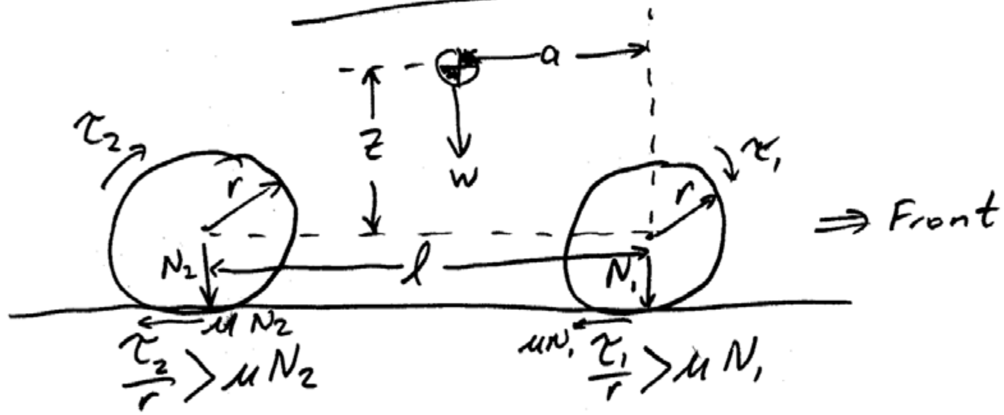
Multi-Wheel Systems

- Obstacle climbing with multiwheel systems
- Planar rocker analysis
- Planar rocker-bogey analysis
- Suspension dynamics

Multi-Wheel Drive Systems



Sum moments around rear wheel

$$N_1 \ell = W(\ell - a) - \mu r(\tau_1 + \tau_2)$$

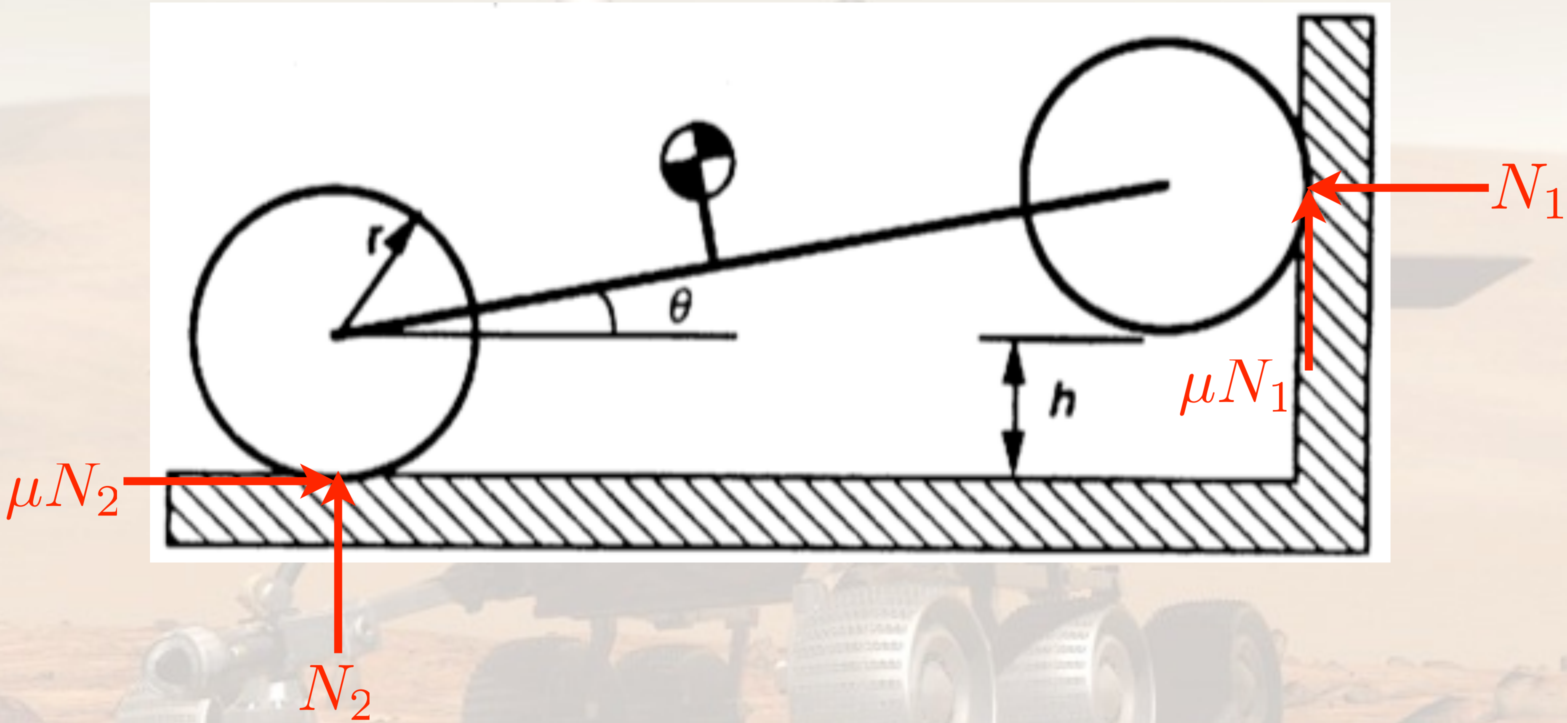
$$N_1 = W\frac{\ell - a}{\ell} - \frac{\mu r}{\ell}(\tau_1 + \tau_2)$$

