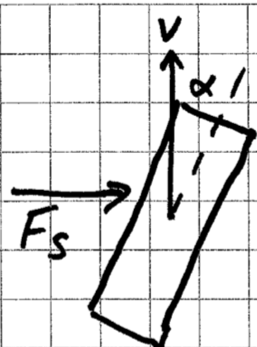
Steering and Tracks

- Side forces on wheel
- Power comparison between skid-steer and ideally steered
- Track systems
- Turning with tracks

Skid Steering

R = rolling resistance $\langle N \rangle$

Drive power $= Rv \quad \langle \frac{Nm}{sec} \rangle = \langle W \rangle$



Skidding wheel

Side force $\quad F_s = \mu_s N$

Slip power $= F_s v_s = \mu_s N v \sin\alpha$

Total power $\quad P_{sw} = v(R + \mu_s N \sin\alpha)$

can define $\quad R = \mu_r N$

$$P_w = vN(\mu_r + \mu_s \sin\alpha)$$

$$= v\mu_r N\left(1 + \frac{\mu_s}{\mu_r}\sin\alpha\right)$$

$$P_w = P_{roll}\left(1 + \frac{\mu_s}{\mu_r}\sin\alpha\right)$$

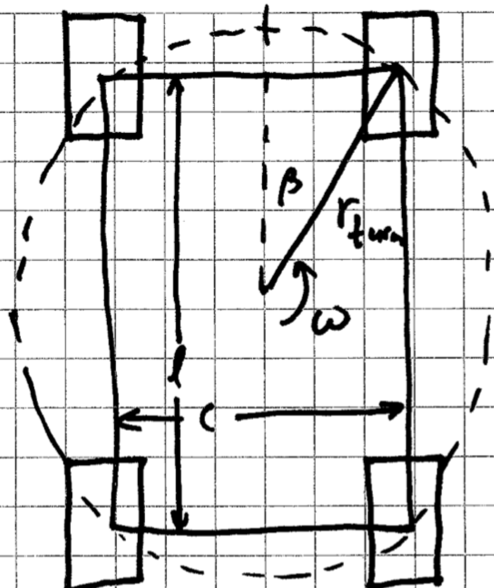
roads $\Rightarrow \mu_r \approx 0.05$ or less

offroad $\Rightarrow \mu_r \sim 0.2 \qquad \mu_s \sim 1$

$$P_{slip} = P_{roll} \quad \text{if} \quad \frac{\mu_s}{\mu_r} \sin\alpha = 1$$

$$\frac{\mu_s}{\mu_r} \sim 5 \Rightarrow \sin\alpha = 0.2 \Rightarrow \alpha = 11.5^\circ$$

Turn in Place (Skid Steer)



$$r_{turn} = \sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$\cos\beta = \frac{c/2}{r_{turn}} \qquad \sin\beta = \frac{l/2}{r_{turn}}$$

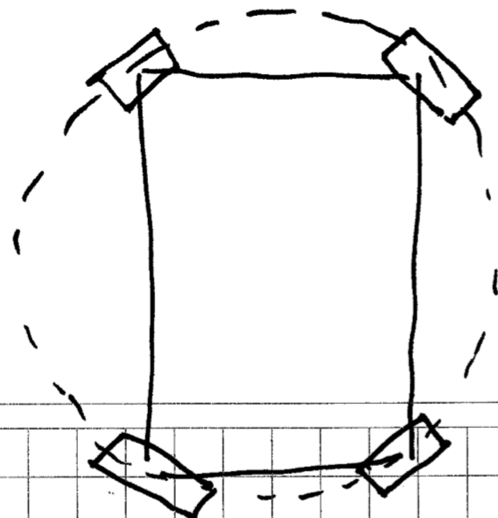
$$V_r = \omega r_{turn} \cos\beta = \frac{\omega c}{2}$$

$$V_s = \omega r_{turn} \sin\beta = \frac{\omega l}{2}$$

$$P_{roll} = V_r \mu_r N = \frac{\omega c}{2} \mu_r N$$

$$P_{slip} = V_s \mu_s N = \frac{\omega l}{2} \mu_s N$$

$$P_w = \omega N \left(\frac{c}{2} \mu_r + \frac{l}{2} \mu_s \right) = \frac{\omega N}{2} (c \mu_r + l \mu_s)$$



Turn-in-Place (steered)

$$P_w = \omega r_{turn} \mu_r N$$

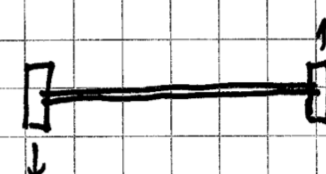
$$\text{T.I.P.:}\quad \frac{P_{skid}}{P_{steer}} = \frac{\frac{\omega N}{2}(c\mu_r + l\mu_s)}{\omega r_{turn} \mu_r N}$$

