Terramechanics 2

- Soil bearing parameters
- Bulldozing
- Tandem wheels

Terzaghi Soil Bearing Capacity Factors

$$N_q = \frac{\exp\left[\left(\frac{3\pi}{2} - \phi\right)\tan\phi\right]}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)}$$

$$N_c = \cot\phi\left\{\frac{\exp\left[\left(\frac{3\pi}{2} - \phi\right)\tan\phi\right]}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} - 1\right\} = \cot\phi\left(N_q - 1\right)$$

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)\tan\phi$$

$$K_{p\gamma} = \left(8\phi^2 - 4\phi + 3.8\right)\tan^2\left(\frac{\pi}{3} + \frac{\phi}{2}\right)$$

$K_c \equiv$ Modulus of density of soil deformation

$$K_c = (N_c - \tan\phi)\cos^2\phi$$

$K_\gamma \equiv$ Modulus of cohesion of soil deformation

$$K_\gamma = \left[\frac{2N_\gamma}{\tan\phi} + 1\right]\cos^2\phi$$

$\alpha \equiv$ Angle of approach of wheel to soil

$$\alpha = \cos^{-1}\left(1 - \frac{2z}{D}\right)$$

$\gamma \equiv$ Weight density of soil $\left(\frac{N}{m^3}\right)$

Soil Bearing Limit

Safe weight on the soil

$$W_s = A\left(cN_c + \gamma z N_q + \frac{1}{2}\gamma b N_\gamma\right)$$

$c \equiv$ Soil cohesion (Pa)

$b \equiv$ Wheel width (m)

Bulldozing Resistance

General case:

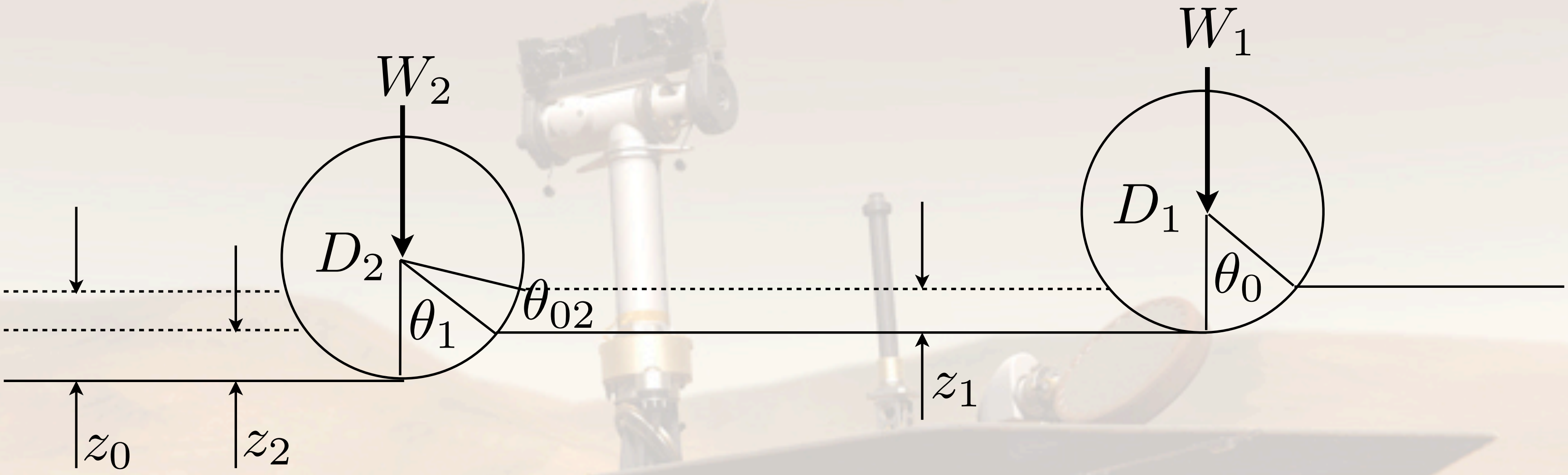
$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin \alpha \cos \phi} \left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\pi \ell_o^3 \gamma (\frac{\pi}{2} - \phi)}{3\pi} + \frac{c\pi \ell_o^2}{\pi} + c\ell_o^2 \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)$$

$$\ell_o = z \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

For tracked vehicles, only the first term applies:

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin \alpha \cos \phi} \left(2zcK_c + \gamma z^2 K_\gamma\right)$$

Tandem Wheels



$$z_0 = z_1 + z_2$$

$$\text{Assume } n = \frac{1}{2} \Longrightarrow P = k\sqrt{z}$$

$$P_1 = k\sqrt{z_1} \qquad P_2 = k\sqrt{z_1 + z_2}$$

Soil Weight Bearing Analysis

In general,

$$W = \int_0^{\theta_0} dF \cos\theta = \int_0^{\theta_0} Pb\ ds \cos\theta$$

$$W = \int_0^{\theta_0} bk\sqrt{z} \cos\theta\ ds$$

$$ds = r\ d\theta \qquad z = r(\cos\theta - \cos\theta_0)$$

$$W = \int_0^{\theta_0} bkr\sqrt{r(\cos\theta - \cos\theta_0)} \cos\theta\ d\theta$$

Generic Wheel Soil Suspension

Assuming small sinkage,

$$z \to \text{ small}, \; \theta \to \text{ small},$$

$$\cos\theta \approx 1 \qquad \cos\theta \; d\theta \approx d\theta$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} + \text{(higher order terms)}$$

$$W = \frac{bkr^{3/2}}{\sqrt{2}} \int_0^{\theta_0} \sqrt{\theta_0^2 - \theta^2} \; d\theta$$

$$W = \frac{bkr^{3/2}}{\sqrt{2}} \frac{1}{2} \left[\theta_0^2 \sin^{-1}\left(\frac{\theta}{\theta_0}\right) + \theta\sqrt{\theta_0^2 - \theta^2} \right]_0^{\theta_0}$$

Weight on the Front Wheel

$$W = \frac{\pi b k r^{3/2}}{4\sqrt{2}} \theta_0^2$$

$$\text{Front wheel:} \quad z_1 = r_1 - r_1 \cos\theta_0$$

$$z_1 = r_1 - r_1 \left(1 - \frac{\theta_0^2}{2} + \cdots \right) \implies \theta_0^2 \cong 2\frac{z_1}{r_1}$$

$$W_1 \cong \frac{\pi b k z_1 \sqrt{r_1}}{2\sqrt{2}}$$

Weight on Back Wheel

Change to limits of integration:

$$0 \longrightarrow \theta_0, \quad r \longrightarrow r_2$$

$$W_2 = \frac{bkr_2^{3/2}}{\sqrt{2}} \int_0^{\theta_0} \sqrt{\theta_{02}^2 - \theta^2}\; d\theta$$

$$\sqrt{\theta_{02}^2 - \theta^2} \cong \theta_{02}\left(1 - \frac{1}{2}\frac{\theta^2}{\theta_{02}^2} + \cdots\right)$$

$$\int_0^{\theta_0} \sqrt{\theta_{02}^2 - \theta^2}\; d\theta \cong \theta_{02}\theta_1 - \frac{1}{6}\frac{\theta_1^3}{\theta_{02}} = \theta_{02}\theta_1\left(1 - \frac{1}{6}\frac{\theta_1^2}{\theta_{02}^2}\right) \cong \theta_{02}\theta_1$$

Weight on Back Wheel

$$W_2 = \frac{bkr^{3/2}}{\sqrt{2}}\theta_{02}\theta_1$$

$$z_0 \cong r_2 \frac{\theta_{02}^2}{2} \qquad z_2 \cong r_2 \frac{\theta_1^2}{2}$$

$$\theta_{02} \cong \sqrt{2\frac{z_0}{r_2}} \qquad \theta_1 \cong \sqrt{2\frac{z_2}{r_2}}$$

$$W_2 = bk\sqrt{2r_2}\sqrt{z_0 z_2}$$

Track Depth of Tandem Wheels

$$\text{Front:}\ \ z_1 = \frac{2\sqrt{2}W_1}{\pi b k \sqrt{r_1}}$$

$$\text{Back:}\ \ z_2 = \left[\frac{W_2}{bk\sqrt{2r_2}}\frac{1}{\sqrt{z_0}}\right]^2$$

$$z_0 = z_1 + z_2 = \frac{2\sqrt{2}W_1}{\pi b k \sqrt{r_1}} + \frac{W_2^2}{(bk)^2 2r_2}\frac{1}{z_0}$$