$$N_1 + N_2 = W \Rightarrow N_2 = W - N_1$$

$$N_2 = W\frac{a}{\ell} + \frac{\mu r}{\ell}(\tau_1 + \tau_2)$$

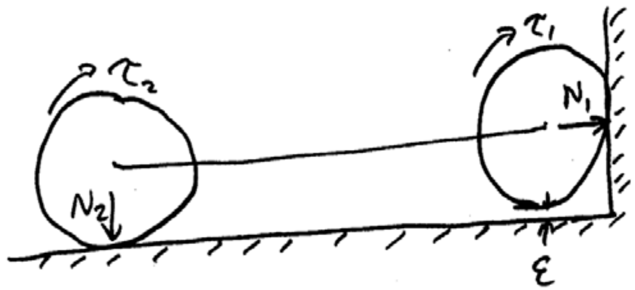
if $(\tau_1 + \tau_2)\mu \ll W$, $\quad N_1 = W\frac{\ell - a}{\ell} \quad N_2 = W\frac{a}{\ell}$ (static case)

Four-Wheeled Vehicle Climbing a Wall



from Howard Eisen, "Scale and Computer Modeling of Wheeled Vehicles for Planetary Exploration" S.M. Thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, May, 1990

Wall Climbing



$$\sum F_{vertical} \Rightarrow N_2 + \mu N_1 = W$$

$$\sum F_{horizontal} \Rightarrow \mu N_2 = N_1$$

$$\sum M_{rear\ axle} \Rightarrow \mu N_2 r + \mu N_1 (l+r) = W(l-a)$$

$$N_2 + \mu^2 N_2 = W \Rightarrow N_2 = \frac{W}{1+\mu^2} \Rightarrow N_1 = \frac{\mu}{1+\mu^2} W$$

$$\frac{\mu}{1+\mu^2} Wr + \frac{\mu^2}{1+\mu^2} W(l+r) = W(l-a)$$

$$(r+a)\mu^2 + r\mu - (l-a) = 0$$

$$\mu = \frac{-r \pm \sqrt{r^2 + 4(r+a)(l-a)}}{2(r+a)}$$

Let $\alpha \equiv \frac{a}{r}$ $\quad \lambda \equiv \frac{l}{r}$

$$\mu = \frac{-1 \pm \sqrt{1 + 4(1+\alpha)(\lambda - a)}}{2(1+\alpha)}$$

Assume $\alpha = \frac{\lambda}{2}$

$$\mu = -\frac{1}{2+\lambda} \pm \frac{\sqrt{1+4\left(1+\frac{\lambda}{2}\right)\left(\frac{\lambda}{2}\right)}}{2+\lambda} \quad \Big\} \rightarrow \quad 1 + 4\frac{\lambda}{2} + 4\frac{\lambda^2}{4}$$