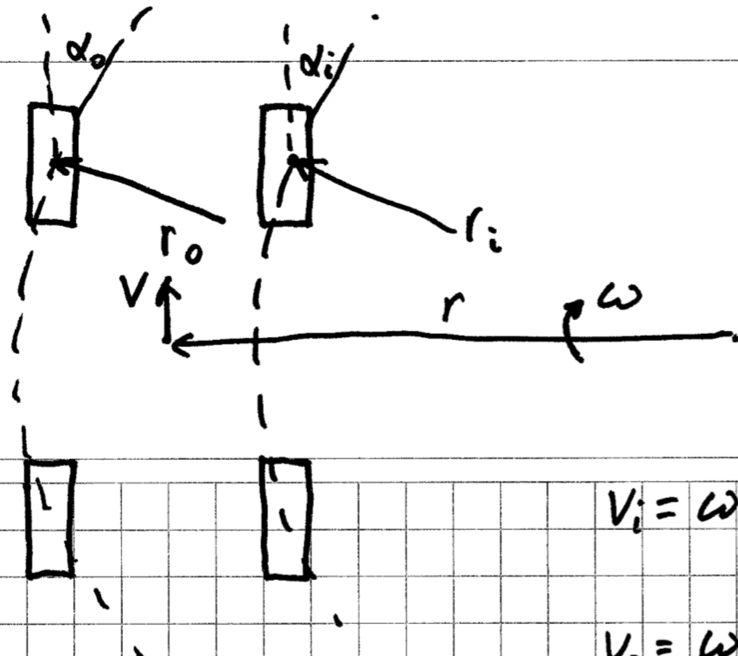
$$= \frac{1}{2}\,\frac{c + l\frac{\mu_s}{\mu_r}}{r_{turn}} = \frac{1}{2}\,\frac{c + l\frac{\mu_s}{\mu_r}}{\sqrt{\left(\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}}$$

$$\frac{P_{skid}}{P_{steer}} = \frac{c + \frac{\mu_s}{\mu_r} l}{\sqrt{c^2 + l^2}}$$

$\frac{\mu_s}{\mu_r} \sim 5 \Rightarrow$ Skid power goes up with l

$\frac{P_{skid}}{P_{steer}} \rightarrow 1$ for $l \rightarrow 0$





$$\omega r = V$$

$$r_i = \sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$r_o = \sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$V_i = \omega r_i = \omega\sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$V_o = \omega r_o = \omega\sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$\sin\alpha_i = \frac{r - \frac{c}{2}}{r_i} \qquad \cos\alpha_i = \frac{l/2}{r_i}$$

$$\sin\alpha_o = \frac{r + \frac{c}{2}}{r_o} \qquad \cos\alpha_o = \frac{l/2}{r_o}$$

Four-Wheel Steer

$$P_{w_i} = \omega r_i \mu_r N = \frac{V}{r}\sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}\,\mu_r N$$

$$P_{w_o} = \omega r_o \mu_r N = \frac{V}{r}\sqrt{\left(r + \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}\,\mu_r N$$

Outer Wheel: $P_{tot} = P_{r_o} + P_{so} = \frac{V}{r}\left[\frac{l}{2}\mu_r + \left(r + \frac{c}{2}\right)\mu_s\right] N$

$$\frac{P_{skid}}{P_{steer}} = \frac{\frac{V}{r}\left[\frac{l}{2}\mu_r + \left(r+\frac{c}{2}\right)\mu_s\right]N}{\frac{V}{r}\sqrt{\left(r+\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}\,\mu_r N} = \frac{\frac{l}{2} + \left(r+\frac{c}{2}\right)\frac{\mu_s}{\mu_r}}{\sqrt{\left(r+\frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}}$$

$$= \frac{l/2}{r_o} + \frac{r+\frac{c}{2}}{r_o}\frac{\mu_s}{\mu_r} = \cos\alpha_o + \sin\alpha_o \frac{\mu_s}{\mu_r}$$

α	$\frac{P_{skid}}{P_{steer}}$
0°	1
2°	1.17
5°	1.43
10°	1.85
20°	2.65
30°	3.37
45°	4.24
60°	4.83
90°	5

Four-Wheel Skid Turn

Inner Wheel: $P_{r_i} = \omega r_i \cos\alpha_i \mu_r N = \frac{V}{r} r_i \frac{l/2}{r_i} \mu_r N = \frac{V}{r}\frac{l}{2}\mu_r N$

