Robotic Mobility – Above the Surface

- Free Space
- Relative Orbital Motion
- Airless Major Bodies (moons)
- Gaseous Environments (Mars, Venus, Titan)
  - Lighter-than-"air" (balloons, dirigibles)
  - Heavier-than-"air" (aircraft, helicopters)

Propulsive Motion in Free Space

- Basic motion governed by Newton's Law
  $F = ma$ (actually, $\overrightarrow{F} = m\ddot{\overrightarrow{x}}$)
- Over a distance d and time t, assuming the motion is predominately coasting,

$$\Delta V = 2\frac{d}{t}$$

  (required to accelerate and decelerate)
- The rocket equation (relates propellant to ΔV)

$$\frac{m_{final}}{m_o} = e^{-\frac{\Delta V}{V_{exhaust}}}$$

Cost of Propulsive Maneuvering

- Assuming $\Delta V \ll V_{exhaust}$
- Use the Taylor's Series expansion of e

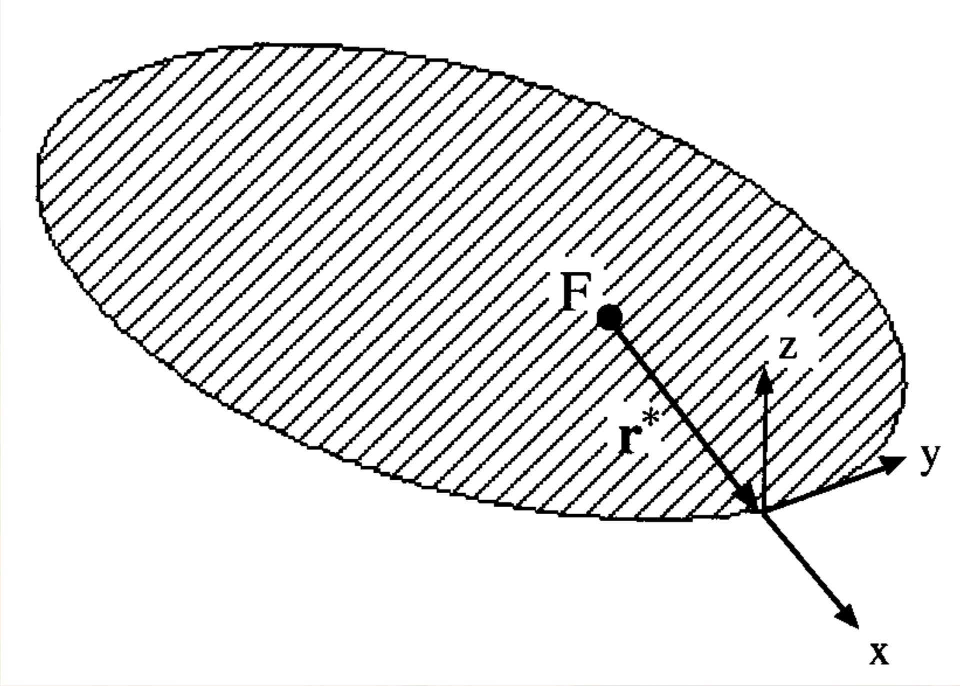
$$\frac{m_{final}}{m_o} \approx 1 - \frac{\Delta V}{V_{exhaust}}$$

- Since $m_o = m_{initial} = m_{prop} + m_{final}$,

$$\frac{m_{prop}}{m_o} \approx \frac{2}{V_{exhaust}} \frac{d}{t} \quad \text{or} \quad \frac{m_{prop}}{m_o} \approx 2 \frac{V_{travel}}{V_{exhaust}}$$

Hill's Equations (Proximity Operations)

Linearized equations of motion relative to a target in circular orbit in a rotating Cartesian reference frame



Ref: J. E. Prussing and B. A. Conway, *Orbital Mechanics*
Oxford University Press, 1993

$$\ddot{x} = 3n^2x + 2n\dot{y} + a_{dx}$$

$$\ddot{y} = -2n\dot{x} + a_{dy}$$

$$\ddot{z} = -n^2z + a_{dz}$$

$$n = \sqrt{\frac{\mu}{a^3}}$$

a_{dx}, a_{dy}, a_{dz} are disturbing accelerations (e.g., thrust, solar pressure)

Clohessy-Wiltshire ("CW") Equations

Force-free solutions to Hill's Equations

$$x(t) = [4 - 3\cos(nt)]x_o + \frac{\sin(nt)}{n}\dot{x}_o + \frac{2}{n}[1 - \cos(nt)]\dot{y}_o$$

$$y(t) = 6[\sin(nt) - nt]x_o + y_o - \frac{2}{n}[1 - \cos(nt)]\dot{x}_o + \frac{4\sin(nt) - 3nt}{n}\dot{y}_o$$

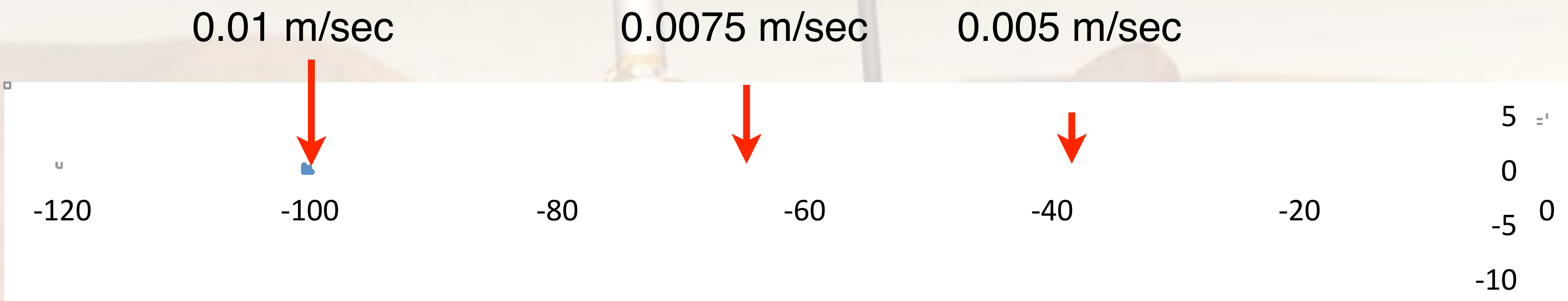
$$\dot{x}(t) = 3n\sin(nt)x_o + cos(nt)\dot{x}_o + 2\sin(nt)\dot{y}_o$$

$$\dot{y}(t) = -6n\left[1 - \cos(nt)\right]x_o - 2sin(nt)\dot{x}_o + \left[4\cos(nt) - 3\right]\dot{y}_o$$

$$z(t) = z_o\cos(nt) + \frac{\dot{z}_o}{n}\sin(nt)$$

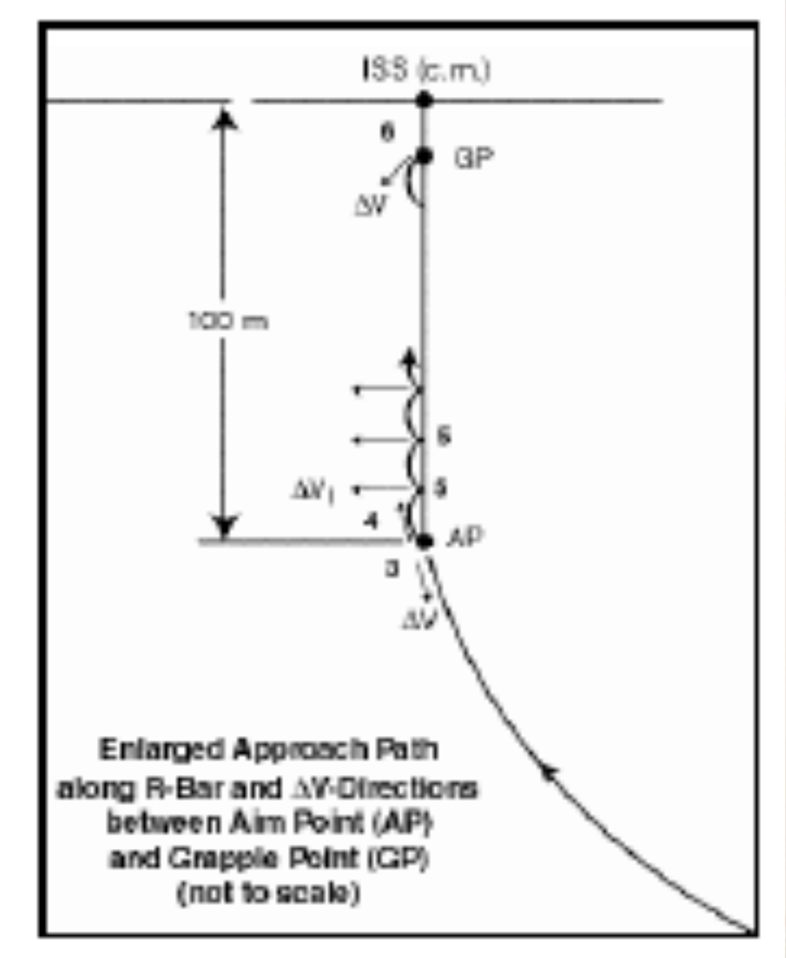
$$\dot{z}(t) = -z_o n\sin(nt) + \dot{z}_o\cos(nt)$$