$$= 1 + 2\lambda + \lambda^2$$

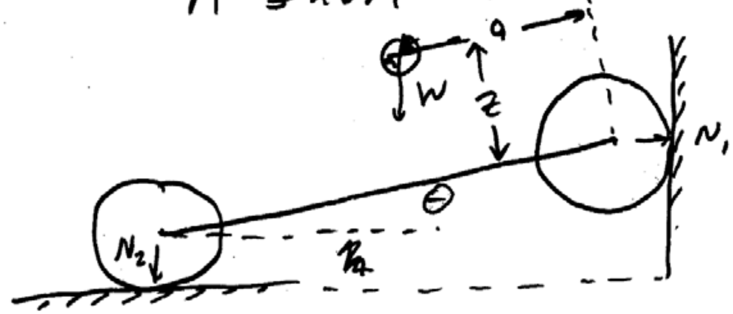
$$= (\lambda + 1)^2$$

$$= -\frac{1 \pm (\lambda+1)}{2+\lambda} = \frac{\lambda}{2+\lambda}, \ -1$$

$\lambda \to 0 \quad \mu_{limit} \to 0 \qquad \lambda \to \infty \quad \mu_{limit} \to 1$

Shorter is better!

A Short Time Later...



$h = l \sin\theta$

$$\sum F_{vert} \Rightarrow \mu N_1 + N_2 = W$$
$$\sum F_{horiz} \Rightarrow \mu N_2 = N_1$$

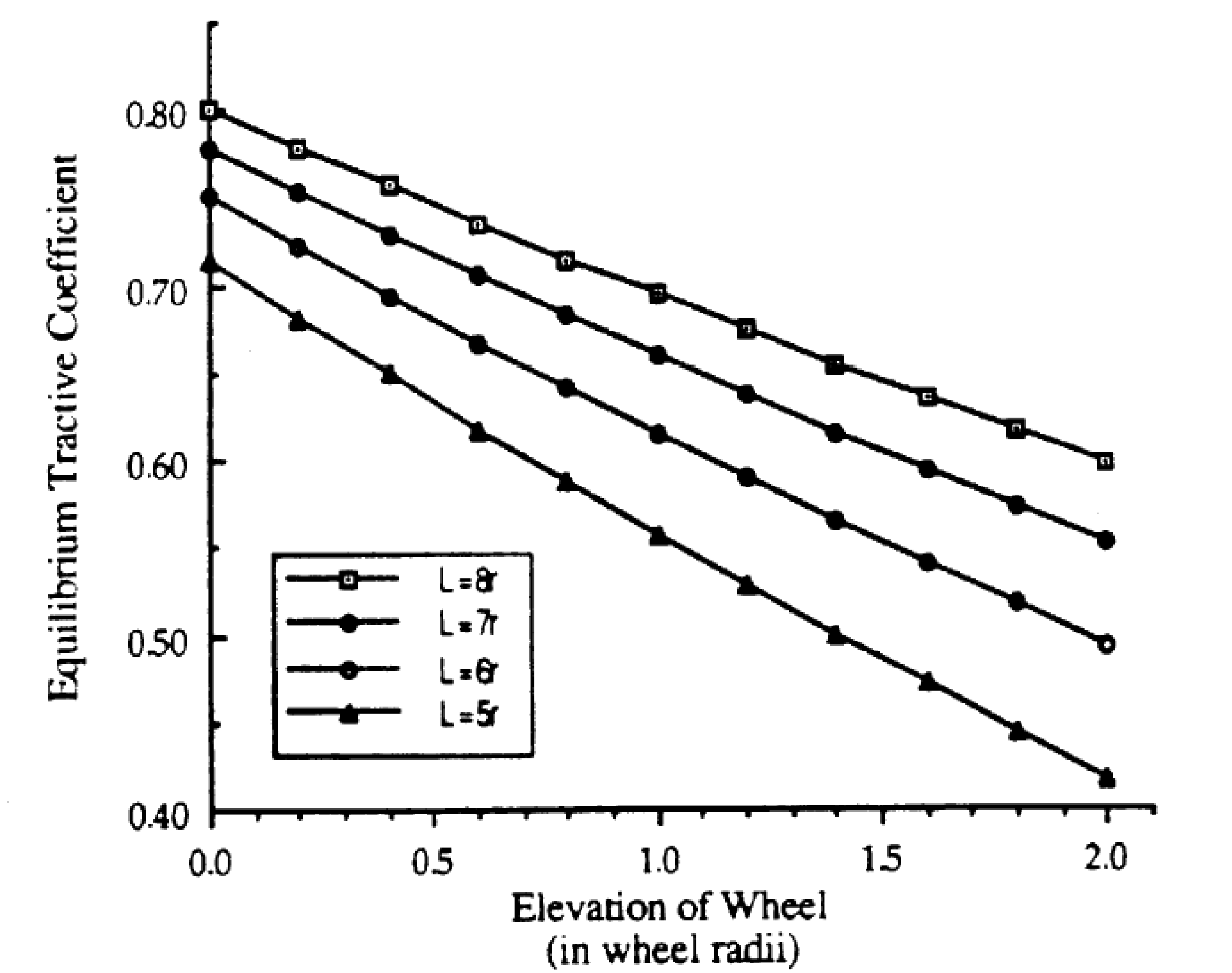
$$N_1 = \frac{\mu}{1+\mu^2} W \qquad N_2 = \frac{W}{1+\mu^2}$$