$P_{s_i} = \omega r_i \sin\alpha_i \mu_s N = \frac{V}{r} r_i \frac{r - c/2}{r_i} \mu_s N = \frac{V}{r}\left(r - \frac{c}{2}\right)\mu_s N$

Outer Wheel: $P_{r_o} = \omega r_o \cos\alpha_o \mu_r N = \frac{V}{r} r_o \frac{l/2}{r_o} \mu_r N = \frac{V}{r}\frac{l}{2}\mu_r N$

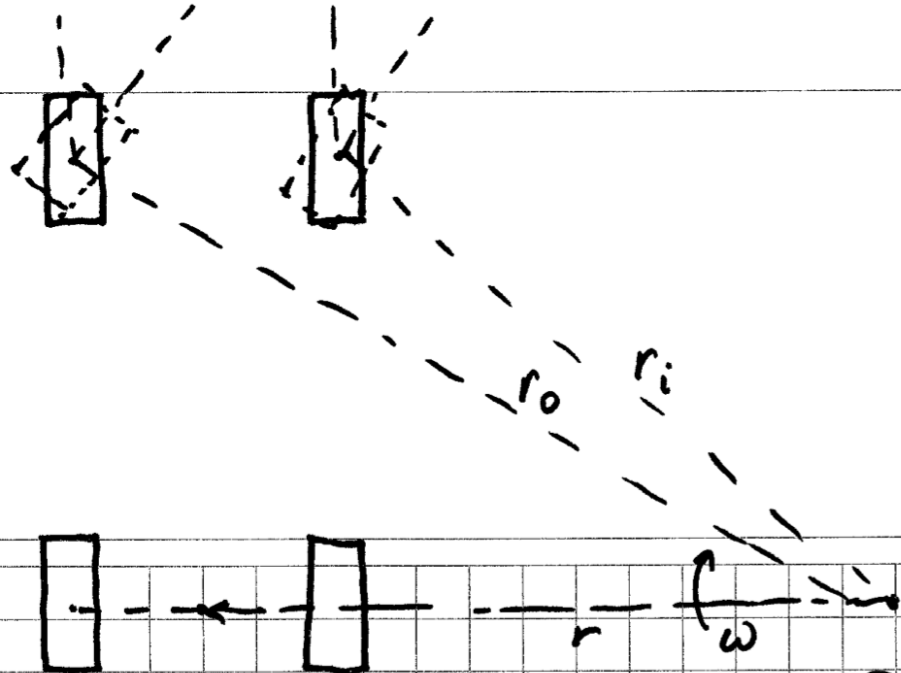
$P_{s_o} = \omega r_o \sin\alpha_o \mu_s N = \frac{V}{r} r_o \frac{r + c/2}{r_o} \mu_s N = \frac{V}{r}\left(r + \frac{c}{2}\right)\mu_s N$

Inner Wheel: $P_{tot} = P_{r_i} + P_{s_i} = \frac{V}{r}\left[\frac{l}{2}\mu_r + \left(r - \frac{c}{2}\right)\mu_s\right] N$

$$\frac{P_{skid}}{P_{steer}} = \frac{\frac{V}{r}\left[\frac{l}{2}\mu_r + \left(r - \frac{c}{2}\right)\mu_s\right] N}{\frac{V}{r}\sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}\,\mu_r N}$$

$$= \frac{\frac{l}{2} + \left(r - \frac{c}{2}\right)\frac{\mu_s}{\mu_r}}{\sqrt{\left(r - \frac{c}{2}\right)^2 + \left(\frac{l}{2}\right)^2}}$$

$$= \frac{l/2}{r_i} + \frac{r - c/2}{r_i}\frac{\mu_s}{\mu_r} = \cos\alpha_i + \sin\alpha_i \frac{\mu_s}{\mu_r}$$



Single-axis Steering

$$r_i = \sqrt{\left(r + \frac{c}{2}\right)^2 + \ell^2}$$

$$r_o = \sqrt{\left(r + \frac{c}{2}\right)^2 + \ell^2}$$

$$P_{i_r} = \frac{V}{r}\left(r - \frac{c}{2}\right) \mu_r N$$

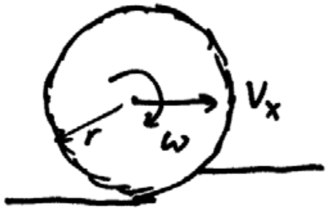
$$P_{o_r} = \frac{V}{r}\left(r + \frac{c}{2}\right) \mu_r N$$

$$P_{i_f} = \frac{V}{r}\sqrt{\left(r - \frac{c}{2}\right)^2 + \ell^2}\, \mu_r N$$

$$P_{o_f} = \frac{V}{r}\sqrt{\left(r + \frac{c}{2}\right)^2 + \ell^2}\, \mu_r N$$

$$P_{tot} = \left(2r + \sqrt{\left(r - \frac{c}{2}\right)^2 + \ell^2} + \sqrt{\left(r + \frac{c}{2}\right)^2 + \ell^2}\right) \frac{V}{r} \mu_r N$$

