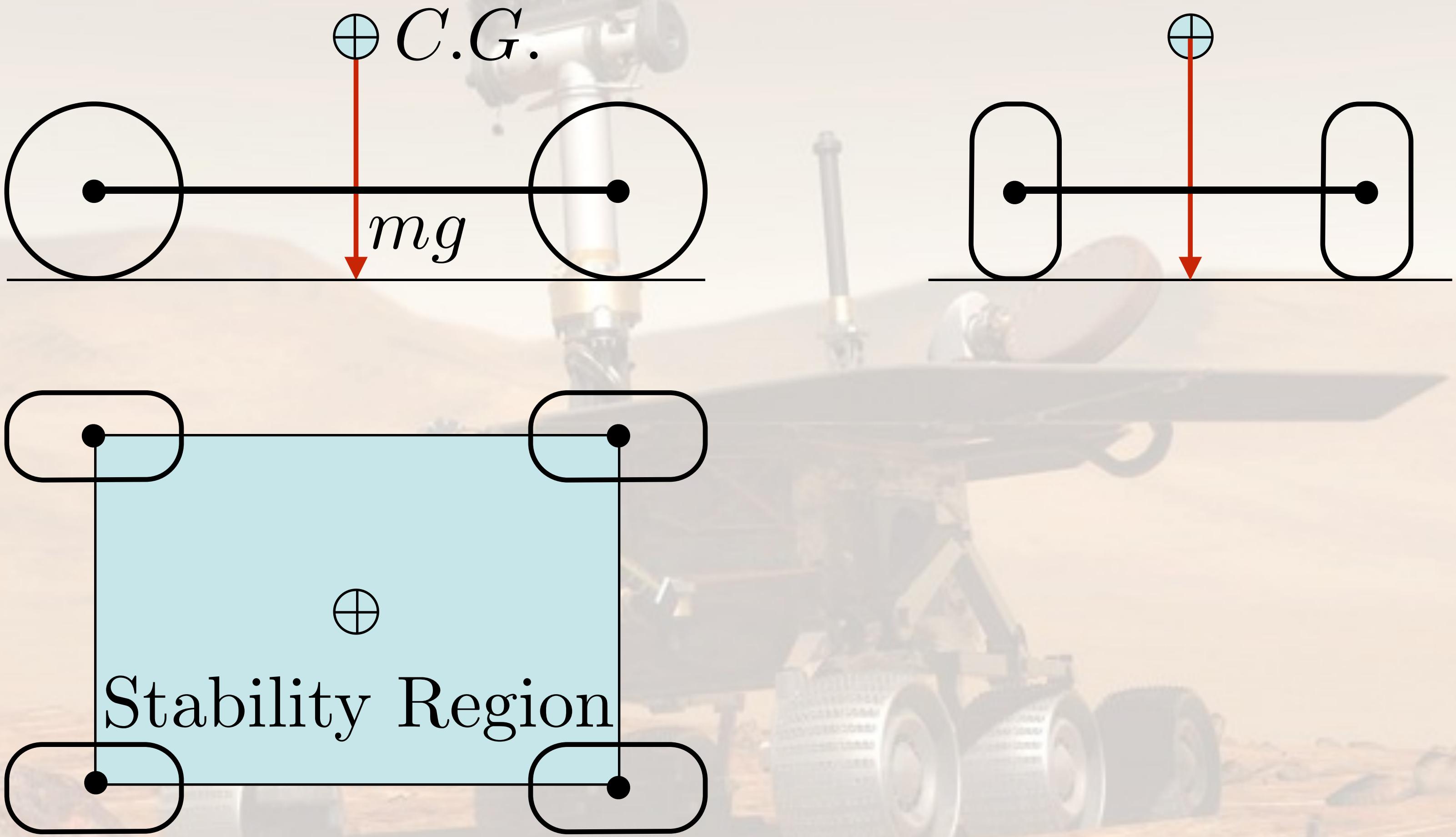
