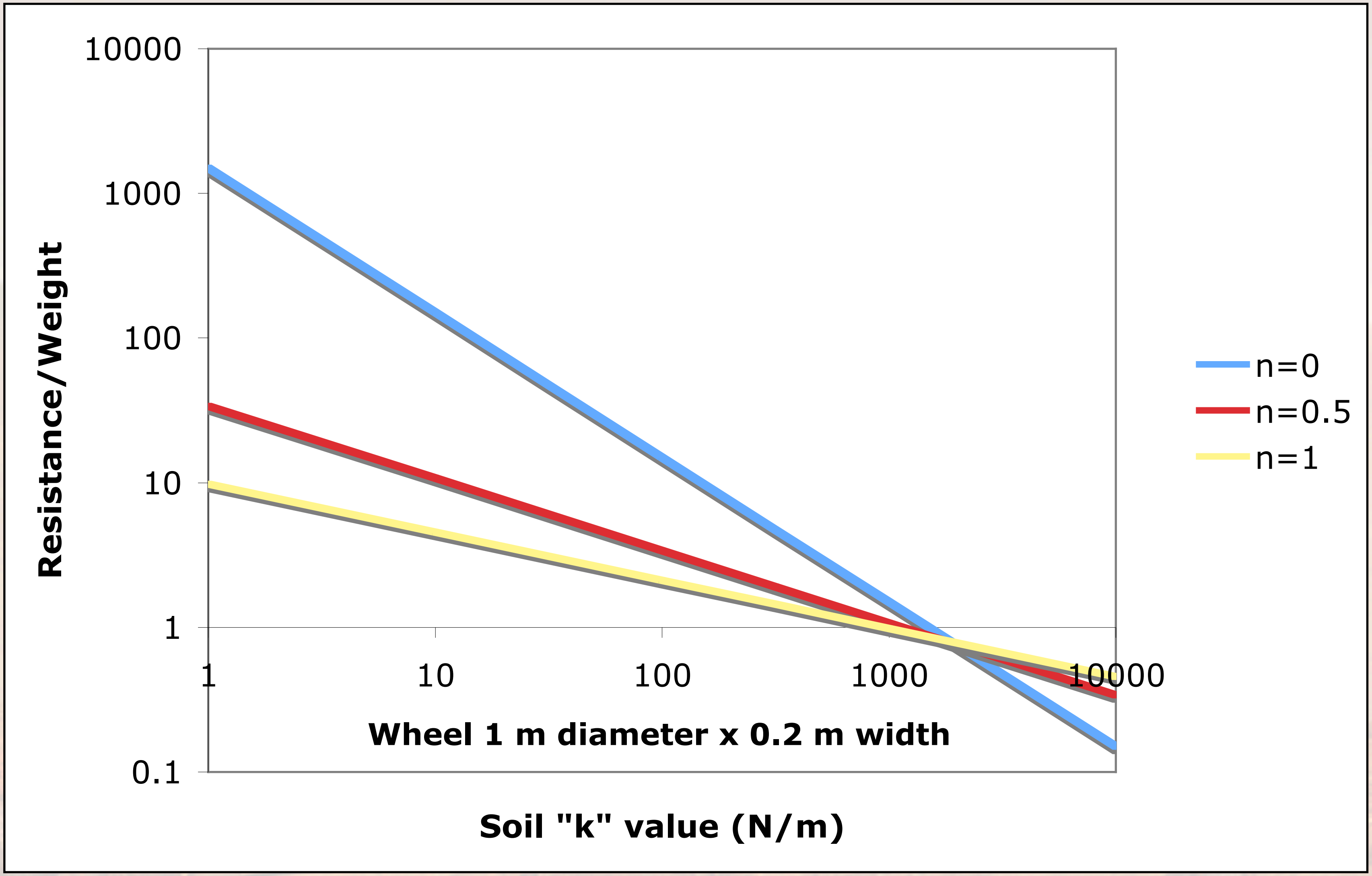
$$R = (k_c + k_\phi b)^{\frac{-1}{2n+1}} W^{\frac{2(n+1)}{2n+1}} \frac{1}{n+1} \left(\frac{3}{3-n} \right)^{\frac{2(n+1)}{2n+1}} D^{\frac{-(n+1)}{2n+1}}$$