$$\sum M_{rear} \Rightarrow \mu N_2 r + N_1 l \sin\theta + \mu N_1 (r + l\cos\theta) = W\left[(l-a)\cos\theta - z\sin\theta\right]$$

As θ increases, effect of W decreases and effect of N_1 increases $\Rightarrow$

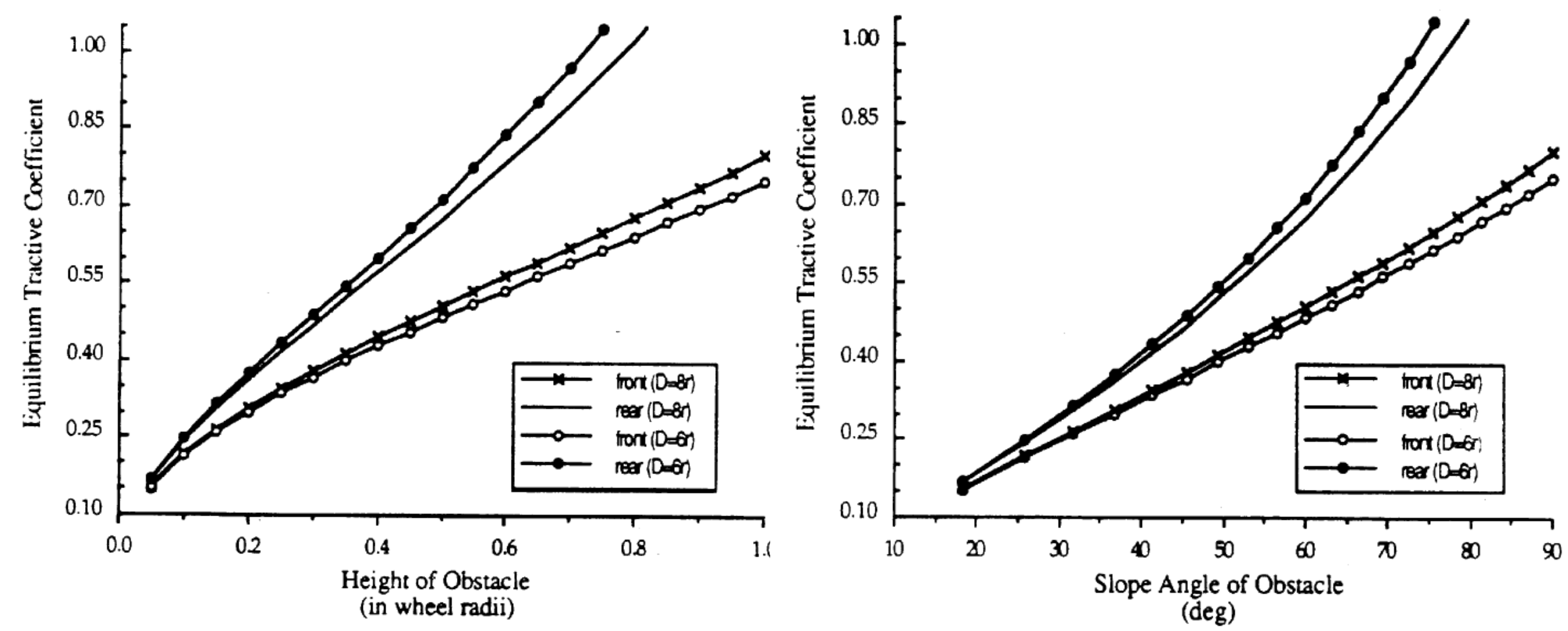
hardest point of the climb is at the start!