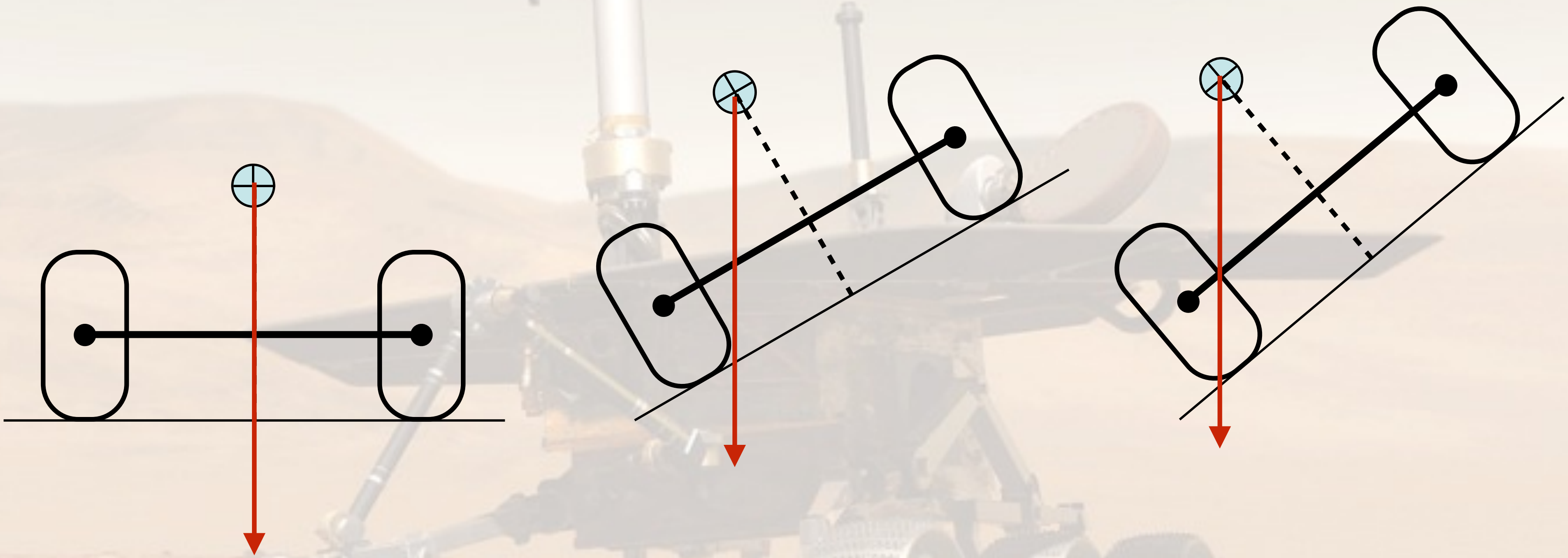
Slopes and Static Stability