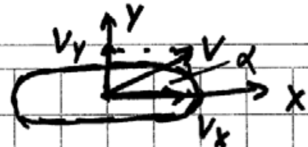
Wheel-Soil Interaction in Turn



Slip ratios

$$s = \frac{r\omega - V_x}{r\omega} \quad \text{for driving: } |r\omega| > |V_x|$$

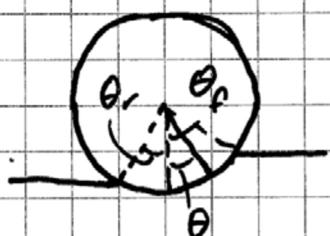
$$s = \frac{r\omega - V_x}{V_x} \quad \text{for braking: } |r\omega| < |V_x|$$



$$-1 \leq s \leq 1$$

Slip angle $\alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$

Wheel sinkage $\quad P(z) = \left(\frac{k_c}{b} + k_\phi\right) z^n$



$$z(\theta) = r(\cos\theta - \cos\theta_r)$$

for static sinkage, $\theta_f = \theta_r = \theta_s$

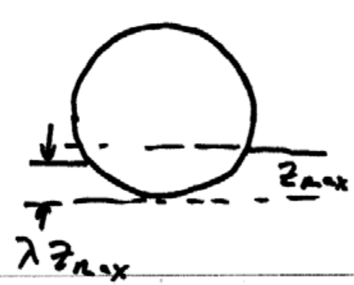
$$z(\theta) = r(\cos\theta - \cos\theta_s)$$

$$P(\theta) = \left(\frac{k_c}{b} + k_\phi\right) r^n (\cos\theta - \cos\theta_s)^n$$

Given weight W on wheel,

$$W = \int_{-\theta_s}^{\theta_s} P(\theta)\, b\, r \cos\theta\, d\theta = r^{n+1}(k_c + k_\phi b) \int_{-\theta_s}^{\theta_s} (\cos\theta - \cos\theta_s)^n \cos\theta\, d\theta$$

Static sinkage $\quad z_s = r(1 - \cos\theta_s)$



Soil entry angle $\theta_f = \cos^{-1}\left(1 - \frac{z_{max}}{r}\right)$

exit angle $\theta_r = \cos^{-1}\left(1 - \lambda \frac{z_{max}}{r}\right)$

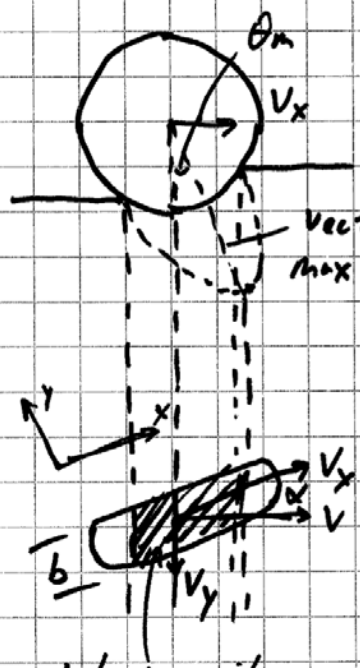
λ = wheel sinkage ratio

= 0 when no soil restitution occurs

= 1 if soil completely rebounds (static sinkage)

can be > 1 if soil is transported to back of wheel

Wheel Stress Distribution



$$\sigma(\theta) = r^n\left(\frac{k_c}{b} + k_\phi\right)\left[\cos\theta - \cos\theta_f\right]^n \qquad \theta_m \le \theta < \theta_f$$

$$\sigma(\theta) = r^n\left(\frac{k_c}{b} + k_\phi\right)\left[\cos\left\{\theta_f - \frac{\theta - \theta_r}{\theta_m - \theta_r}(\theta_f - \theta_m)\right\} - \cos\theta_f\right]^n$$

$$\theta_r < \theta \le \theta_m$$

$$\theta_m = (a_0 + a_1 s)\,\theta_f$$

$a_0 \sim 0.4 \quad 0 \le a_1 \le 0.3$

Soil Shear Stresses

$$\tau_x(\theta) = (c + \sigma(\theta)\tan\phi)\left[1 - e^{-j_x(\theta)/k_x}\right]$$

$$\tau_y(\theta) = (c + \sigma(\theta)\tan\phi)\left[1 - e^{-j_y(\theta)/k_y}\right]$$