Required Traction for Wall Climbing



from Howard Eisen, “Scale and Computer Modeling of Wheeled Vehicles for Planetary Exploration” S.M. Thesis, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, May, 1990

Bump/Slope Traction Requirements



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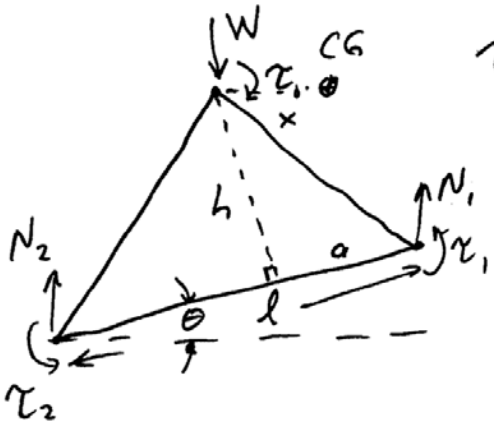


$\tau_0 = xW$

Planar Rocker Analysis

$\sum$ Forces: $N_1 + N_2 = W$

$\sum$ Moment (rear axle)

$$\tau_1 + \tau_2 + N_1 l \cos\theta = \tau_0 + W[(l-a)\cos\theta \mp h\sin\theta]$$

$$N_1 l \cos\theta = \tau_0 - \tau_1 - \tau_2 + W[(l-a)\cos\theta - h\sin\theta]$$

$$\boxed{N_1 = \frac{\tau_0 - \tau_1 - \tau_2}{l\cos\theta} + W\left(\frac{l-a}{l} - \frac{h}{l}\tan\theta\right)}$$

$$N_2 = W - N_1 = W\left(1 - \frac{l-a}{l} + \frac{h}{l}\tan\theta\right) - \frac{\tau_0 - \tau_1 - \tau_2}{l\cos\theta}$$

$$\boxed{N_2 = W\left(\frac{a}{l} + \frac{h}{l}\tan\theta\right) + \frac{\tau_1 + \tau_2 - \tau_0}{l\cos\theta}}$$

Nondimensionalize

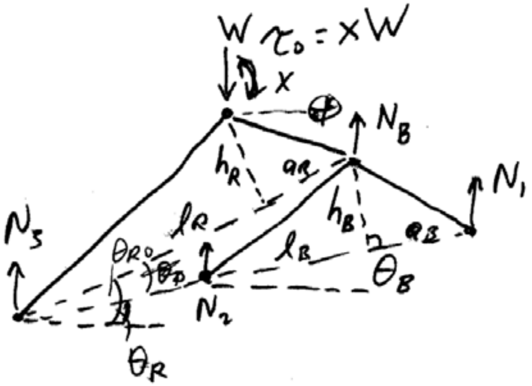
$$\tau_0 = W x \Rightarrow \frac{\tau_0}{W\ell} = \frac{x}{\ell}$$

$$\tau_{wheel} = T r \Rightarrow \frac{\tau_{wheel}}{W\ell} = \frac{T}{W}\frac{r}{\ell}$$

$$\frac{N_1}{W} = 1 - \frac{a}{\ell} - \frac{h}{\ell}\tan\theta + \left(\frac{x}{\ell} - \frac{T_1}{W}\frac{r_1}{\ell} - \frac{T_2}{W}\frac{r_2}{\ell}\right)\frac{1}{\cos\theta}$$

$$\frac{N_2}{W} = \frac{a}{\ell} + \frac{h}{\ell}\tan\theta + \left(\frac{T_1}{W}\frac{r_1}{\ell} + \frac{T_2}{W}\frac{r_2}{\ell} - \frac{x}{\ell}\right)\frac{1}{\cos\theta}$$

Planar Rocker-Bogey Analysis



For bogey,

$$N_1 = N_B\left(\frac{l_B - a_B}{l_B} - \frac{h_B}{l_B}\tan\theta_B\right) - \frac{\tau_1 + \tau_2}{l_B\cos\theta_B}$$

$$N_2 = N_B\left(\frac{a_B}{l_B} + \frac{h_B}{l_B}\tan\theta_B\right) + \frac{\tau_1 + \tau_2}{l_B\cos\theta_B}$$

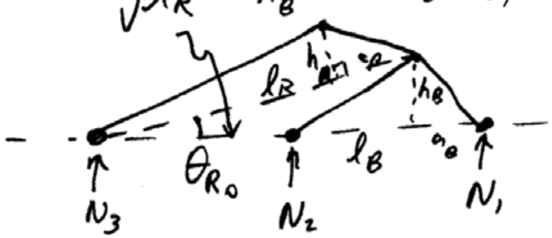
($\tau_B = 0$ always)