- Stability across and along slopes
- Forces and torques on wheels
- Acceleration/deceleration
- Turning
- Hitting obstacles
- Rigid suspensions and obstacles

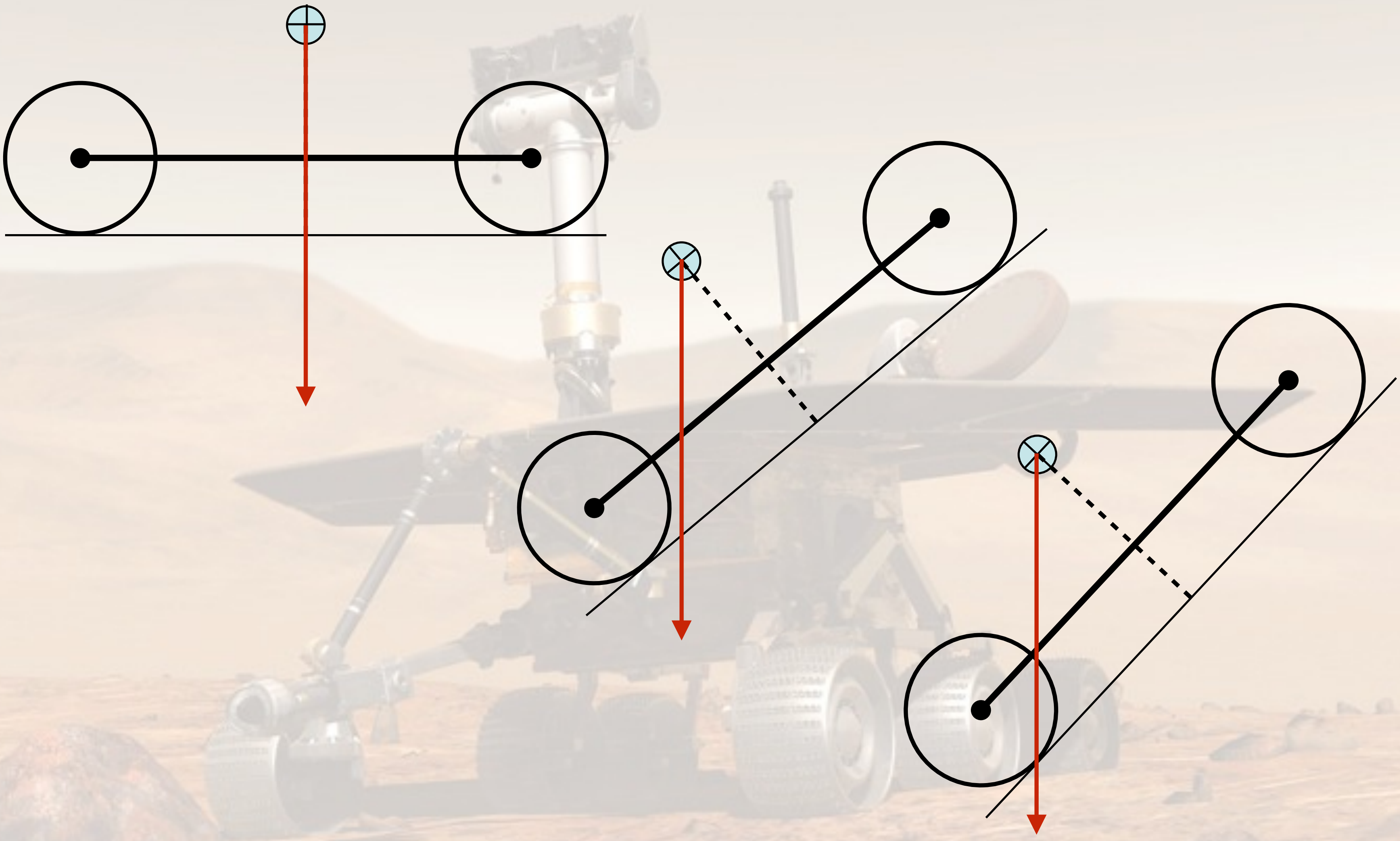
Rover with CG and Force Vector



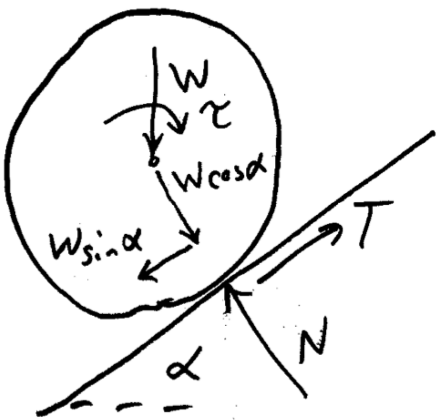
Rover on Cross Slope



Rover Climbing/Descending Slope



Slopes & Obstacles



wheel coefficient of friction with ground $= \mu$

$N \equiv$ normal force to surface

$$\tau = \mu r N = \mu r W \sin\alpha$$

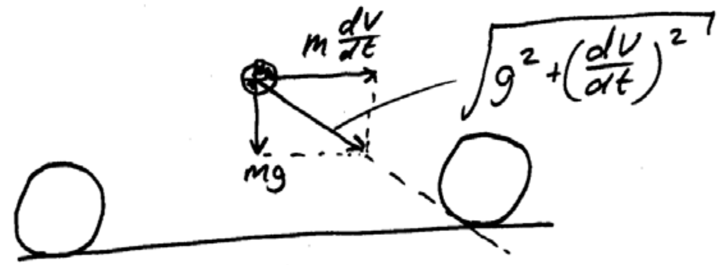
$$T = \mu N = \frac{\tau}{r} = W \sin\alpha$$

$$\mu W \cos\alpha = W \sin\alpha$$

$$\tan\alpha = \mu$$

Assume $\tau > \mu_{limit} N r$ (friction limited, not torque limited)