"V-Bar" Approach



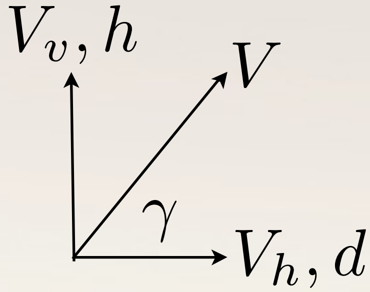
"R-Bar" Approach

- Approach from along the radius vector ("R-bar")
- Gravity gradients decelerate spacecraft approach velocity - low contamination approach
- Used for Mir, ISS docking approaches



Ref: Collins, Meissinger, and Bell, *Small Orbit Transfer Vehicle (OTV) for On-Orbit Satellite Servicing and Resupply*, 15th USU Small Satellite Conference, 2001

Hopping (Airless Flat Planet)



Use *F=ma* for vertical motion

$$\dot{V}_v = -g \qquad h = V_v t - \frac{1}{2} g t^2$$

$$t_{flt} = 2V_v/g$$

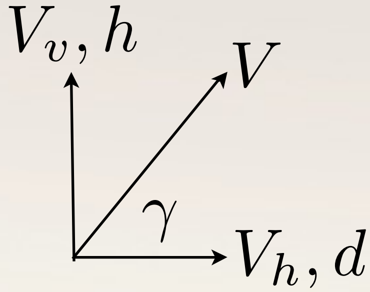
Constant velocity in horizontal direction produces

$$d = V_h t_{flt} = 2\frac{V_h V_v}{g}$$

$$V_h = V\cos\gamma; V_v = V\sin\gamma$$

$$d = 2\frac{V^2 \sin\gamma\cos\gamma}{g} = \frac{V^2}{g}\sin(2\gamma)$$

Hopping (Airless Flat Planet)



Horizontal distance is maximized when $\sin(2\gamma) = 1$

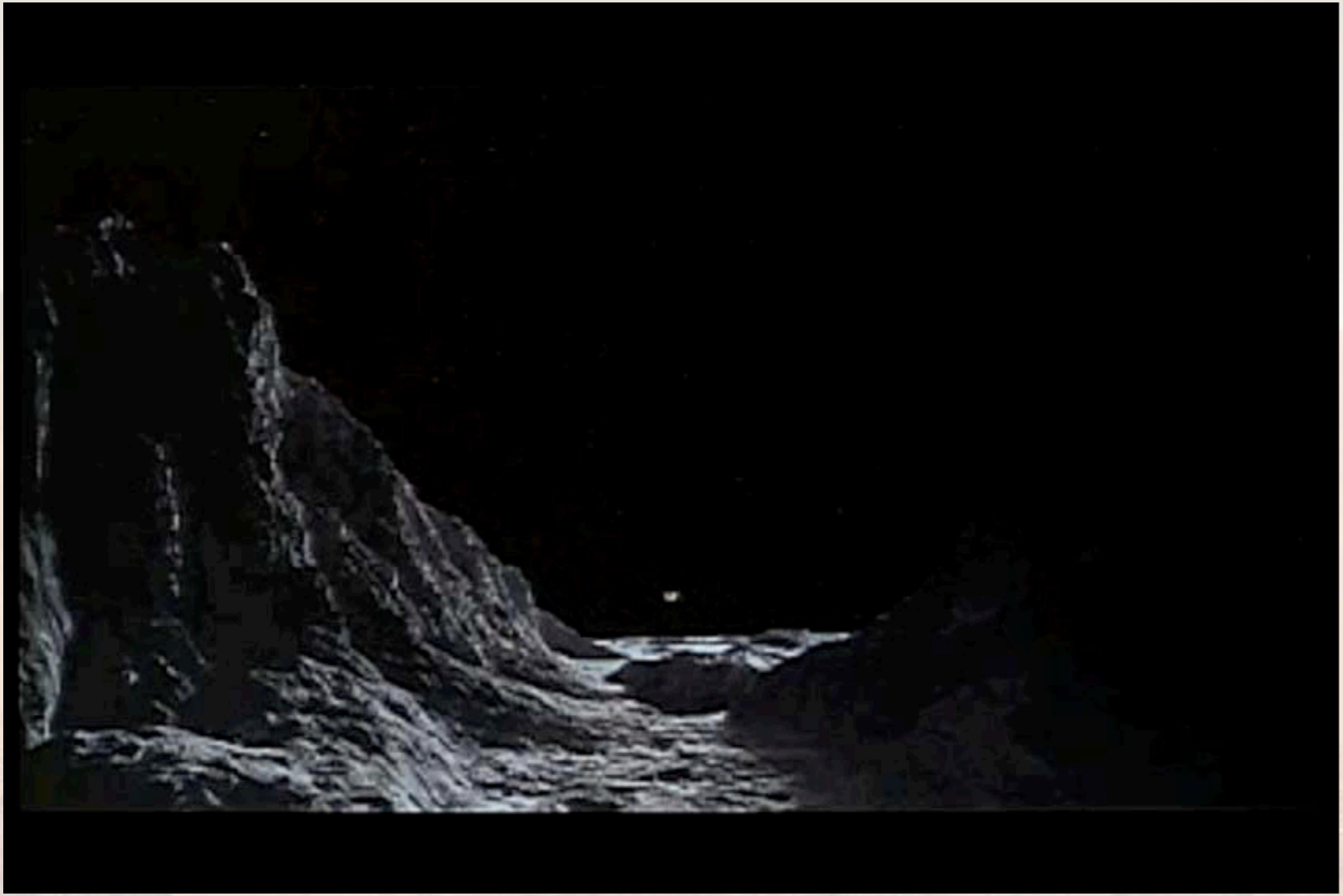
$$\gamma_{opt} = \frac{\pi}{2} = 45^o \qquad d_{max} = \frac{V^2}{g}$$

$$V = \sqrt{gd} \qquad \Delta V_{total} = 2V = \boxed{2\sqrt{gd}}$$

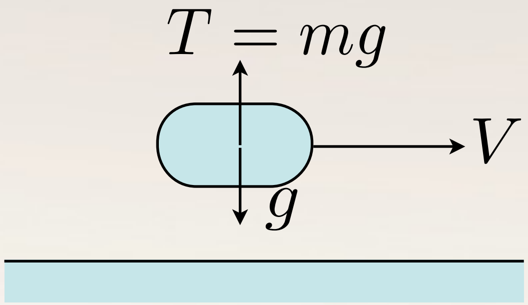
$$h_{max} = V_v \frac{V_v}{g} - \frac{1}{2} g \left(\frac{V_v}{g} \right)^2 \qquad V_v = \frac{V}{\sqrt{2}}$$

$$h_{max} = \frac{V^2}{4g} = \frac{\sqrt{gd}^2}{4g} = \frac{d}{4}$$

An Example of Propulsive Gliding



Propulsive Gliding (Airless Flat Planet)



Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V$$

$$\Delta V_v = g t_{flt} = \frac{gd}{V}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V}$$

Propulsive Gliding (Airless Flat Planet)