Soil Deformations

$$j_x(\theta) = r\left[\theta_f - \theta - (1-s)(\sin\theta_f - \sin\theta)\right]$$

$$j_y(\theta) = r(1-s)(\theta_f - \theta)\tan\alpha$$

Resolve into orthogonal forces

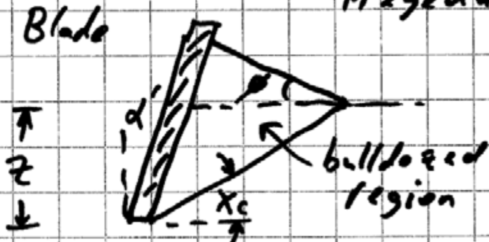
Drawbar pull $$F_x = rb\int_{\theta_r}^{\theta_f}\{\tau_x(\theta)\cos\theta - \sigma(\theta)\sin\theta\}\,d\theta$$

Side force $$F_y = \int_{\theta_r}^{\theta_f}\{\underbrace{r_b\,\tau_y(\theta)}_{F_u} + \underbrace{R_b\,(r - z(\theta)\cos\theta)}_{F_s}\}\,d\theta$$

F_u are side loads generated under the wheel

F_s are side loads generated by bulldozing by side of wheel

Hegedus bulldozing theory



$$R_b(z) = D_1\left[c\,z(\theta) + D_2\,\frac{\rho\,z^2(\theta)}{2}\right]$$

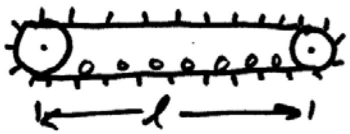
$$D_1(X_c, \phi) = \cot X_c + \tan(X_c + \phi)$$

$$D_2(X_c, \phi) = \cot X_c + \cot^2 X_c / \cot\phi$$

Bekker destructive angle $X_c = \frac{\pi}{4} - \frac{\phi}{2}$

Vertical Force $$F_z = rb\int_{\theta_r}^{\theta_f}\{\tau_x(\theta)\sin\theta + \sigma(\theta)\cos\theta\}\,d\theta$$

Tracked Vehicles



Bekker soil sinkage equation

$$p = k z_0^n = \left(\frac{k_c}{b} + k_\phi\right) z_0^n \Rightarrow z_0 = \left(\frac{p}{k_c/b + k_\phi}\right)^{1/n}$$

Uniform Pressure $\Rightarrow$ $p = \frac{W}{A} = \frac{W}{bl}$ $\quad (A = bl)$

$$z_0 = \left[\frac{W/bl}{(k_c/b + k_\phi)}\right]^{1/n}$$

$$\text{Work} = bl\int_0^{z_0} p\,dz = bl\int_0^{z_0}\left(\frac{k_c}{b} + k_\phi\right) z^n\,dz$$

$$= bl\left(\frac{k_c}{b} + k_\phi\right)\frac{z_0^{n+1}}{n+1}$$

Substituting, $\text{Work} = \frac{bl}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)^{1/n}}\left(\frac{W}{bl}\right)^{\frac{n+1}{n}} = R_c\, l$

$$R_c = \frac{b}{(n+1)\left(\frac{k_c}{b} + k_\phi\right)^{1/n}}\left(\frac{W}{bl}\right)^{\frac{n+1}{n}}$$

$$= \frac{1}{(n+1)\, b^{1/n}\left(\frac{k_c}{b} + k_\phi\right)^{1/n}}\left(\frac{W}{l}\right)^{\frac{n+1}{n}}$$

$$= \frac{1}{(n+1)(k_c + k_\phi b)^{1/n}}\left(\frac{W}{l}\right)^{\frac{n+1}{n}}$$

$\Leftarrow$ motion resistance due to soil compaction by uniformly loaded tread

Maximum tractive effort F_{max} determined by terrain shear strength τ_{max} and contact area A

$$F_{max} = A\tau_{max} = A(c + P\tan\phi) = Ac + W\tan\phi$$

Sandy soil: $c \to 0$ $\quad F_{max} \cong W\tan\phi$ $\quad \phi \sim 35^\circ$

$\tan\phi \sim 0.7$

$F_{max} \sim 0.7\,W$

Clay soil: $\phi \to 0$ $\quad F_{max} \cong Ac$ no effect of weight

larger contact area $\Rightarrow$ larger force

Slip definition for track

$$i = 1 - \frac{V}{r\omega} = 1 - \frac{V}{V_t} = \frac{V_t - V}{V_t} = \frac{V_j}{V_t}$$

r, ω refer to drive sprocket

V_j is speed of slip $\rightarrow$ opposite to vehicle motion

(skidding - same direction as vehicle motion)

since tread cannot stretch, local slip velocity V_j constant everywhere

Distance of slip $j = V_j t$ $\quad t = \dfrac{x}{V_j}$ $\quad j = \dfrac{V_j x}{V_t} = ix$ $\quad x =$ vehicle distance traveled