Much algebra then ensues...

$$z_0^2 - \frac{2\sqrt{2}W_1}{\pi b k \sqrt{r_1}} z_0 + \frac{W_2^2}{(bk)^2 2r_2} = 0$$

Rolling Resistance of Tandem Wheels

Solve the quadratic equation to get

$$z_0 = \frac{1}{bk}\left(\frac{\sqrt{2}W_1}{\pi\sqrt{r_1}} + \sqrt{\frac{2W_1^2}{\pi^2 r_1} + \frac{W_2^2}{2r_2}}\right)$$

This was all done for $n = \frac{1}{2} \implies R = \frac{2}{3}bkz_0^{3/2}$

$$R = \frac{2}{3}\frac{1}{\sqrt{bk}}\left(\frac{\sqrt{2}W_1}{\pi\sqrt{r_1}} + \sqrt{\frac{2W_1^2}{\pi^2 r_1} + \frac{W_2^2}{2r_2}}\right)^{3/2}$$

$$R = \frac{2}{3}\frac{1}{\sqrt{bk}}\left(\frac{2W_1}{\pi\sqrt{D_1}} + \sqrt{\frac{4W_1^2}{\pi^2 D_1} + \frac{W_2^2}{D_2}}\right)^{3/2}$$

Nondimensional Forms

Total wheel load $W = W_1 + W_2$ Wheel weight ratio $a \equiv \frac{W_1}{W_2}$

$$\text{For } W_1 = W_2 = \frac{W}{2} \implies a = 1$$

$$W_1 = \frac{a}{1+a} W \qquad W_2 = \frac{1}{1+a} W$$

$$R = \frac{2}{3} \frac{1}{(a+1)^{3/2}} \frac{W^{3/2}}{\sqrt{bk}} \left(\frac{2a}{\pi\sqrt{D_1}} + \sqrt{\frac{4a^2}{\pi^2 D_1} + \frac{1}{D_2}} \right)^{3/2}$$

Define wheel diameter ratio $\rho \equiv \frac{D_1}{D_2}$

Nondimensional Forms

$$R = \frac{2}{3}\frac{1}{(a+1)^{3/2}}\frac{W^{3/2}}{D_2^{3/4}\sqrt{bk}}\left(\frac{2a}{\pi\sqrt{\rho}} + \sqrt{1+\frac{4a^2}{\pi^2\rho}}\right)^{3/2}$$

$$\text{Let } \xi \equiv \frac{2}{3}\frac{1}{(a+1)^{3/2}}\left(\frac{2a}{\pi\sqrt{\rho}} + \sqrt{1+\frac{4a^2}{\pi^2\rho}}\right)^{3/2}$$

$$R = \frac{\xi}{\sqrt{bk}}\frac{W^{3/2}}{D_2^{3/4}}$$

Simple Example Case

$$\text{Consider } \rho = 1 \quad (D_1 = D_2 = D)$$

$$a = 1 \quad \left(W_1 = W_2 = \frac{W}{2} \right)$$

$$\text{For tandem wheels, } R = \frac{0.580}{\sqrt{bk}} \frac{W^{3/2}}{D_2^{3/4}}$$

$$\text{For single wheel (n=1/2), } R = \frac{0.876}{\sqrt{bk}} \frac{W^{3/2}}{D_2^{3/4}}$$

Tandem wheels reduce rolling resistance by 34%

Dual Wheels

Equivalent to single wheel case twice as wide $b \Longrightarrow 2b$

$$R = \frac{0.876}{\sqrt{2}} \frac{1}{\sqrt{bk}} \frac{W^{3/2}}{D_2^{3/4}}$$

$$R_{dual} = \frac{0.619}{\sqrt{bk}} \frac{W^{3/2}}{D_2^{3/4}}$$

Dual wheel rolling resistance 29% less than single,

7% higher than tandem

ATHLETE in Partial Gravity Suspension