For rocker,

$$N_B = \frac{\tau_0 - \tau_3}{l_R\cos(\theta_R + \theta_{R_0})} + W\left(\frac{l_R - a_R}{l_R} - \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0})\right)$$

$$N_3 = W\left(\frac{a_R}{l_R} + \frac{h_R}{l_R}\tan(\theta_R + \theta_{R_0})\right) + \frac{\tau_3 - \tau_0}{l_R\cos(\theta_R + \theta_{R_0})}$$

$$\sqrt{l_R^2 - h_B^2} - (l_B - a_B) \equiv l_3 \qquad \theta_{R_0} = \sin^{-1}\frac{h_B}{l_R}$$



$$\Rightarrow \sqrt{(l_3 + l_B - a_B)^2 + h_B^2} = l_R \Rightarrow \frac{l_R}{l_B} = \sqrt{\left(\frac{l_3}{l_B} + 1 - \frac{a_B}{l_B}\right)^2 + \left(\frac{h_B}{a_B}\right)^2}$$

Normalize by W and l_B

$$\frac{N_3}{W} = \frac{a_R}{l_R} + \frac{h_R}{l_R}\tan(\theta_R+\theta_{R0}) + \left(\frac{T_3}{W}\frac{r_3}{l_B}\frac{l_B}{l_R} - \frac{x}{l_B}\frac{l_B}{l_R}\right)\frac{1}{\cos(\theta_R+\theta_{R0})}$$

$$\frac{N_B}{W} = 1 - \frac{a_R}{l_R} - \frac{h_R}{l_R}\tan(\theta_R+\theta_{R0}) + \left(\frac{x}{l_B}\frac{l_B}{l_R} - \frac{T_3}{W}\frac{r_3}{l_B}\frac{l_B}{l_R}\right)\frac{1}{\cos(\theta_R+\theta_{R0})}$$

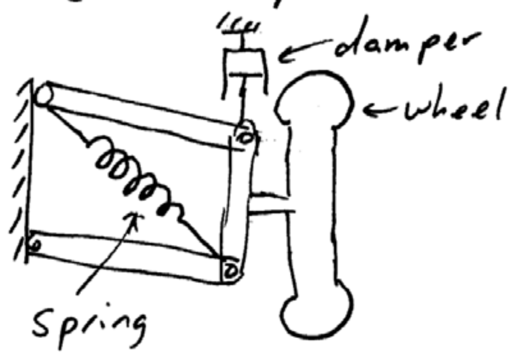
$$\frac{N_1}{W} = \frac{N_B}{W}\left(1 - \frac{a_B}{l_B} - \frac{h_B}{l_B}\tan\theta_B\right) - \left(\frac{T_1}{W}\frac{r_1}{l_B} + \frac{T_2}{W}\frac{r_2}{l_B}\right)\frac{1}{\cos\theta_B}$$

$$\frac{N_2}{W} = \frac{N_B}{W}\left(\frac{a_B}{l_B} + \frac{h_B}{l_B}\tan\theta_B\right) + \left(\frac{T_1}{W}\frac{r_1}{l_B} + \frac{T_2}{W}\frac{r_2}{l_B}\right)\frac{1}{\cos\theta_B}$$

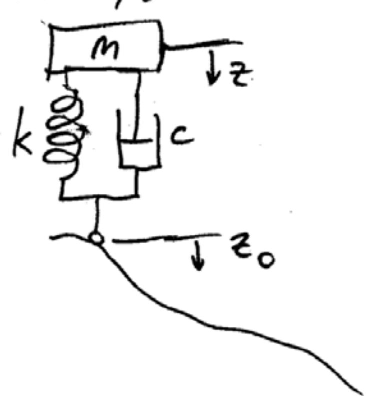
Suspension Systems

- Current planetary rovers (e.g., MER, MSL) have little or no shock absorption

- Notional car suspension



Analyze one wheel first



$$m\ddot{z} + c\dot{z} + kz = c\dot{z}_0 + kz_0$$

Undamped force-free solution

$$m\ddot{z} + kz = 0 \qquad z = Z_1 \cos \omega_n t$$

$$-m\omega_n^2 + k = 0 \qquad \ddot{z} = -Z_1 \omega_n^2 \cos \omega_n t$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Rover Example