From soil mechanics,

$$\tau = \tau_{max}(1-e^{-j/K}) = (c + p \tan\phi)(1-e^{-j/K})$$

τ = shear stress j = shear displacement

K = shear deformation modulus

Total tractive effort $F = b\int_0^l \tau\, dx$

$$F = b\int_0^l (c + p \tan\phi)(1-e^{-j/K})\, dx$$

For normal distribution $p = \frac{W}{bl}$ everywhere

$$F = b\int_0^l \left(c + \frac{W}{bl}\tan\phi\right)(1-e^{-ix/K})\, dx$$

$$= (Ac + W\tan\phi)\left[1 - \frac{K}{il}\left(1-e^{-il/K}\right)\right]$$

(Taylor's Series: for l/K small, $i \sim 1 \Rightarrow e^{-il/K} \cong 1 - \frac{l}{K}$

$[\ ]$ becomes $\left[1 - \frac{K}{l}\left(1 - 1 + \frac{l}{K}\right)\right] = 1$ for 100% slip (?)

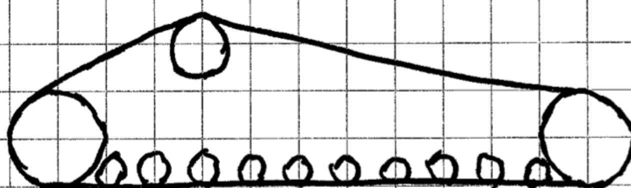
100% slip $F \cong Ac + W\tan\phi$

Consider two vehicles: $A_1 = A_2$, $W_1 = W_2$, but $l_1 = 2l_2$ so $b_1 = b_2/2$

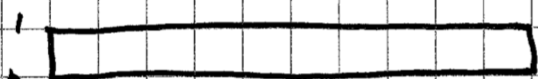
Set tractive forces to be the same

$$1 - \frac{K}{i_1 l_1}\left(1 - e^{-i_1 l_1/K}\right) = 1 - \frac{K}{i_2 l_1/2}\left(1 - e^{-i_2 l_2/K}\right)$$

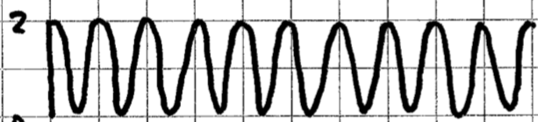
$e^{-il/K}$ small $\Rightarrow$ $i_1 = \frac{1}{2} i_2$ longer track slips less



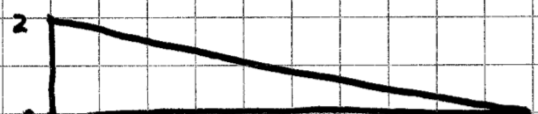
Different Loading Patterns



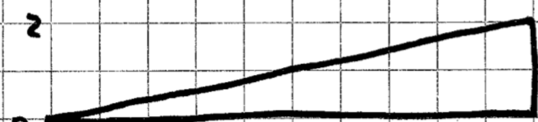
Linear



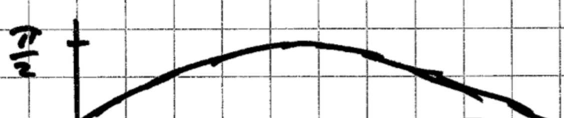
Multipeak Sinusoidal



Linear increasing front $\rightarrow$ back



Linear increasing back $\rightarrow$ front



Single sinusoidal

Example: 2 vehicles @ 135 kN = W

Terrain: $n = 1.6$ $k_c = 4.37\ kN/m^{2.6}$ $k_\phi = 196.72\ kN/m^{3.6}$

$K = 5\ cm$ $c = 1.0\ kPa$ $\phi = 19.7°$

$A = 7.2\ m^2$ (both) Unit A: $b = 1m$ $l = 3.6m$

Unit B: $b = 0.8m$ $l = 4.5m$

Unit A: Sinkage $z_0 = \left(\frac{p}{k_c/b + k_\phi}\right)^{1/n} = \left(\frac{135/7.2}{4.37/1 + 196.72}\right)^{0.625} = 0.227m$

$$R_c = 2b\left(\frac{k_c}{b} + k_\phi\right)\frac{z_0^{n+1}}{n+1} = 3.28\ kN$$

Unit B: $z_0 = 0.226m$ $R_c = 2.6\ kN$

F_{max} (unit A) $= 2blc + W\tan\phi = 2 \times 1 \times 3.6 \times 1 + 135 \times 0.358 = 55.54\ kN$