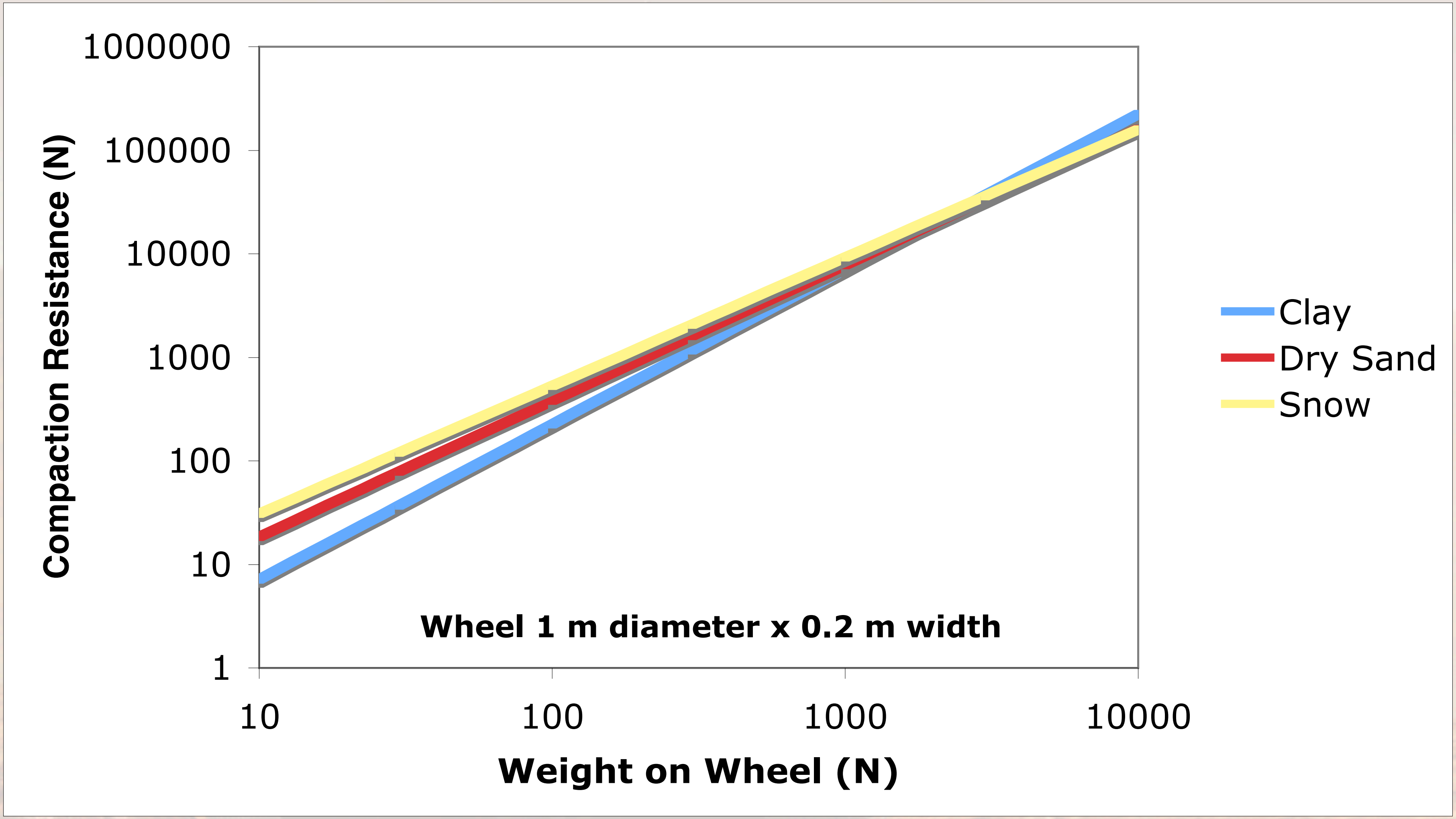
Effects of Wheel Parameters



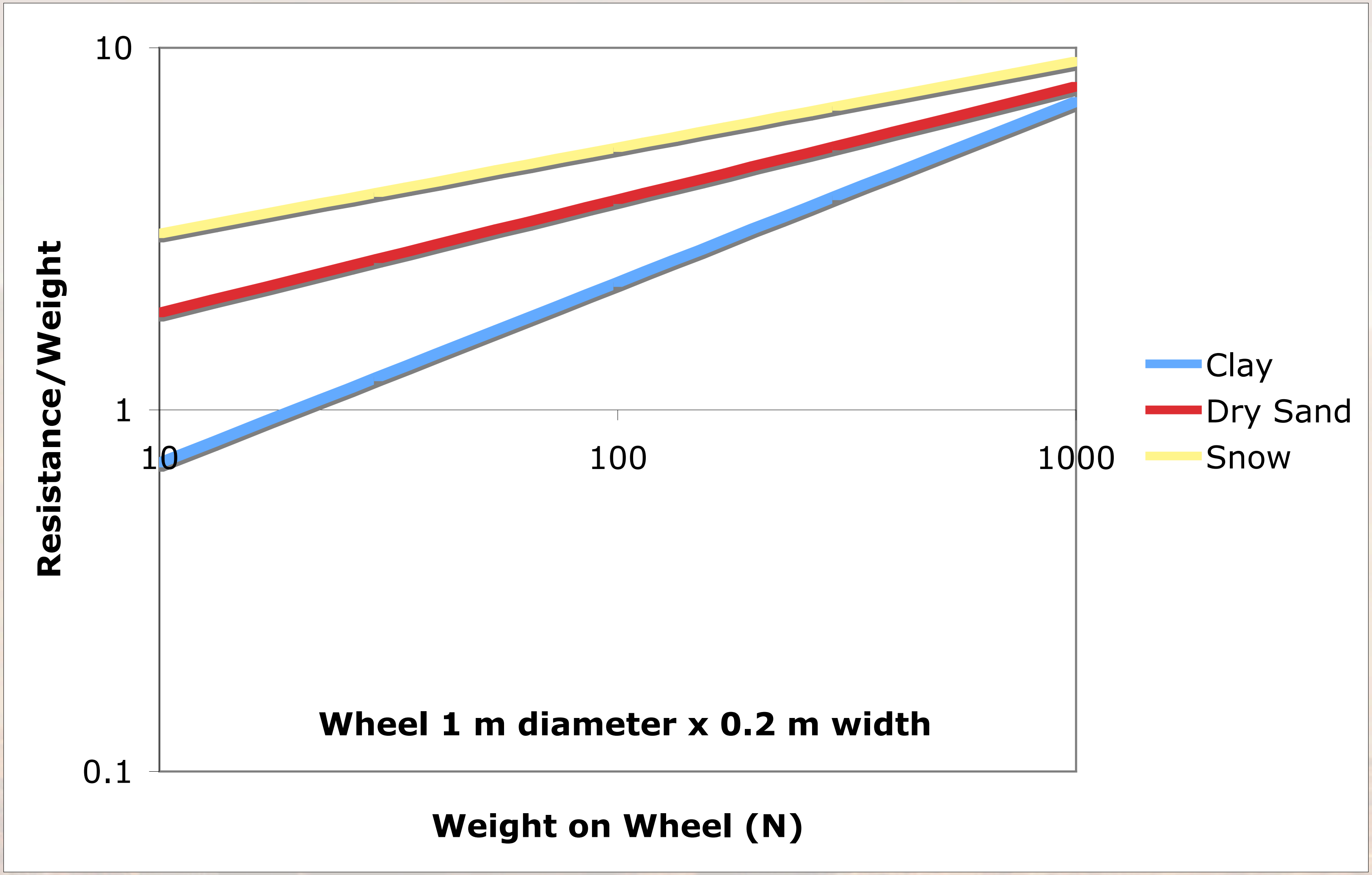