$M_{TOT} = 500\,kg \Rightarrow$ wheel $m = \frac{M_{TOT}}{4} = 125\,kg$

d = deflection of suspension (at rest) ~ 0.1m

$$k = \frac{F}{d} = \frac{mg}{d}$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$f_n = \frac{\omega_n}{2\pi}$$

	Earth	Moon
k	12,250 N/m	2000 N/m
ω_n	9.9 rad/sec	4 rad/sec
f_n	1.6 Hz	0.64 Hz

l_{crit} = critical distance between bumps

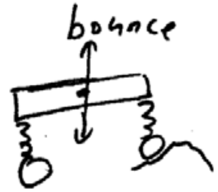
$$= \frac{V}{f_n}$$

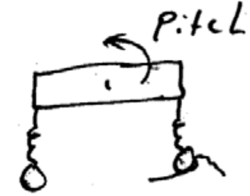
@10kph (2.8 m/sec)

	Earth	Moon
l_{crit}	1.8m	4.3m

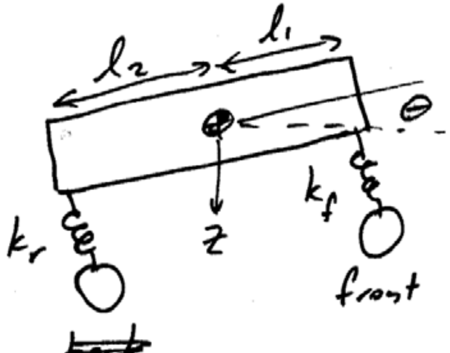
Multiwheel Analysis

Responses to two-wheels hitting a bump





We can excite both of these modes



Equation of Motion
(assuming no damping)

Bounce: $m\ddot{z} + k_f(z - l_1\theta) + k_r(z + l_2\theta) = 0$

(small angles)

Pitch: $I_y\ddot{\theta} + k_f l_1(z - l_1\theta) + k_r l_2(z + l_2\theta) = 0$

let $I_y = m r_y^2$ $\quad r_y$ = radius of gyration

Solve this set of coupled differential eqns

let $D_1 \equiv \dfrac{k_f + k_r}{m}$ $\quad D_2 = \dfrac{k_r l_2 - k_f l_1}{m}$ $\quad D_3 \equiv \dfrac{k_f l_1^2 + k_r l_2^2}{I_y}$

Rewrite in terms of D_1, D_2, D_3

$$\ddot{z} + D_1 z + D_2 \theta = 0$$

$$\ddot{\theta} + D_3 \theta + \frac{D_2}{r_y^2} z = 0$$

D_2 = coupling coefficient

Equations are independent if $D_2 = 0 \Rightarrow k_f l_1 = k_r l_2$

If $D_2 = 0$, force @ CG only produces bounce $\omega_{n_z} = \sqrt{D_1}$

force elsewhere produces pitch $\omega_{n_\theta} = \sqrt{D_3}$

Assume $D_2 \neq 0$

$z = Z_1 \cos \omega_n t \qquad \theta = \Theta \cos \omega_n t$

$$\left.\begin{aligned} (D_1 - \omega_n^2) Z_1 + D_2 \Theta &= 0 \\ \frac{D_2}{r_y^2} Z_1 + (D_3 - \omega_n^2)\Theta &= 0 \end{aligned}\right\} \quad \begin{vmatrix} D_1 - \omega_n^2 & D_2 \\ \frac{D_2}{r_y^2} & D_3 - \omega_n^2 \end{vmatrix} = 0$$

$$\omega_n^4 - (D_1 + D_3)\,\omega_n^2 + \left(D_1 D_3 - \frac{D_2^2}{r_y^2}\right) = 0$$

$$\omega_{n_1}^2 = \frac{D_1 + D_3}{2} \pm \frac{1}{2}\sqrt{(D_1 + D_3)^2 - 4\left(D_1 D_3 - \frac{D_2^2}{r_y^2}\right)}$$

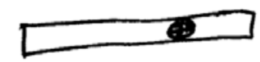
$\vdots$ (Insert algebra here)

$$\omega_{n_1}^2 = \frac{D_1 + D_3}{2} - \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

$$\omega_{n_2}^2 = \frac{D_1 + D_3}{2} + \sqrt{\frac{1}{4}(D_1 - D_3)^2 + \frac{D_2^2}{r_y^2}}$$

Example: $k_f = k_r = 2000\ \text{N/m}$ (moon)