Thrust Values

Slip	A	B
5%	40.54	43.32
10%	47.84	49.37
20%	51.68	52.46
40%	53.62	54.0
60%	54.25	54.51
80%	54.57	54.77

Effect of Track Loading Patterns

Assume frictional soil ($c \to 0$)

Multipeak sinusoidal pressure

$$P = \frac{W}{bl}\left(1 + \cos\frac{2n\pi x}{l}\right) \qquad n = \# \text{ of peaks}$$

$$\tau = \frac{W}{bl}\tan\phi\left(1 + \cos\frac{2n\pi x}{l}\right)\left(1 - e^{-\frac{ix}{K}}\right)$$

$$F = b\int_0^l \frac{W}{bl}\tan\phi\left(1 + \cos\frac{2\pi n x}{l}\right)\left(1 - e^{-\frac{ix}{K}}\right)dx$$

$$F = W\tan\phi\left[1 + \frac{K}{il}\left(e^{-\frac{il}{K}} - 1\right) + \frac{K\left(e^{-il/K} - 1\right)}{il\left(1 + 4n^2K^2\frac{\pi^2}{i^2}l^2\right)}\right]$$

Increasing linearly front $\to$ back

$$P = 2\frac{W}{bl}\frac{x}{l}$$

$$F = W\tan\phi\left[1 - 2\left(\frac{K}{il}\right)^2\left(1 - e^{-\frac{il}{K}} - \frac{il}{K}e^{-\frac{il}{K}}\right)\right]$$

Increasing linearly Back-front

$$P = 2\frac{W}{bl}\,\frac{l-x}{l}$$

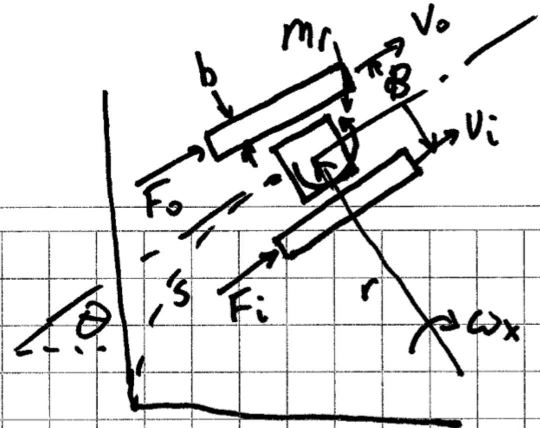
$$F = 2W\tan\phi\left[1-\frac{K}{il}\left(1-e^{-il/k}\right)\right]-W\tan\phi\left[1-2\left(\frac{K}{il}\right)^2\left(1-e^{-il/k}-\frac{il}{K}e^{-il/k}\right)\right]$$

Single sinusoid, 0 at front and back

$$P = \frac{W}{bl}\,\frac{\pi}{2}\sin\left(\frac{\pi x}{l}\right)$$

$$F = W\tan\phi\left[1-\frac{e^{-\frac{il}{k}}+1}{2\left(1-i^2l^2/\pi^2k^2\right)}\right]$$

Steering of Tracked Vehicles



$$m\frac{d^2 s}{dt^2} = F_o + F_i - R_{tot}$$

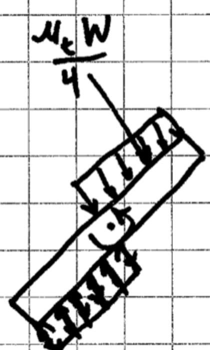
$$I_z \frac{d^2\theta}{dt^2} = \frac{B}{2}(F_o - F_i) - m_r$$

M_r = moment due to turning resistance

Under steady-state conditions (no acceleration)

$$F_o + F_i - R_{tot} = 0$$

$$\frac{B}{2}(F_o - F_1) - M_r = 0$$



$$F_o = \frac{R_{tot}}{2} + \frac{m_r}{2} = \frac{\mu_r W}{2} + \frac{m_r}{B}$$

$$F_i = \frac{R_{tot}}{2} - \frac{m_r}{c} = \frac{\mu_r W}{2} - \frac{m_r}{B}$$

For a normal pressure profile uniformly distributed

Lateral resistance $R_l = \dfrac{\mu_t W}{2l}$

μ_t depends on soil and track design:

Steel Tracks, Hard ground: 0.55-0.58 ~~hard ground~~ 0.87-1.11 grass

Rubber Tracks, hard ground: 0.65-0.66 grass 0.67-1.14

$$M_r = 4\,\frac{W\mu_t}{2l}\int_0^{l/2} x\,dx = \frac{\mu_t W l}{4}$$

$$F_o = \frac{\mu_r W}{2} + \frac{\mu_t W l}{4B}$$

$$F_i = \frac{\mu_r W}{2} - \frac{\mu_t W l}{4B}$$

Vehicle with uniform normal pressure distribution turning at low speed on level ground.