Effect of Soil "Spring Constant" on R/W



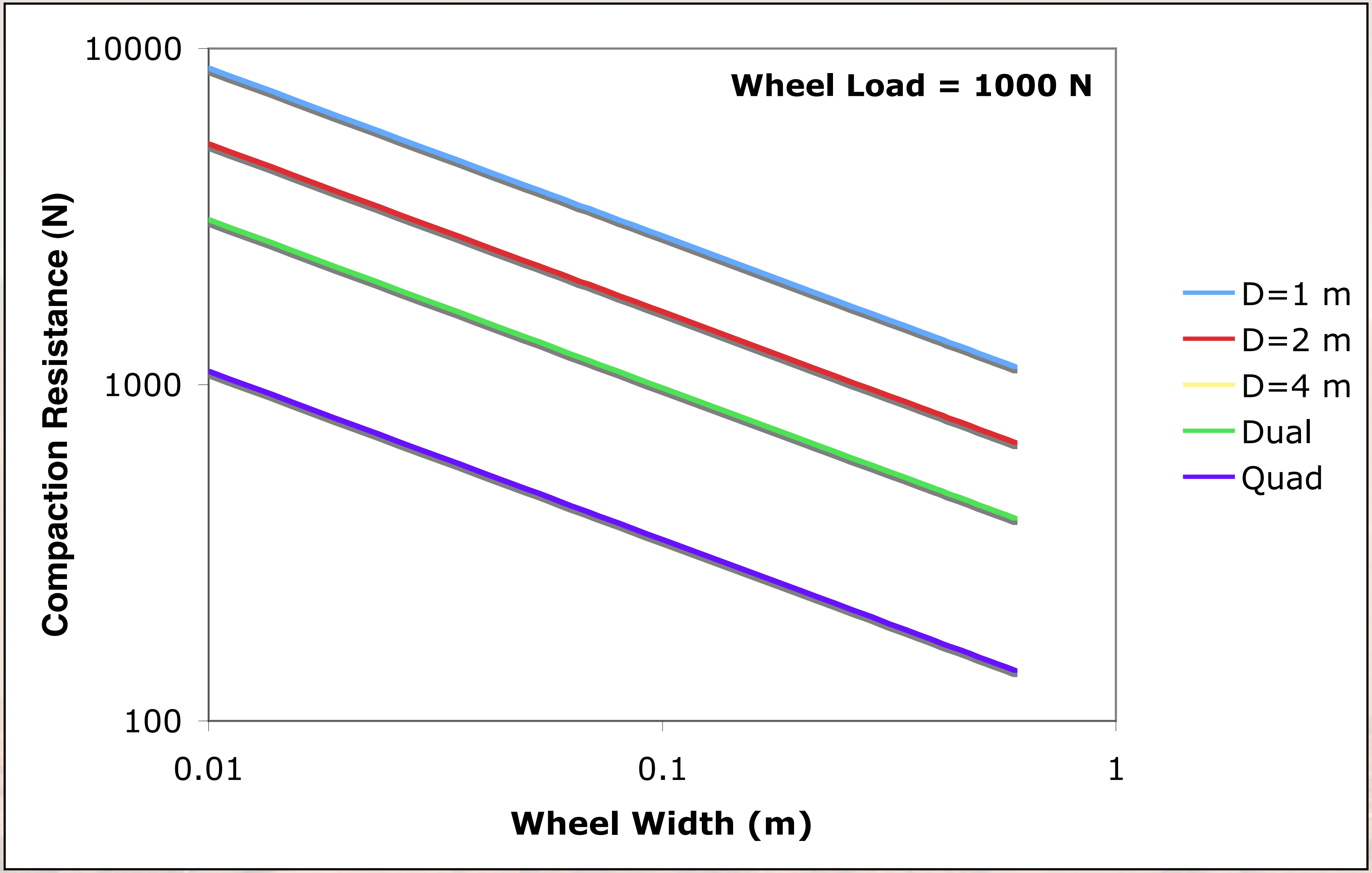
Soil Type and Wheel Load



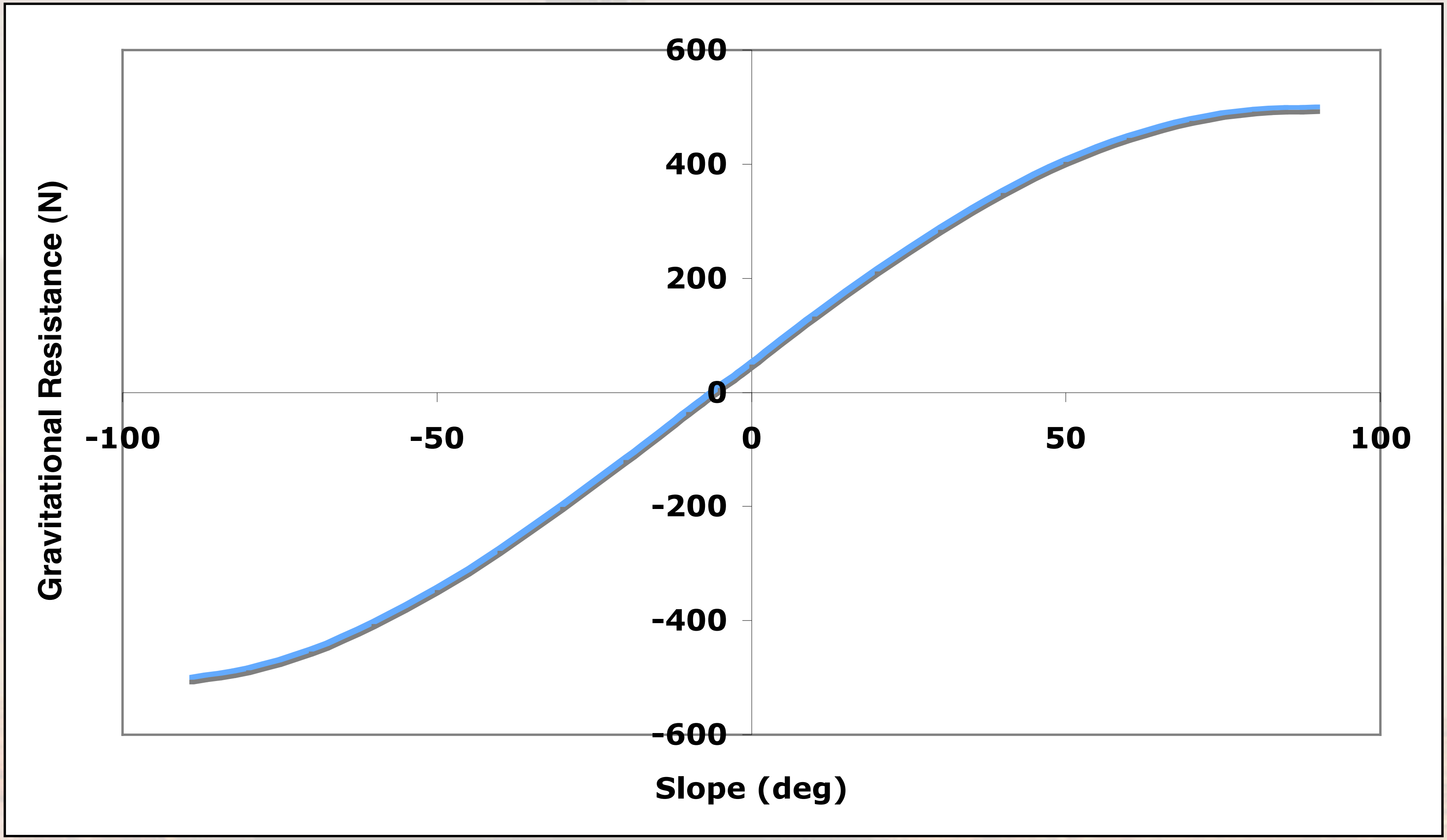