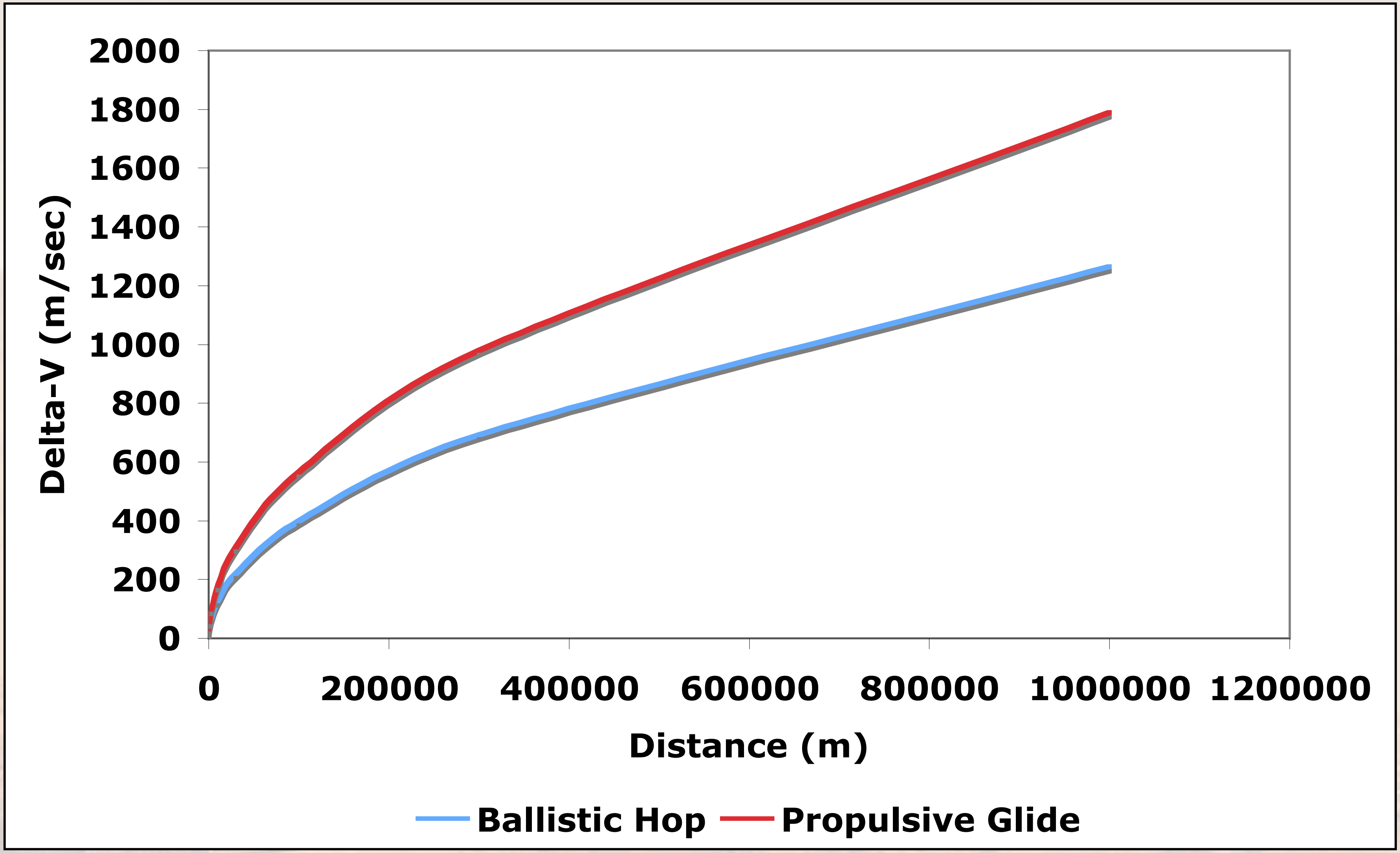
Want to choose V to minimize

$$\frac{\partial}{\partial V}\left(2V+\frac{gd}{V}\right)=0 \qquad 2-\frac{gd}{V^2}=0$$

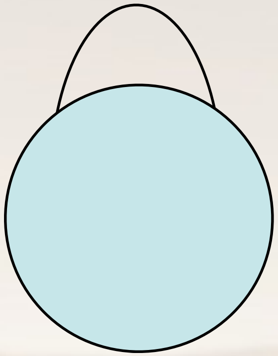
$$V_{opt}=\sqrt{\frac{gd}{2}}$$

$$\Delta V_{total}=2\sqrt{\frac{gd}{2}}+gd\sqrt{\frac{2}{gd}}=\boxed{2\sqrt{2}\sqrt{gd}}$$

Delta-V for Hopping and Gliding



Hopping (Spherical Planet)



$$\Delta v = 2v_o$$

$$v = \sqrt{\mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$r = \frac{p}{1 + e\cos\nu} = \frac{a(1-e^2)}{1 - e\cos\theta}$$

$$a = r\left(\frac{1 - e\cos\theta}{1-e^2}\right) \qquad v = \sqrt{\mu\left(\frac{2}{r} - \frac{1-e^2}{r(1-e\cos\theta}\right)}$$

$$\frac{\partial v}{\partial e} = 0 \Rightarrow \frac{-r(1-e\cos\theta)(-2e) + (1-e^2)r(-\cos\theta)}{r^2(1-e\cos\theta)^2} = 0$$

Hopping (Spherical Planet)

$$2er - 2e^2 r\cos\theta - r\cos\theta + re^2\cos\theta = 0$$

$$\cos\theta e^2 - 2e + \cos\theta = 0$$

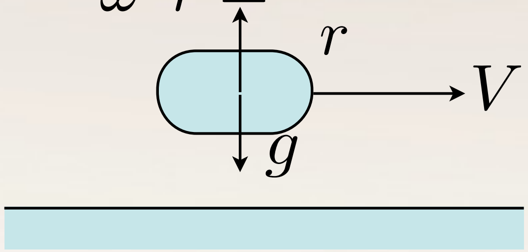
$$e_{opt} = \frac{2 \pm \sqrt{2^2 - 4\cos^2\theta}}{2\cos\theta} = \frac{1 \pm \sin\theta}{\cos\theta}$$

$+$ produces $e > 1$ (hyperbolic orbit); $-$ gives elliptical orbit

$$e_{opt} = \frac{1 - \sin\theta}{\cos\theta} \qquad a_{opt} = r\left(\frac{1 - e_{opt}\cos\theta}{1 - e_{opt}^2}\right)$$

Propulsive Gliding (Airless Round Planet)

$$\omega^2 r = \frac{V^2}{r}$$



Assume horizontal velocity is V

$$\Delta V_h = 2V$$

(includes acceleration and deceleration)

$$t_{flt} = d/V \qquad \Delta V_v = \left(g - \frac{V^2}{r}\right) t_{flt} = \frac{gd}{V} - \frac{dV}{r}$$

Total ΔV becomes

$$\Delta V_{total} = \Delta V_v + \Delta V_h = 2V + \frac{gd}{V} - \frac{dV}{r}$$

Propulsive Gliding (Airless Round Planet)

Want to choose V to minimize

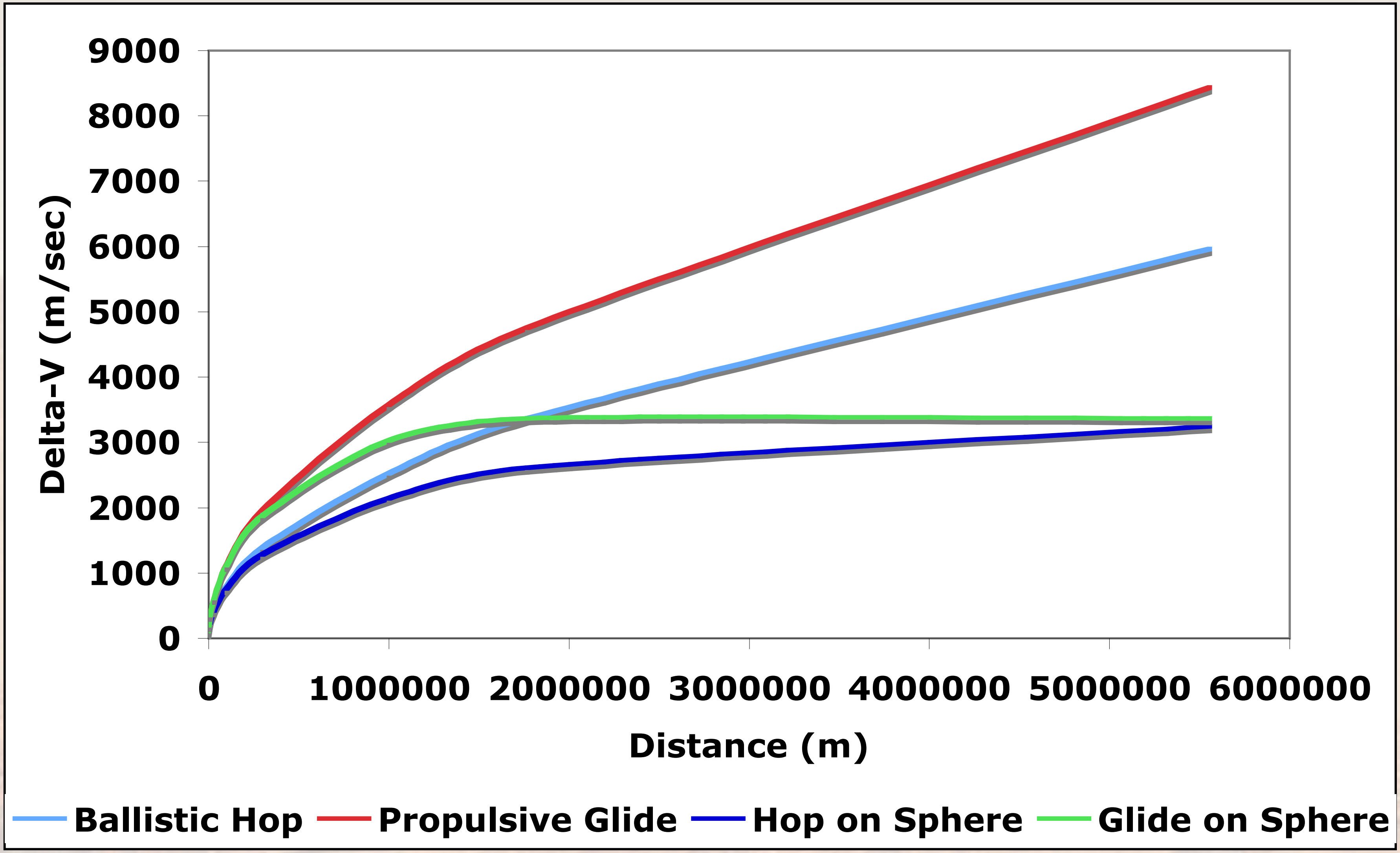
$$\frac{\partial}{\partial V}\left(2V + \frac{gd}{V} - \frac{dV}{r}\right) = 0 \qquad 2 - \frac{gd}{V^2} - \frac{d}{r} = 0$$

$$V_{opt} = \sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{\frac{gd}{2 - \frac{d}{r}}} + gd\sqrt{\frac{2 - \frac{d}{r}}{gd}} - \frac{d}{r}\sqrt{\frac{gd}{2 - \frac{d}{r}}}$$

$$\Delta V_{total} = 2\sqrt{2 - \frac{d}{r}}\sqrt{gd}$$

Hopping on Flat and Round Bodies



Nondimensional Forms

$$\text{Define } \nu \equiv \frac{V}{\sqrt{dg}} \qquad \rho \equiv \frac{d}{r} \qquad \eta \equiv \frac{h_{max}}{d}$$

$$\nu_{flat\ glide} = 2\sqrt{2}$$

$$\nu_{flat\ hop} = 2 \qquad \eta = \frac{1}{4}$$

$$\nu_{spherical\ glide} = 2\sqrt{2-\rho} \qquad (0 \leq \rho \leq 1)$$

Multiple Hops

- Assume n hops between origin and destination
- At each intermediate "touchdown", v_v has to be reversed

$$\Delta V_{total} = 2V + 2(n-1)V_v$$

$$t_{peak} = \frac{V_v}{g} \qquad t_{total} = 2nt_{peak} = 2n\frac{V_v}{g}$$

$$d = V_h t_{total} = \frac{2n}{g} V_h V_v \qquad V_v = \sqrt{2gh_{max}} \qquad \nu_v = \sqrt{\frac{2\eta}{n}}$$

$$\nu \equiv \frac{V}{\sqrt{dg}} \qquad \eta \equiv \frac{h_{max}}{d/n} \qquad V_h = \frac{dg}{2nV_v} \qquad \nu_h = \frac{1}{2}\sqrt{\frac{1}{2n\eta}}$$