Acceleration / Deceleration



Time (sec)	dV/dt (m/sec^2)	Earth accel total (g's)
7	3.83	1.07
6	4.47	1.10
5	5.36	1.14
4	6.70	1.21

Accel: "0-60 mph in ___ seconds"

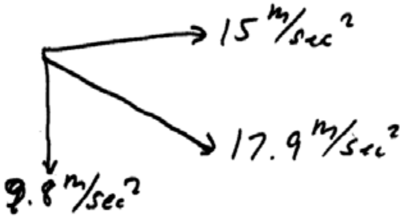
60 mph = 88 ft/sec = 26.8 m/sec

Decel: "1 car length / 10 mph"

60 mph, $\ell = 4m \Rightarrow 24\,m$ to stop

$s = \frac{1}{2}at^2 \quad v = at \Rightarrow s = \frac{1}{2}\frac{v^2}{a} \quad a = \frac{1}{2}\frac{v^2}{s}$

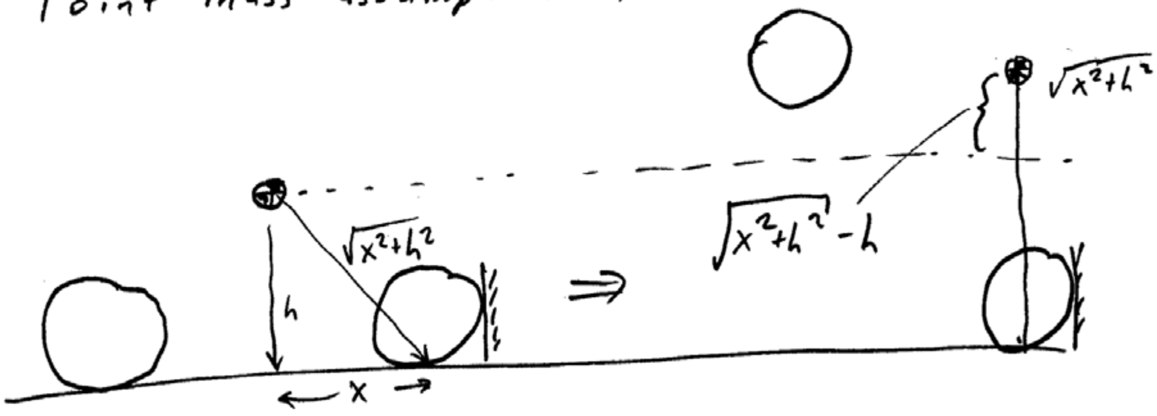
$a_{decel} = \frac{(26.8\,m/sec)^2}{2(24m)} = 14.96\,m/sec^2$



$\boxed{a_{decel} = 1.84\,g}$

Impulsive Pitchover Criteria

Point mass assumption $\Rightarrow$ conservative estimate



$$KE = PE$$

$$mg\left(\sqrt{x^2+h^2}-h\right) = \frac{1}{2} m V^2$$

$$\sqrt{x^2+h^2} = \frac{V^2}{2g} + h$$

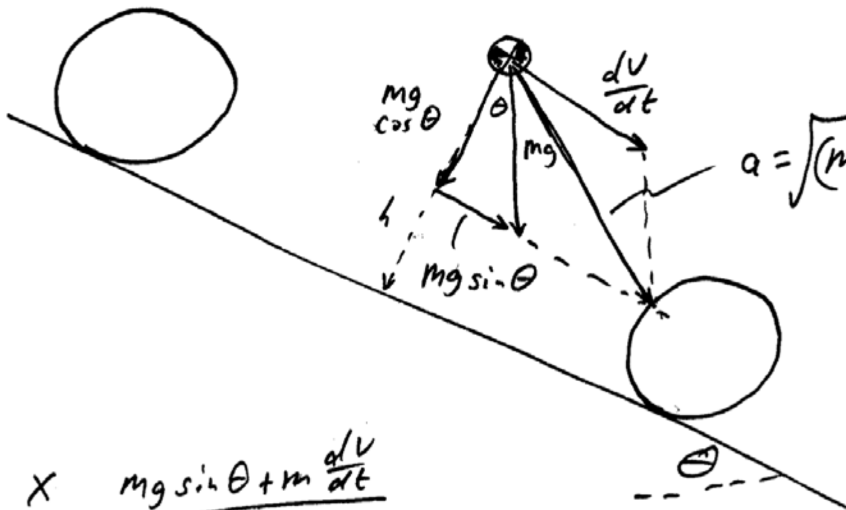
$$x^2+h^2 = \left[\frac{V^2}{2g}+h\right]^2$$