Soil Type and Specific Resistance



Effect of Wheel Diameter and Width



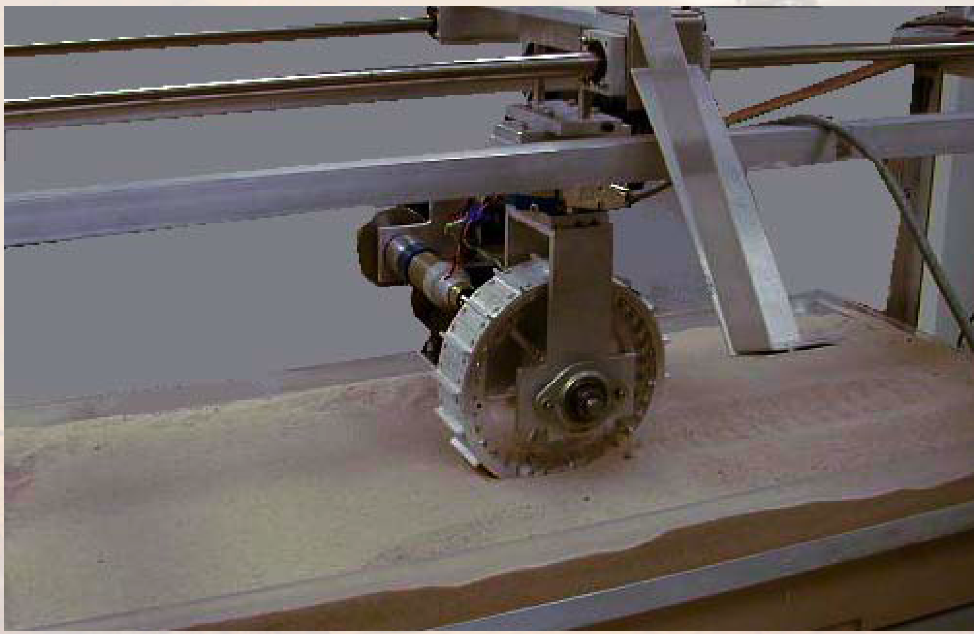