Multiple Hop Analysis

$$\Delta\nu = 2\nu + 2(n-1)\nu_v$$

$$\Delta\nu = 2\sqrt{\nu_v^2 + \nu_h^2} + 2(n-1)\nu_v$$

$$\Delta\nu = 2\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}} + 2(n-1)\sqrt{\frac{2\eta}{n}}$$

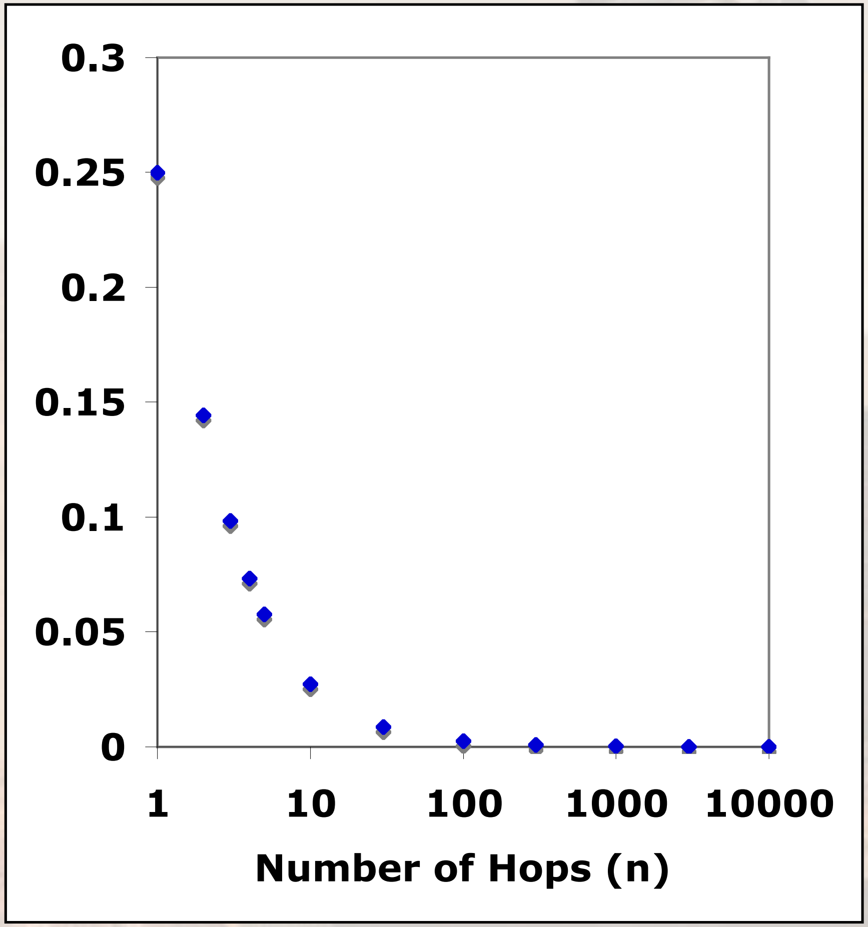
$$\frac{\partial \Delta\nu}{\partial \eta} = \left[\frac{1}{\sqrt{\frac{2\eta}{n} + \frac{1}{8n\eta}}}\left(\frac{2}{n} - \frac{1}{8n\eta^2}\right)\right] + (n-1)\sqrt{\frac{2}{n\eta}} = 0$$

Analytically messy, but note that for $n = 1 \Rightarrow \eta_{opt} = \frac{1}{4}$

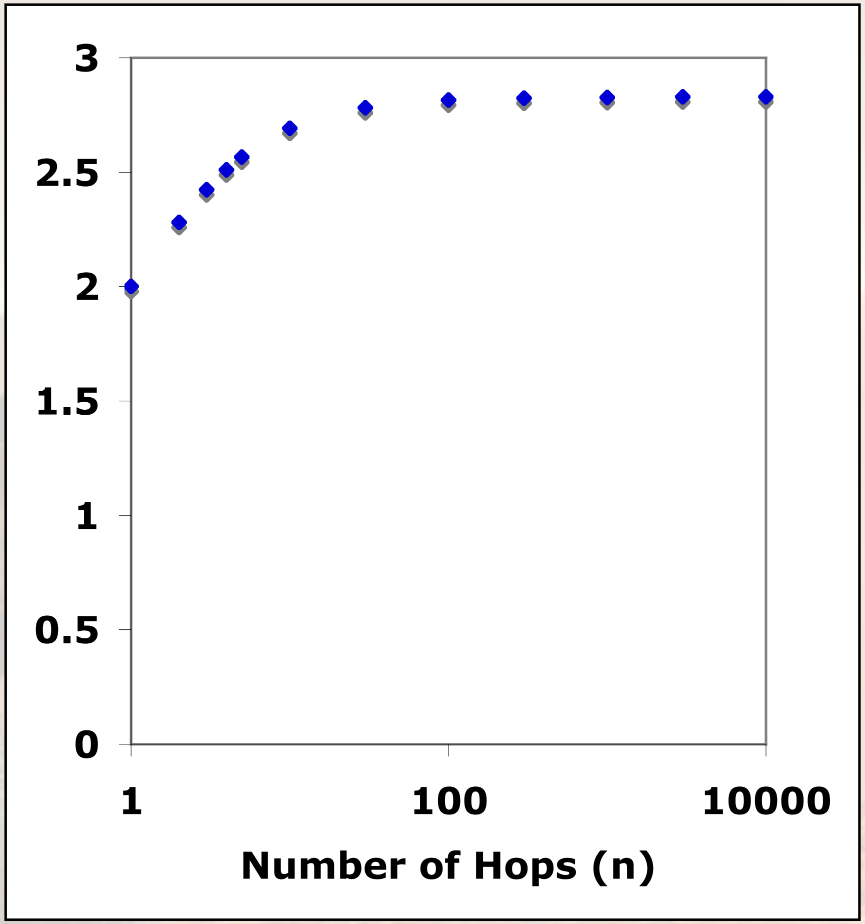
(In general, solve numerically)

Optimal Solutions for Multiple Hops

η_{opt}



$\Delta\nu$



Hopping Between Different Altitudes

Relative to starting point, landing elevation $\equiv h_2$

$$v_1 = (v_h, v_{v1}) \qquad v_2 = (v_h, v_{v2}) \qquad v_{v1} \neq v_{v2}$$

$$h = v_{v1}t - \frac{1}{2}gt^2 \qquad t_{peak} = \frac{v_{v1}}{g} \qquad h_{peak} = \frac{1}{2}\frac{v_{v1}^2}{g}$$