$$\frac{x}{h} = \sqrt{\frac{1}{h^2}\left[\frac{V^2}{2g}+h\right]^2-1} = \sqrt{\frac{V^2}{2gh}\left(\frac{V^2}{2gh}+2\right)}$$

moon: $\left.\frac{x}{h}\right|_{crit} = 1.969$

$\Uparrow$

Accel/Decel on Slopes



$$a = \sqrt{(mg\cos\theta)^2 + \left(mg\sin\theta + m\frac{dV}{dt}\right)^2}$$

$$\frac{X}{h} = \frac{mg\sin\theta + m\frac{dV}{dt}}{mg\cos\theta}$$

$$= \frac{g\sin\theta + \frac{dV}{dt}}{\cancel{m}g\cos\theta}$$

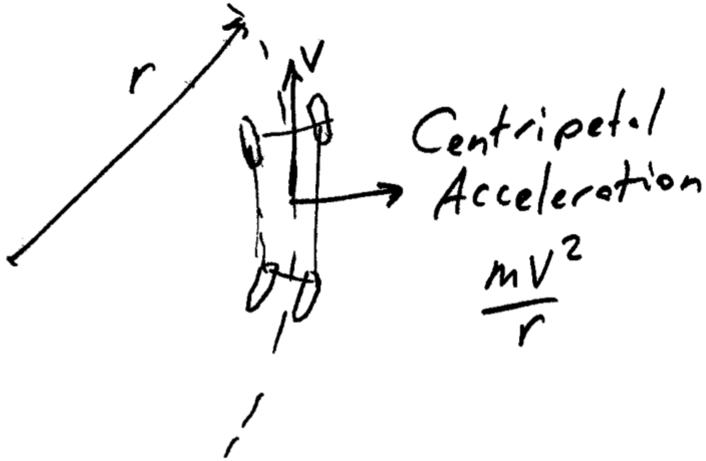
if $\theta = 0 \Rightarrow \frac{X}{h} = \frac{\frac{dV}{dt}}{g}$ ✓

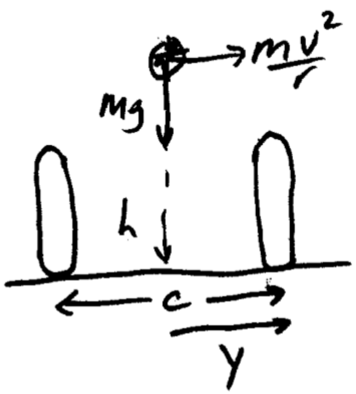
$$\frac{X}{h} g\cos\theta = g\sin\theta + \frac{dV}{dt} \Rightarrow \frac{dV}{dt} = g\left(\frac{X}{h}\cos\theta - \sin\theta\right)$$

Moon: $\frac{X}{h} = 1$

$\frac{dV}{dt}\Big\vert_{limit}$	θ	S_{stop} (10 km/hr)
1.6 m/sec²	0	2.42 m
1.3 m/sec²	10°	3.0 m
0.96 m/sec²	20°	4.0 m
0.59 m/sec²	30°	6.5 m

Lateral Stability





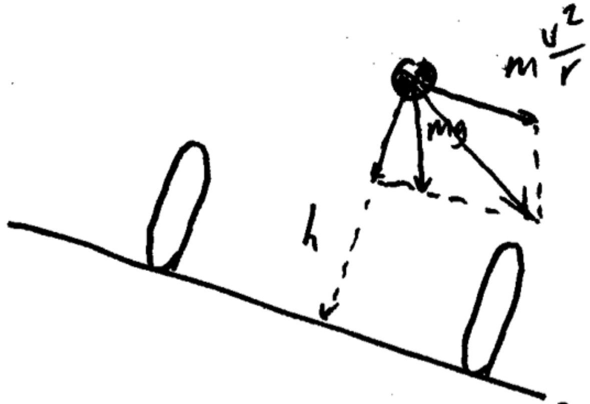
$$\frac{y}{h} = \frac{m\frac{V^2}{r}}{mg} = \frac{V^2}{gr}$$

set $\frac{y}{h}$, $g \Rightarrow \frac{V^2}{r} = \text{constant} \left(= g\frac{y}{h}\right)$

Doubling $V \Rightarrow$ increasing $r \times 4$!!