Effect of Slope

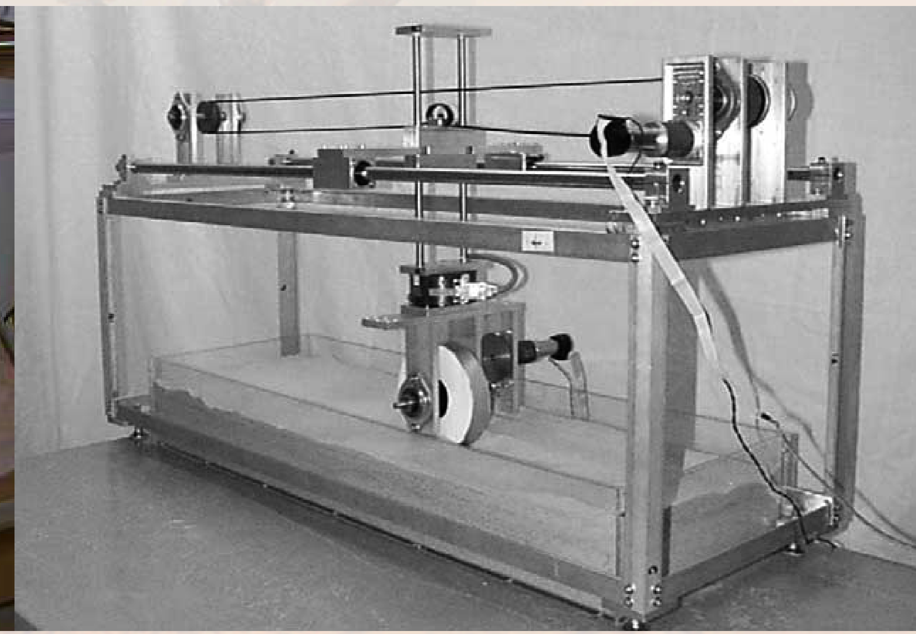


Wheel Test Apparatus

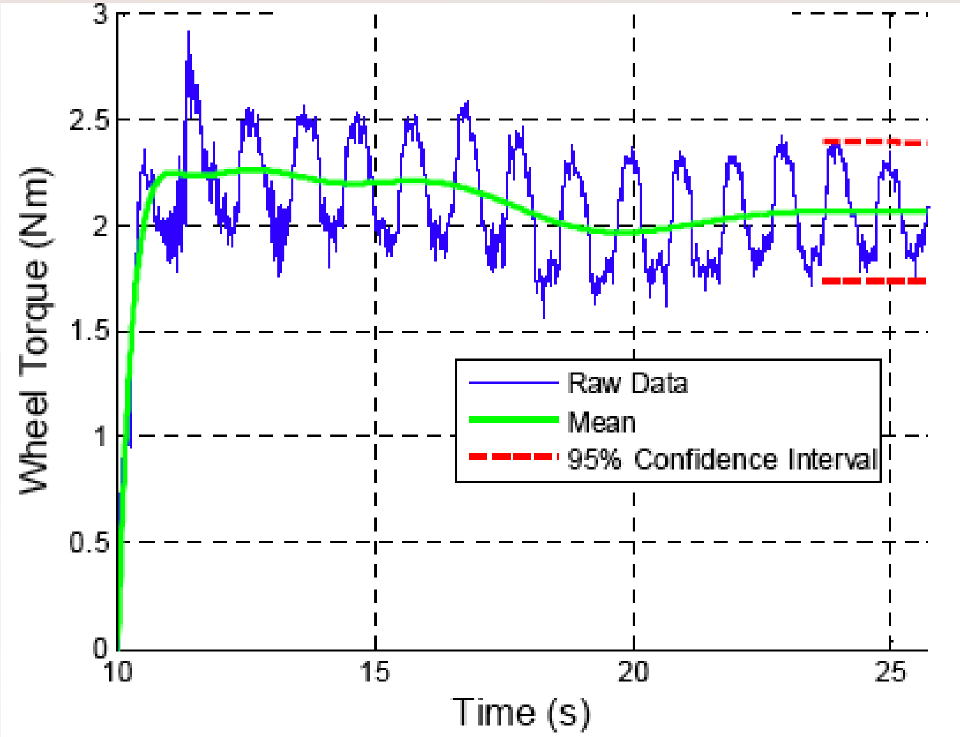
- Wheel testing done at MIT Field and Space Robotics Laboratory
- Independent control of motion and wheel velocity provides controllable slip