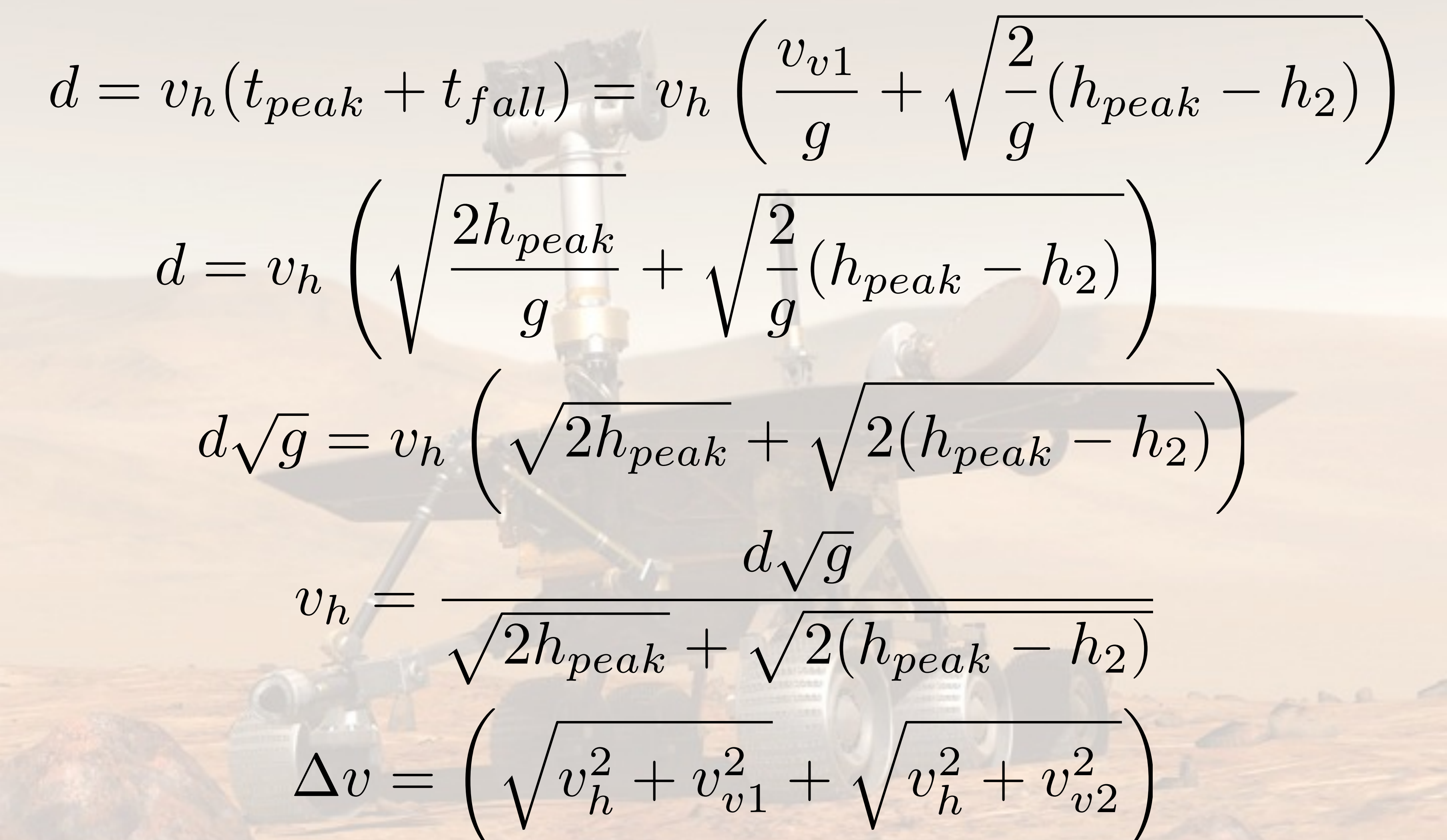
$$v_{v1} = \sqrt{2gh_{peak}}$$

From peak, $v_v = -gt_{fall}$; $h = h_{peak} - \frac{1}{2}gt_{fall}^2$

$$h_2 = h_{peak} - \frac{1}{2}\frac{v_{v2}^2}{g} \qquad t_{fall} = \sqrt{\frac{2}{g}(h_{peak} - h_2)}$$

$$v_{v2} = \sqrt{2g(h_{peak} - h_2)}$$

Optimal Hop with Altitude Change



$$d = v_h(t_{peak} + t_{fall}) = v_h \left(\frac{v_{v1}}{g} + \sqrt{\frac{2}{g}(h_{peak} - h_2)} \right)$$

$$d = v_h \left(\sqrt{\frac{2h_{peak}}{g}} + \sqrt{\frac{2}{g}(h_{peak} - h_2)} \right)$$

$$d\sqrt{g} = v_h \left(\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)} \right)$$

$$v_h = \frac{d\sqrt{g}}{\sqrt{2h_{peak}} + \sqrt{2(h_{peak} - h_2)}}$$

$$\Delta v = \left(\sqrt{v_h^2 + v_{v1}^2} + \sqrt{v_h^2 + v_{v2}^2} \right)$$

Nondimensional Form of Equations

$$\text{Remember that } \nu \equiv \frac{v}{\sqrt{dg}};\ \eta \equiv \frac{h_{peak}}{d};\ \lambda \equiv \frac{h_2}{d}$$

$$\Delta\nu = \left(\sqrt{\nu_h^2 + \nu_{v1}^2} + \sqrt{\nu_h^2 + \nu_{v2}^2}\right)$$

$$\nu_{v1} = \sqrt{2\eta} \qquad \nu_{v2} = \sqrt{2(\eta - \lambda)}$$

$$\nu_h = \frac{1}{\sqrt{2\eta} + \sqrt{2(\eta - \lambda)}}$$

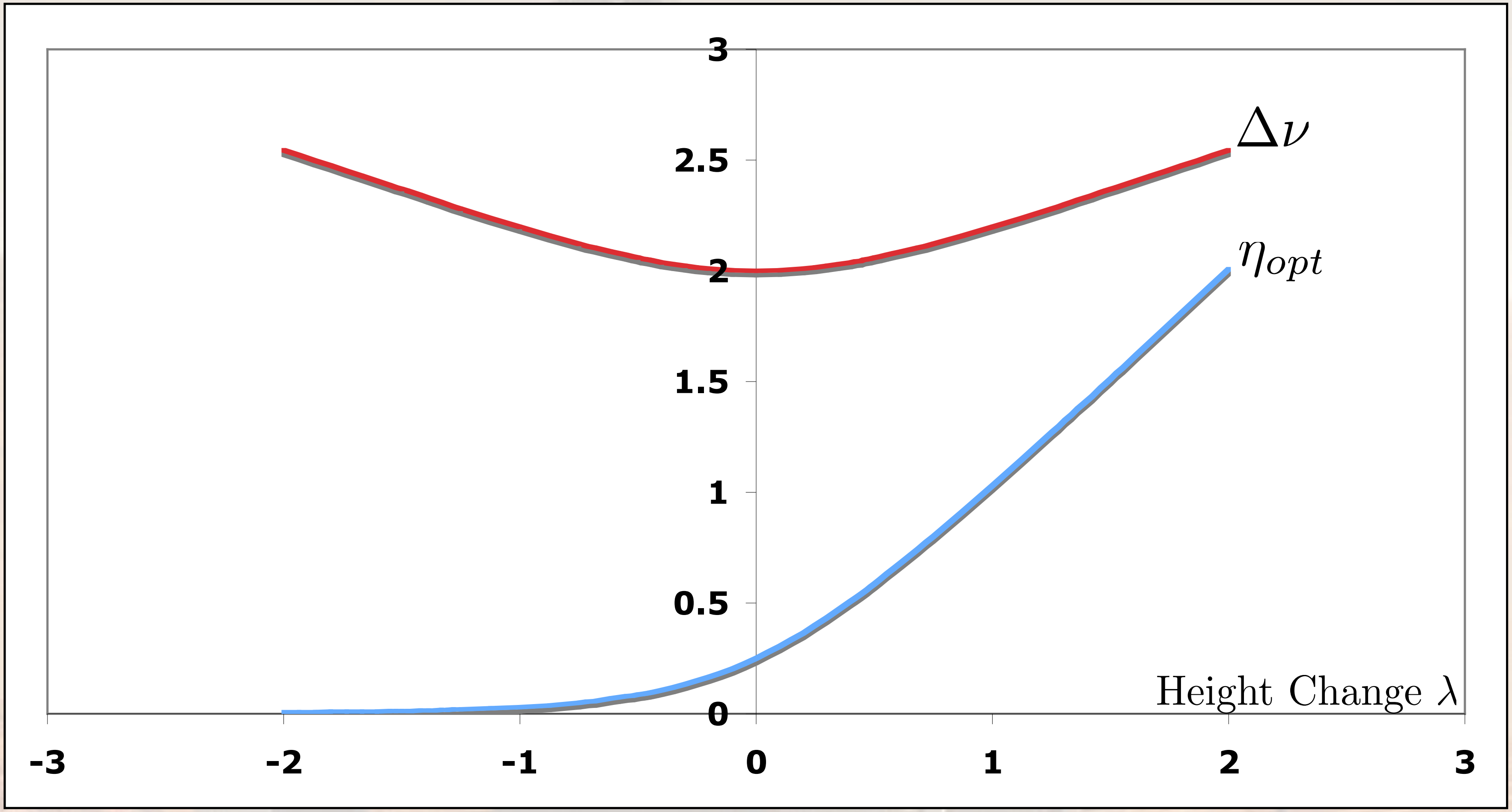
$$\Delta\nu = \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta-\lambda)}}\right)^2 + 2\eta} + \sqrt{\left(\frac{1}{\sqrt{2\eta} + \sqrt{2(\eta-\lambda)}}\right)^2 + 2(\eta - \lambda)}$$