$r \propto \frac{1}{g}$ at same velocity

Turning on a slope



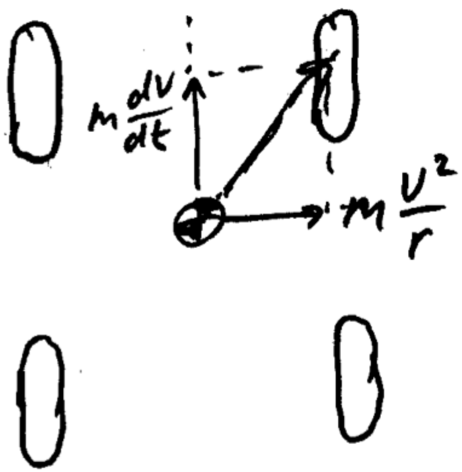
$$\frac{y}{h} = \frac{mg\sin\theta + m\frac{v^2}{r}}{mg\cos\theta}$$

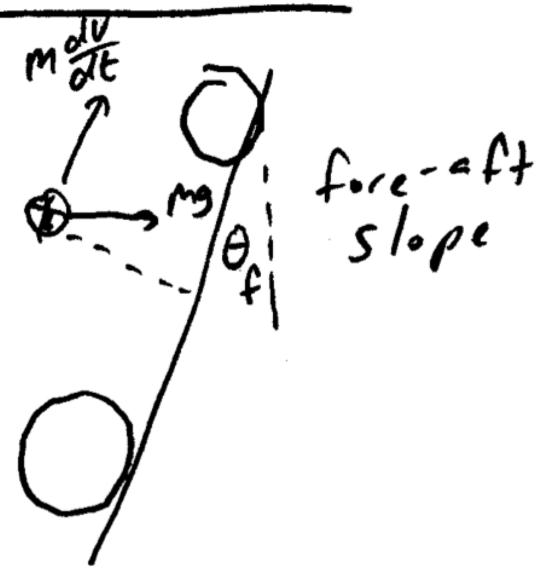
$$= \frac{\sin\theta + \frac{v^2}{gr}}{\cos\theta}$$

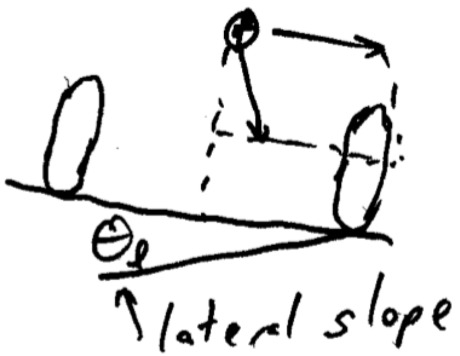
$$\frac{y}{h}\cos\theta - \sin\theta = \frac{v^2}{gr}$$

$$r = \frac{v^2}{g}\left[\frac{y}{h}\cos\theta - \sin\theta\right]^{-1}$$

Composite Stability



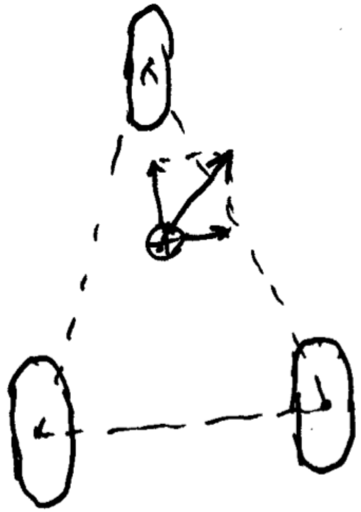




For conventional configuration,
(rectangular stability region)
lateral and longitudinal
stability are decoupled

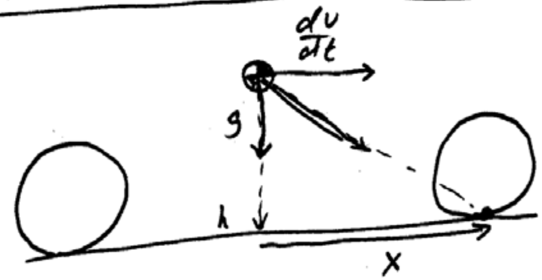
Alternative Configurations

Wagon Steering



Lateral and longitudinal forces are stable alone, but unstable together.