$$s = 1 - \frac{V}{\omega r}$$

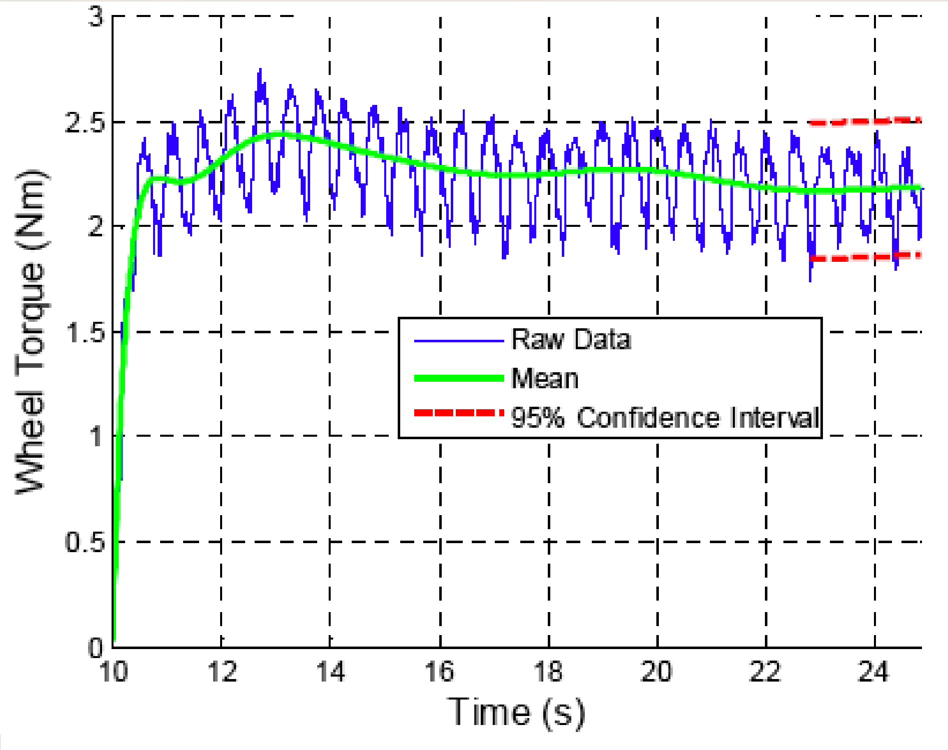




Wheel Torque vs. Time



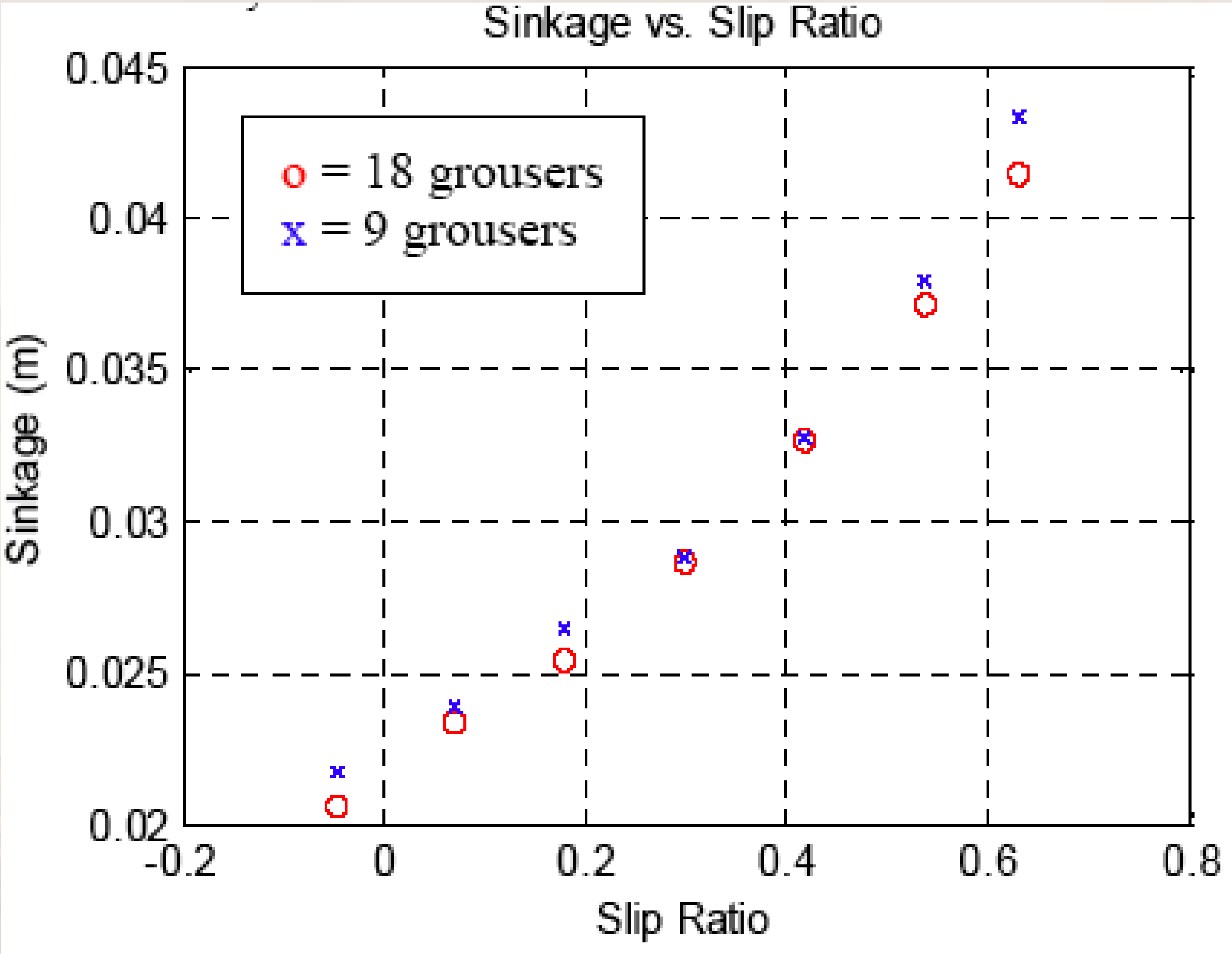
9 grousers