Optimization of Height-Changing Hop

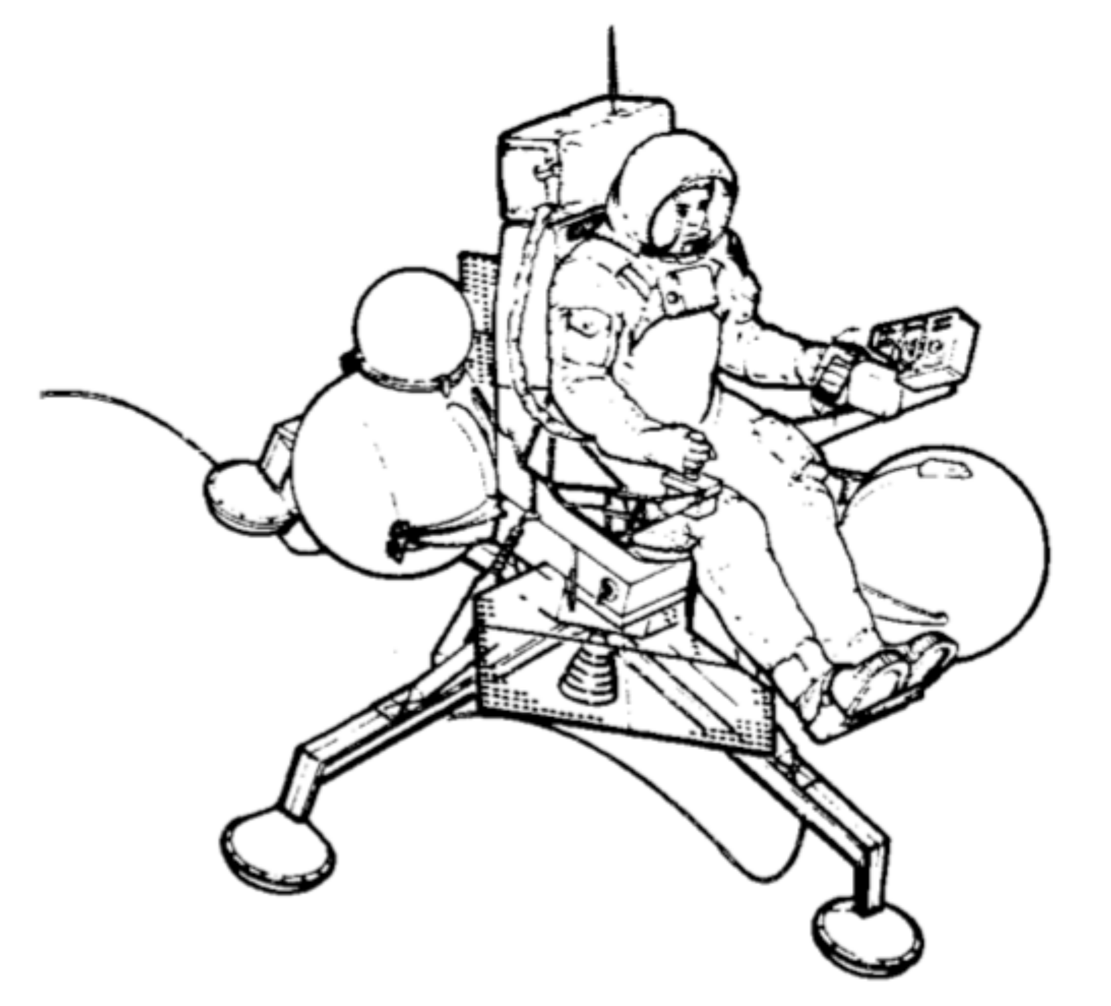
- This is not going to be one where you can take the derivative and set equal to zero, so use the equation to find a numerical optimization
- Set $\lambda = 0$ to check for plain hop solution

$$\Delta\nu = 2\sqrt{\frac{1}{8\eta} + 2\eta} \Rightarrow \eta_{opt} = \frac{1}{4}$$

Trajectory Design for Height Change

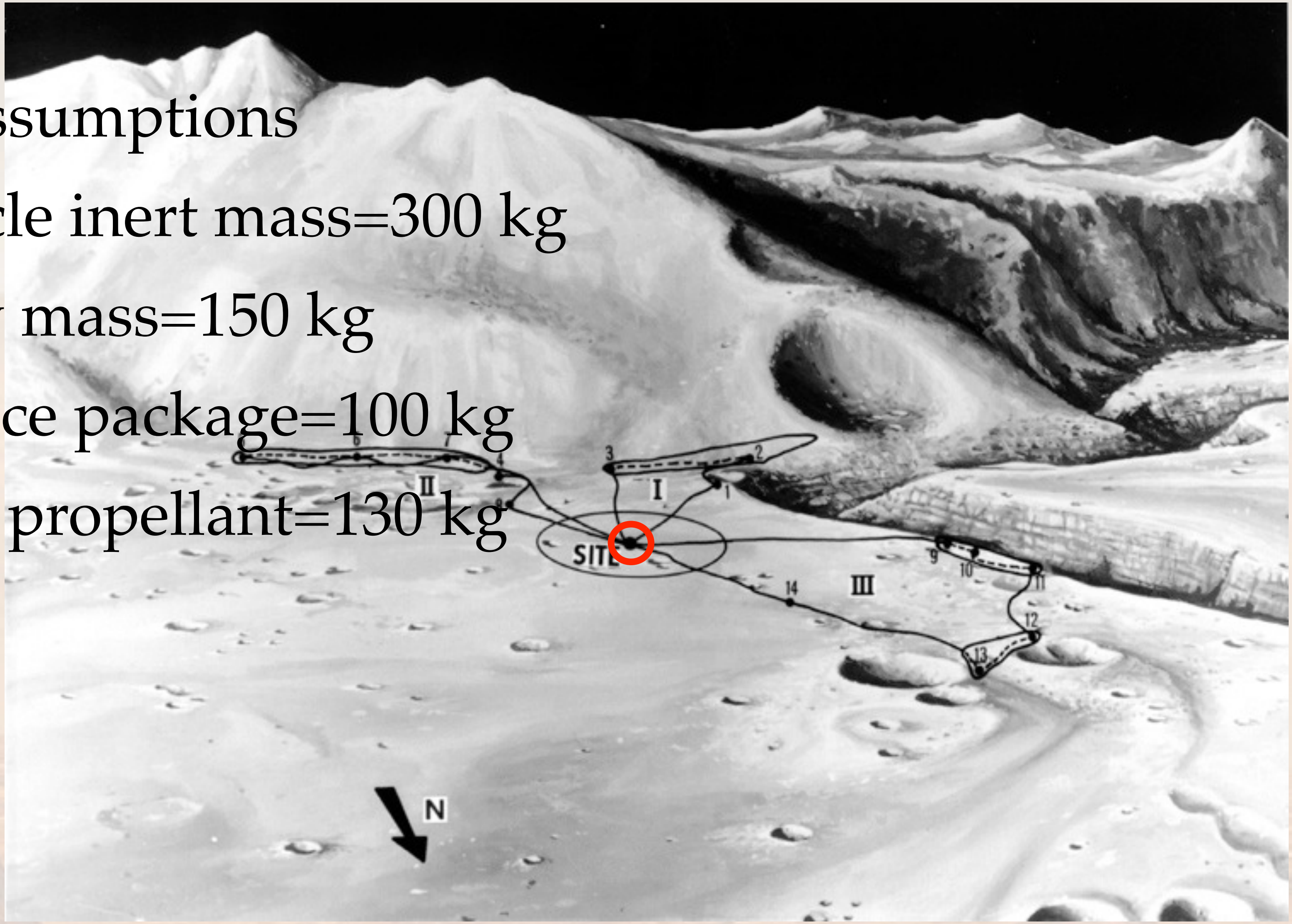


Apollo Concept of Lunar Flying Vehicle



from "Study of One-Man Lunar Flying Vehicle - Final Report Volume 1: Summary" North American Rockwell, NASA CR-101922, August 1969

Apollo 15 Revisited: LFV Sortie



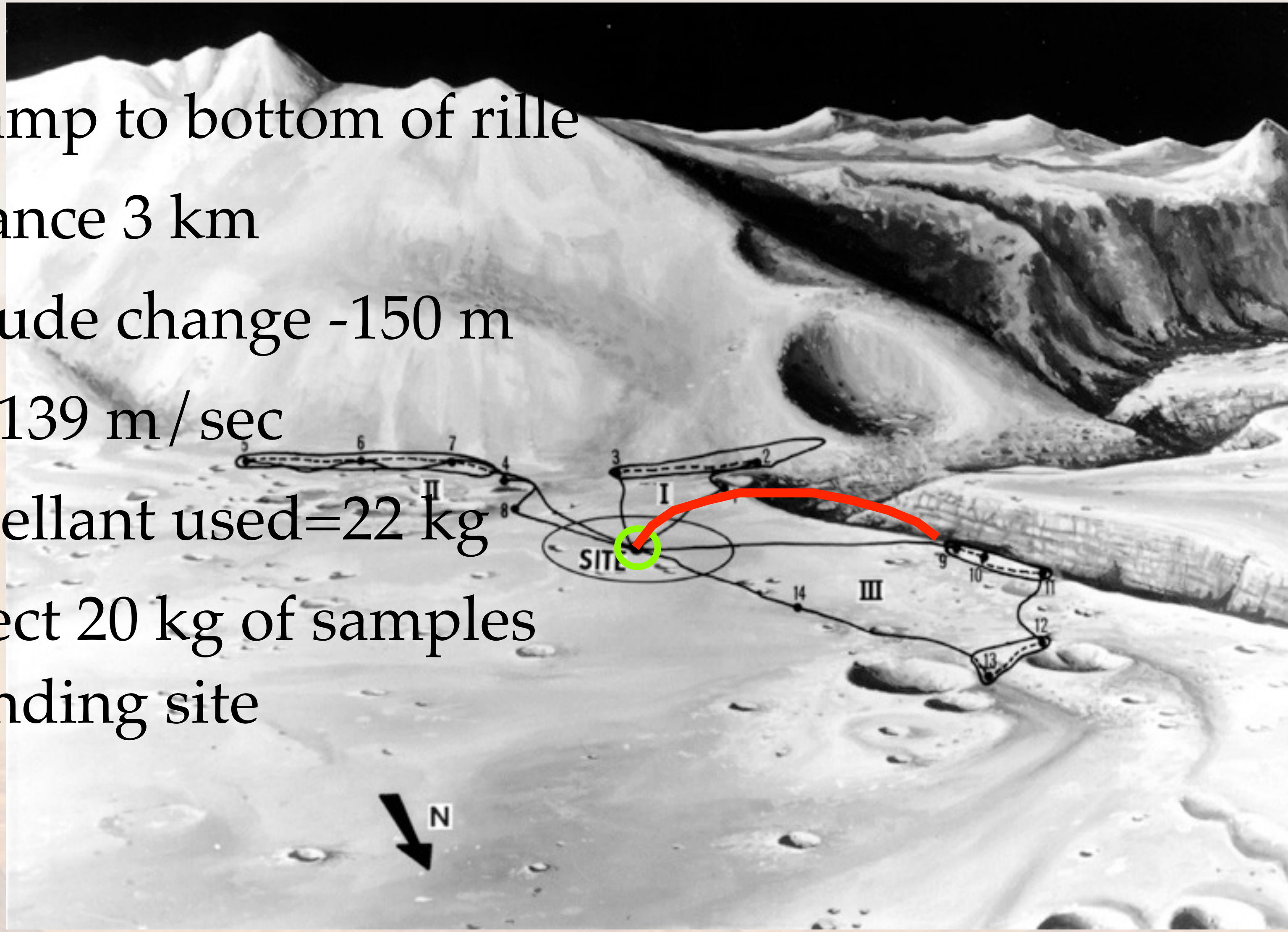
Basic assumptions

- Vehicle inert mass=300 kg
- Crew mass=150 kg
- Science package=100 kg
- Total propellant=130 kg