Deceleration Stability Limits



$$\frac{x}{h} = \frac{dV/dt}{g}$$

	V = 60 mph, $dV/dt = 15\ m/sec^2$, $(x/h)_{limit}$	V = 10 mph, $dV/dt = 2.5\ m/sec^2$
Earth	1.53	0.255
Mars	4.03	0.658
Moon	9.38	1.56

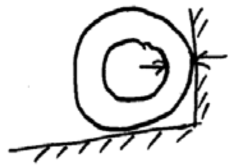
Less stable in lower gravity

60 mph panic stop on Earth = 10 mph panic stop on Moon

for given $\frac{x}{h}$: $\left.\frac{dV}{dt}\right|_{limit} \propto g \qquad S_{min} \propto \frac{1}{g}$

60 mph panic stop on Moon requires 147 m !!

"Hitting a Wall"



$S_{stop} \sim$ tire thickness

$\sim 0.3\,m$

e.g., pressurized rover @ 4000 kg

$$10\ kg/hr = 2.78\ m/sec\ (\cong 6\ mph)$$

$$KE = \frac{1}{2} m v^2 = 15.4\ KJ$$

$$a = \frac{v^2}{2S} = 12.9\ m/sec^2$$

$$\text{Moon:}\quad \frac{X}{h} = \frac{\frac{dv}{dt}}{g} = \frac{12.9}{1.6} = 8.1$$

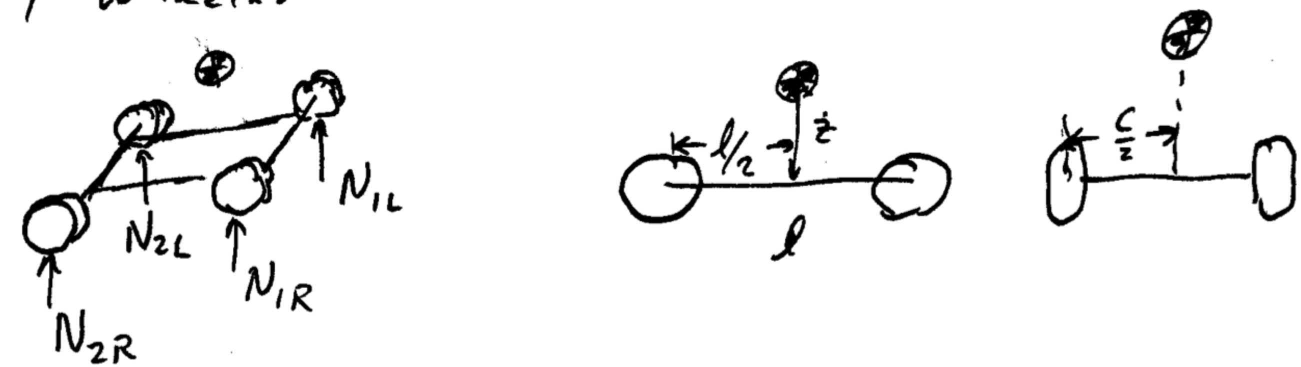
Assuming constant deceleration $F = ma =$

$$(4000\ kg)(12.9\ m/sec^2) = 51.6\ kN$$

$$(11{,}600\ lbs)$$

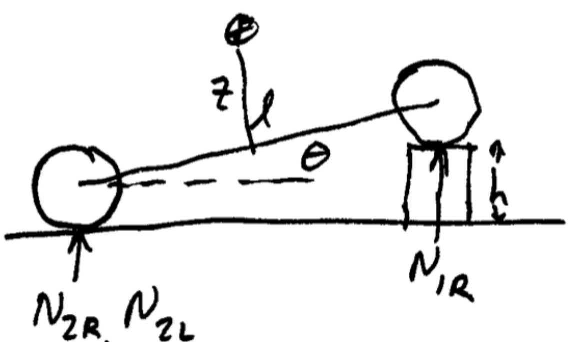
Static Forces on Rigid Suspension

4-wheeled cart



Level Ground: $N_{1R} = N_{2R} = N_{1L} = N_{2L} = \frac{W}{4}$

Obstacle under right front wheel



$N_{1L} = 0$

(left front wheel dangling in air)