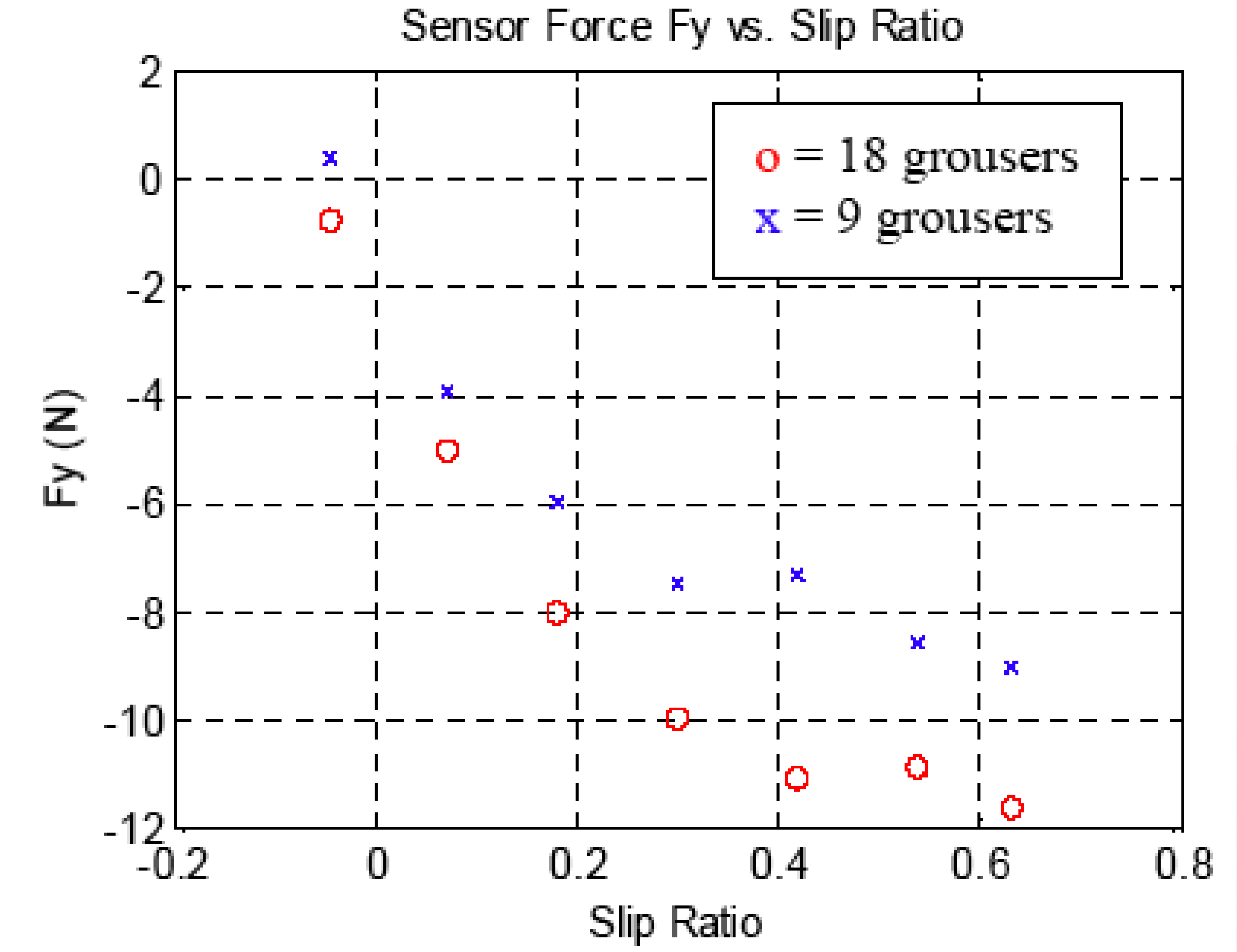
18 grousers

$\varphi=0.24$

Sinkage vs. Slip Ratio



Drawbar Pull vs. Slip Ratio



Motor Torque vs. Slip Ratio