$l_1 = 1\text{m} \quad l_2 = 2\text{m}$



$I = \frac{ml^2}{12} \Rightarrow I_y = 375\ \text{kg m}^2$

$r_y = 0.75\ \text{m}$

$$D_1 = \frac{4000}{500} = 8\ \frac{\text{N/m}}{\text{kg}} = \left\langle \frac{1}{\text{sec}^2} \right\rangle$$

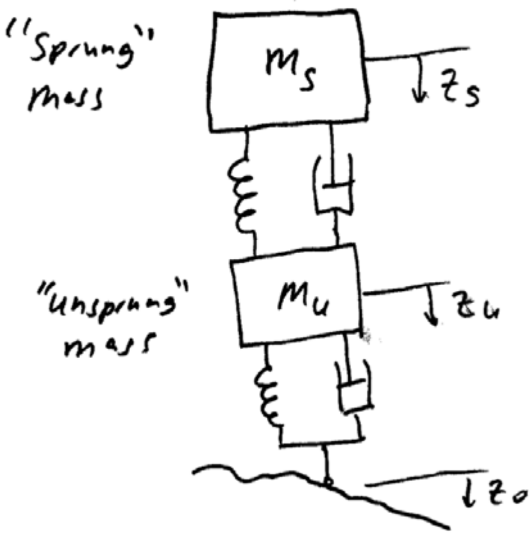
$$D_2 = \frac{2000\ \text{N/m}\,(2\text{m}) - 2000\ \text{N/m}\,(1\text{m})}{500\ \text{kg}} = 4\ \frac{\text{N}}{\text{kg}} = 4\ \text{m/sec}^2$$

$$D_3 = \frac{2000\ \text{N/m}\,(1\text{m})^2 + 2000\ \text{N/m}\,(2\text{m})^2}{375\ \text{kg m}^2} = 26.7\ \frac{\frac{\text{N}}{\text{m}}\text{m}^2}{\text{kg m}^2} = \left\langle \frac{1}{\text{sec}^2} \right\rangle$$

$$\omega_n^2 = 17.33 \pm 10.43$$

$$\boxed{\begin{aligned} \omega_{n_1} &= 2.63 \ \text{rad/sec} \Rightarrow 0.42 \ \text{Hz} \\ \omega_{n_2} &= 5.67 \ \text{rad/sec} \Rightarrow 0.84 \ \text{Hz} \end{aligned}}$$

Add in Tire Mass & Stiffness



Sprung Mass

$$m_s \ddot{z}_s + C_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = 0$$

Unsprung Mass

$$m_u \ddot{z}_u + C_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s) + C_u \dot{z}_u + k_u z_u = F(t) = C_u \dot{z}_o + k_u z_o$$

Undamped Force-Free Solutions

$$m_s \ddot{z}_s + k_s(z_s - z_u) = 0$$

$$m_u \ddot{z}_u + k_s(z_u - z_s) + k_u z_u = 0$$

$$z_s = Z_s \cos \omega_n t \qquad z_u = Z_u \cos \omega_n t$$

$$\begin{vmatrix} k_s - m_s \omega_n^2 & -k_u \\ -k_s & k_s + k_u - m_u \omega_n^2 \end{vmatrix} = 0$$

$$\omega_n^4 (m_u m_s) + \omega_n^2 (-m_s k_s - m_s k_u - m_u k_s) + k_s k_u = 0$$

$$\omega_{n_1} = \frac{+B - \sqrt{B^2 - 4AC}}{2A} \qquad \omega_{n_2} = \frac{B + \sqrt{B^2 - 4AC}}{2A}$$

$$A = m_u m_s \qquad B = m_s (k_s + k_u) + m_u k_s \qquad C = k_s k_u$$

Example: $m_s = 100\ kg$ $\quad m_u = 25\ kg$

$k_s = 2000\ N/m$ $\quad k_u = 10{,}000\ N/m$

$A = 2500\ kg^2$ $\quad B = 1.25 \times 10^6\ kg^2/sec^2$ $\quad C = 2 \times 10^7\ N^2/m^2$

$\omega_{n_1} = 4.76\ \frac{rad}{sec} \Rightarrow 0.8\ Hz \Leftarrow$ suspension frequency

$\omega_{n_2} = 21.8\ \frac{rad}{sec} \Rightarrow 3.5\ Hz \Leftarrow$ wheel ~~response~~ stiffness frequency