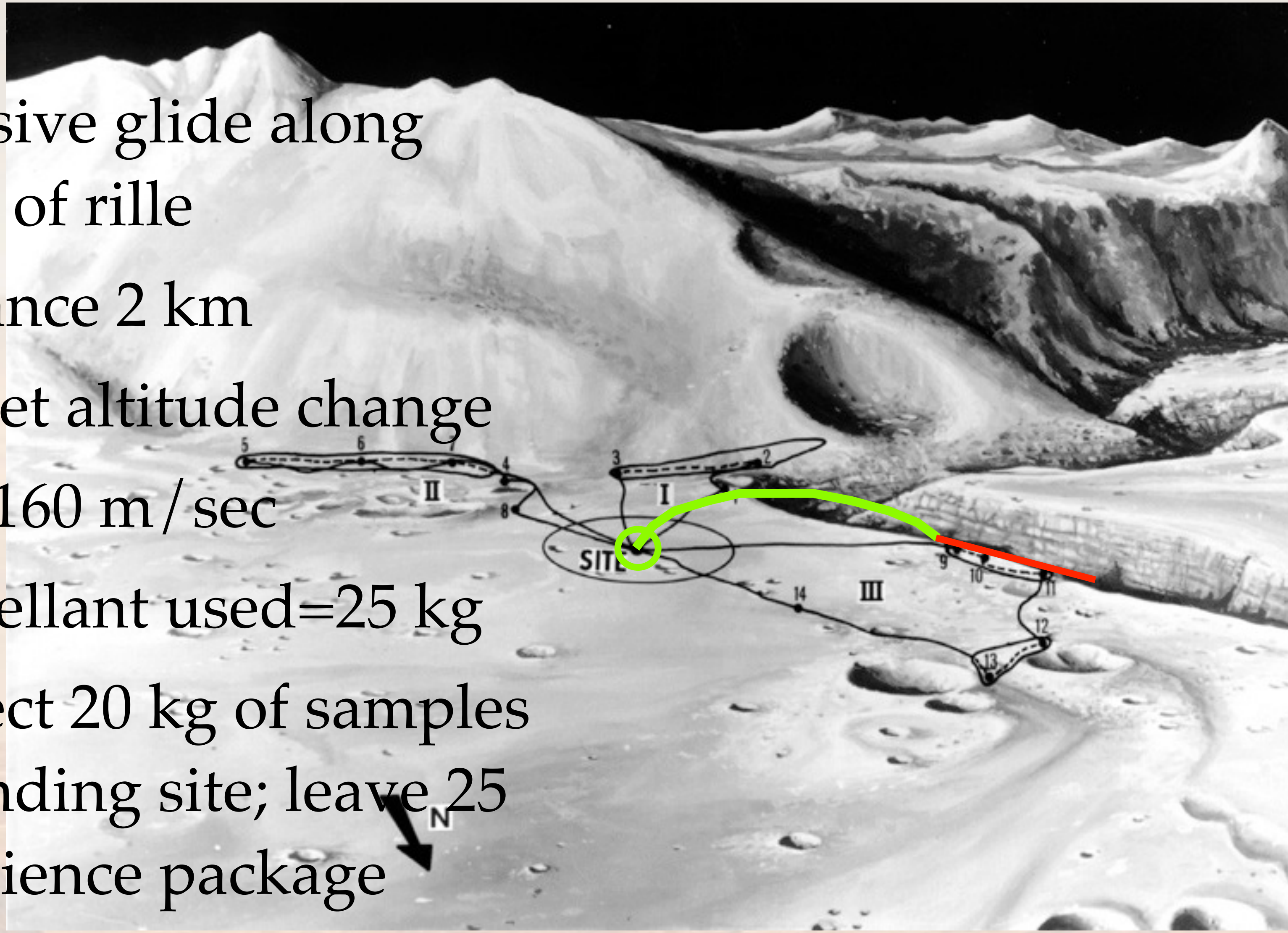
Apollo 15 Revisited: Leg 1

Base camp to bottom of rille

- Distance 3 km
- Altitude change -150 m
- ΔV=139 m/sec
- Propellant used=22 kg
- Collect 20 kg of samples at landing site



Apollo 15 Revisited: Leg 2



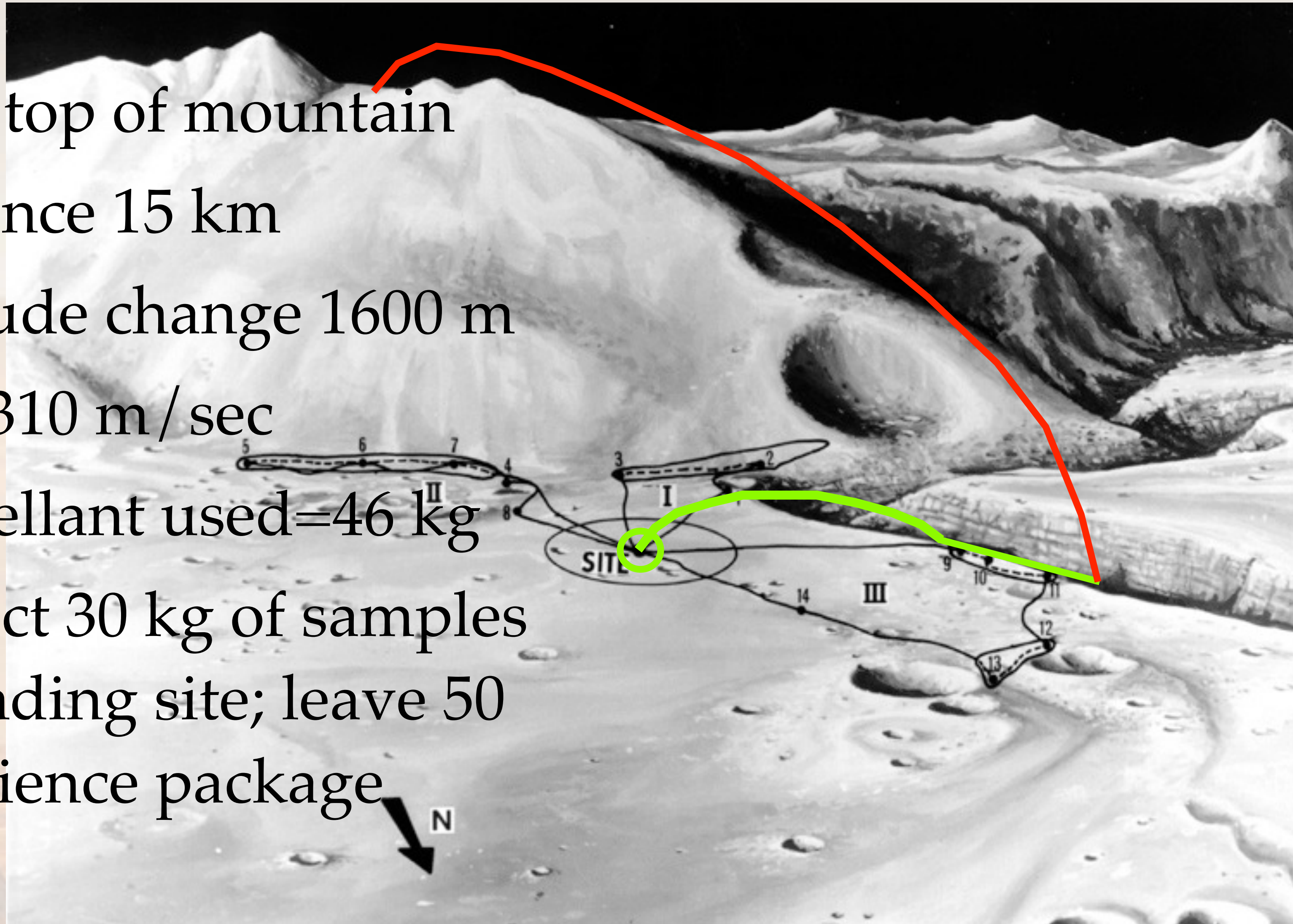
Propulsive glide along bottom of rille

- Distance 2 km
- No net altitude change
- ΔV=160 m/sec
- Propellant used=25 kg
- Collect 20 kg of samples at landing site; leave 25 kg science package