Force Equilibrium over an Obstacle

$$\sum M_{\substack{back\\axle}} = N_{1R}\, l \cos\theta = W\left(\frac{l}{2}\cos\theta - z\sin\theta\right)$$

$$\sin\theta = \frac{h}{l} \qquad \cos\theta = \sqrt{1 - \frac{h^2}{l^2}}$$

Say, $h = \frac{3}{5} l$ (arbitrary)

$$\sin\theta = \frac{3}{5} \qquad \cos\theta = \frac{4}{5} \qquad z = \frac{l}{2}$$

$$N_{1R}\, \cancel{l} \left(\frac{4}{5}\right) = W\left(\frac{\cancel{l}}{2}\,\frac{4}{5} - \frac{\cancel{l}}{2}\,\frac{3}{5}\right)$$

$$N_{1R} = \frac{5}{4} W \left(\frac{4}{10} - \frac{3}{10}\right) = \frac{W}{8}$$ (was $\frac{W}{4}$)

$$N_{2R} + N_{2L} = \frac{7}{8} W$$

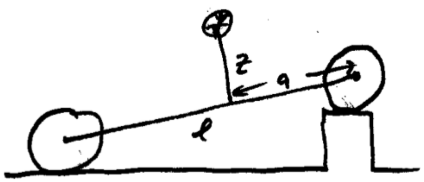
Lateral Equilibrium

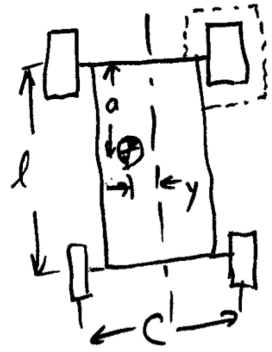
$$N_{2L} = N_{2R} + N_{1R} = \frac{1}{8} W + N_{2R} = \frac{W}{2}$$

$$\boxed{N_{1R} = \frac{W}{8};\quad N_{2R} = \frac{3W}{8};\quad N_{2L} = \frac{W}{2}}$$

Rigid Suspension - General Case

(Still has front right wheel on obstacle; front left wheel off the ground)





Σ Moments about rear axle

$$N_{1R}\,\ell\cos\theta = W\left[(\ell-a)\cos\theta - z\sin\theta\right]$$

$$N_{1R} = W\left(\frac{\ell-a}{\ell} - \frac{z}{\ell}\tan\theta\right)$$

Σ Moments about centerline

$$N_{2L}\,\frac{c}{2} = (N_{1R}+N_{2R})\frac{c}{2} + Wy$$

$$N_{2L} = N_{1R} + N_{2R} + \frac{2Wy}{c}$$

$$N_{2L} = W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) + N_{2R} + \frac{2Wy}{c}$$

Σ Forces

$$N_{1R} + N_{2R} + N_{2L} = W$$

$$2W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) + 2N_{2R} + \frac{2Wy}{c} = W$$

$$\boxed{\begin{aligned} N_{2R} &= \frac{W}{2} - W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) - \frac{Wy}{c} \\ N_{2L} &= \frac{W}{2} + \frac{Wy}{c} \\ N_{1R} &= W\left(\frac{l-a}{l} - \frac{z}{l}\tan\theta\right) \end{aligned}}$$

$a = \frac{l}{2}$ $\quad z = 0 \quad y = 0$ (CG in plane of axles)

$$N_{2R} = \frac{W}{2} - \frac{W}{2} = 0 \qquad N_{2L} = \frac{W}{2} \qquad N_{1R} = \frac{W}{2} \qquad N_{1L} = 0$$

$y \neq 0$

$$N_{2R} = -W\frac{y}{c} \qquad N_{2L} = \frac{W}{2} + \frac{Wy}{c} \qquad N_{1R} = \frac{W}{2}$$

if $y > 0$, $N_{2R} \to 0$