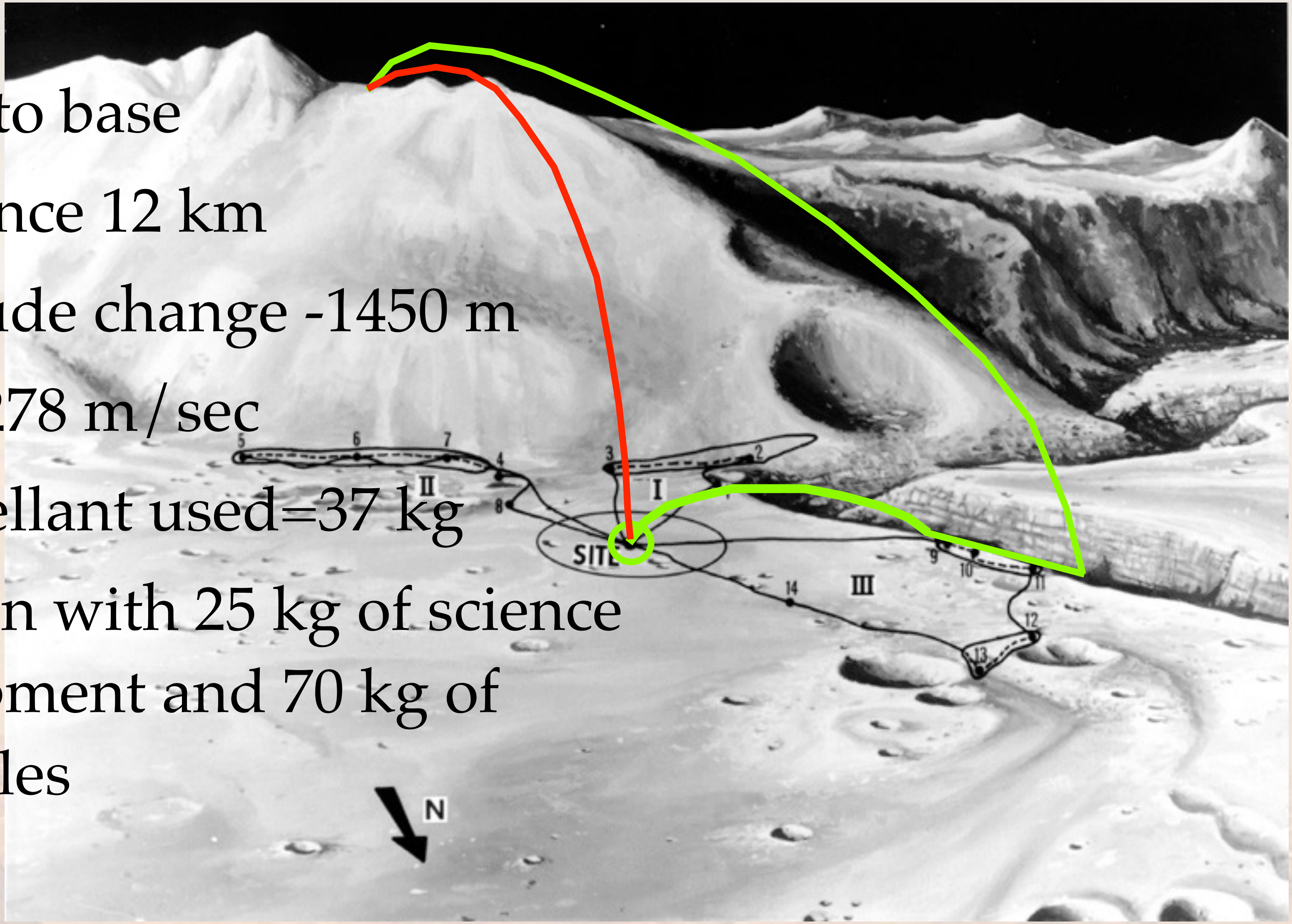
Apollo 15 Revisited: Leg 3

Hop to top of mountain

- Distance 15 km
- Altitude change 1600 m
- ΔV=310 m/sec
- Propellant used=46 kg
- Collect 30 kg of samples at landing site; leave 50 kg science package



Apollo 15 Revisited: Leg 4



Return to base

- Distance 12 km
- Altitude change -1450 m
- ΔV=278 m/sec
- Propellant used=37 kg
- Return with 25 kg of science equipment and 70 kg of samples

Apollo 15 Revisited: Discussion

- Current minimum estimates are for 400 kg of residual propellants in Altair at landing - would support three equivalent sorties
- Presence of water ice or ISRU propellant production at outpost would easily support moderate flier mission requirements
- Challenges in routine refueling of cryogenic propellants on the lunar surface, reliable flight and landing control system

Landing Impact Attenuation

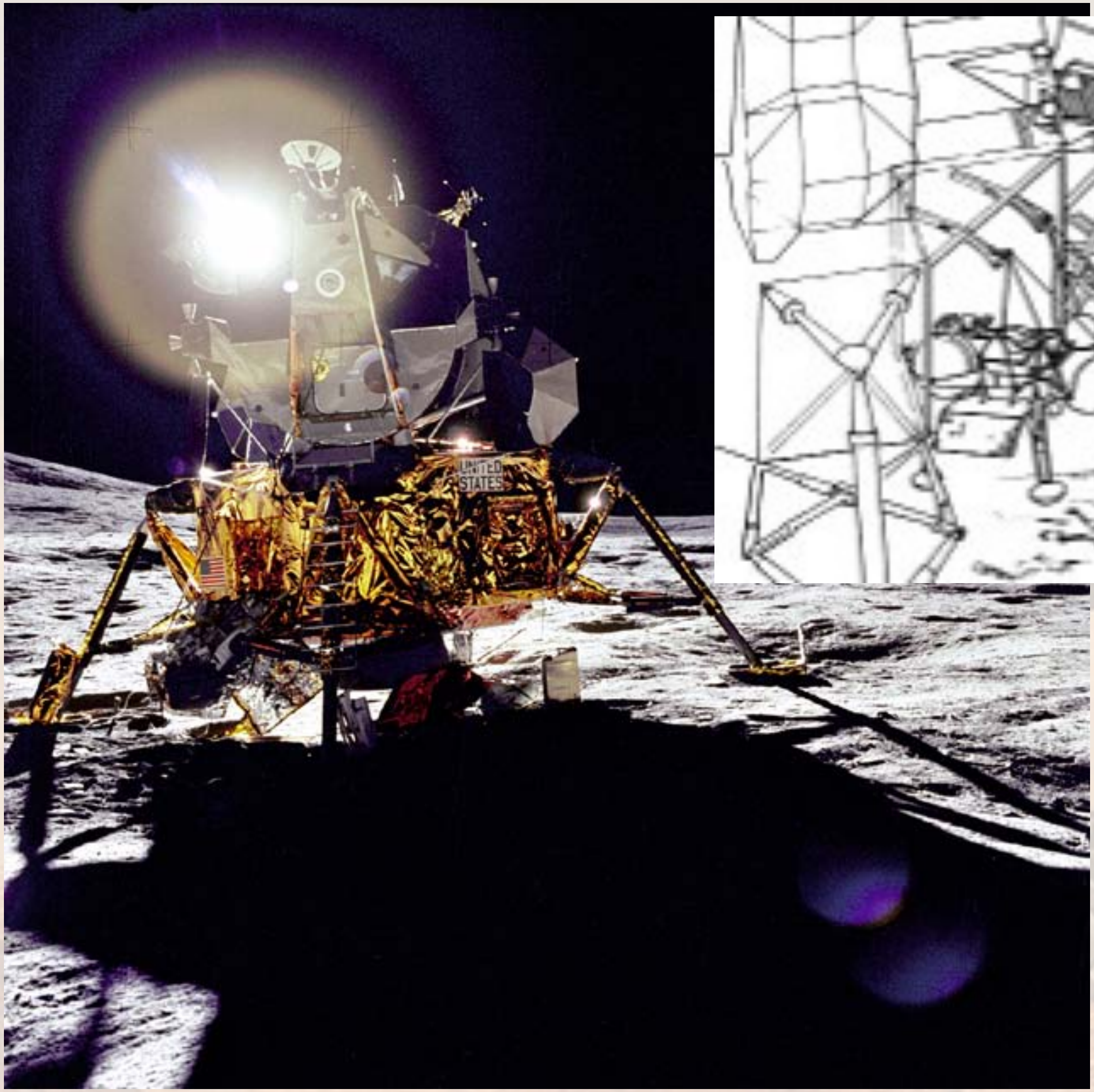
- Cannot rely on achieving perfect zero velocity at touchdown
- Specifications for landing conditions
  - Vertical velocity ≤ 3 m/sec
  - Horizontal velocity ≤ 1 m/sec

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{1}{2}m(v_h^2 + v_v^2)$$

$$\text{Max case 500 kg vehicle} \implies E = 2500Nm$$

Mars Phoenix Lander

Apollo Lunar Module



Landing Deceleration

- Look at 3 m/sec vertical velocity
- Constant force deceleration

$$\frac{1}{2}mv^2 = Fd \qquad \frac{1}{2}v^2 = \frac{F}{m}d = a_{desired}d \qquad d = \frac{1}{2}\frac{v^2}{a_{desired}}$$

$$t_{decel} = \frac{v}{a_{desired}}$$

$a_{desired}$	$d\langle cm\rangle$	$t_d\langle sec\rangle$
1/6 g	281	1.88
1/2 g	92	0.61
1 g	46	0.31
2 g	23	0.15
3 g	15	0.10

- Spring deceleration

$$F = kx \qquad \int F dx = \frac{1}{2}mv^2$$

$$k = \frac{mv^2}{d^2} \qquad a_{peak} = \frac{kd}{m}$$