Maximum thrust of vehicle is limited by terrain and vehicle properties -

Outside track: $F_o \leq cbl + \frac{W\tan\phi}{2}$

$$\frac{\mu_r W}{2} + \frac{\mu_t W l}{4B} \leq cbl + \frac{W\tan\phi}{2}$$

Massage to get

$$\frac{l}{B} \leq \frac{1}{\mu_t}\left(\frac{4cA}{W} + 2\tan\phi - 2\mu_r\right) \qquad A = bl$$

To steer without spinning outside track,

$$\frac{l}{B} \leq \frac{2}{\mu_t}\left(\frac{c}{p} + \tan\phi - \mu_r\right) = \frac{2}{\mu_t}\left(\frac{2cbl}{W} + \tan\phi - \mu_r\right)$$

Examples:

Sandy terrain $c=0$ $\phi=30^\circ$ $\mu_t=0.5$ $\mu_r=0.1$

$$\frac{l}{B}\leq 1.9$$

Clay soil $c=3.45\ kPa$ $\phi=10^\circ$ $P=6.9\ kPa$ $(=1\ psi)$

$\mu_t=0.4$ $\mu_r=0.1$

$$\frac{l}{B}\leq 2.88$$

$$F_i=\frac{\mu_r W}{2}-\frac{\mu_t W l}{4B} \Rightarrow \frac{\mu_t l}{2B}<\mu_r \text{ for } F_i \geq 0$$

$\mu_t=0.5$ $\mu_r=0.1$ $\frac{l}{B}=1.5$ $\frac{0.5}{2}(1.5)=0.375>0.1$

$\Rightarrow$ inside track has to brake to turn

Max thrust of outside track is limited by terrain

$\Rightarrow$ braking inner track diminishes steering, DP

Tracked Vehicle Steering Example

$W = 155.7\,kN$ (75,000 lb) $B = 203.2\,cm$

$l = 304.8\,cm$ $b = 76.2\,cm$ $c = 3.45\,kPa$

$\phi = 25°$ $\mu_r = 0.15$ $\mu_t = 0.5$

1) Determine steerability – $\frac{l}{B} \leq \frac{2}{\mu_t}\left(\frac{c}{p} + \tan\phi - \mu_r\right) \leq 1.67$

Actual $\frac{l}{B} = 1.5$ so it __is__ steerable

2) Analyze thrust during turn

$$F_o = \frac{\mu_r W}{2} + \frac{\mu_t W l}{4B} = 40.87\,kN$$

$$F_i = \frac{\mu_r W}{2} - \frac{\mu_t W l}{4B} = -17.52\,kN$$

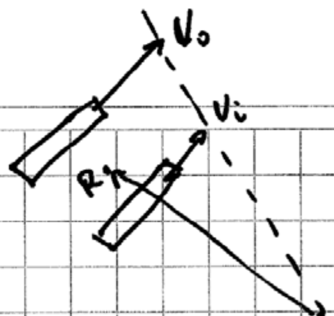
Kinematics of Tracked Skid Steering

Outside drive wheel turns at ω_o

Inside " " " " ω_i

Assume tracks do not skid or slip

Turn radius R yaw velocity Ω_z



By similar triangles $\dfrac{V_o}{R+\frac{B}{2}} = \dfrac{V_i}{R-\frac{B}{2}} \Rightarrow V_o R - V_o \frac{B}{2} = V_i R + V_i \frac{B}{2}$

$$R = \frac{B}{2}\,\frac{V_o+V_i}{V_o-V_i}$$

drive sprocket radius r $\quad V = \omega r$

$$R = \frac{B}{2}\,\frac{\omega_o r + \omega_i r}{\omega_o r - \omega_i r} \qquad K_s = \frac{\omega_o}{\omega_i}$$

$$= \frac{B}{2}\,\frac{K_s+1}{K_s-1}$$

$$\frac{\omega_o r + \omega_i r}{2R} = \frac{\omega_o r - \omega_i r}{B}$$

$$\Omega_z = \frac{r\omega_o + r\omega_i}{2R} = \frac{r\omega_i (K_s-1)}{B}$$

Outside track always thrusts $\Rightarrow$ always slips

$$R' = \frac{B[r\omega_o(1-i_o) + r\omega_i(1-i_i)]}{2[r\omega_o(1-i_o) - r\omega_i(1-i_i)]}$$

$$= \frac{B[K_s(1-i_o) + (1-i_i)]}{2[K_s(1-i_o) - (1-i_i)]}$$

$$\Omega'_z = \frac{r\omega_o(1-i_o) + r\omega_i(1-i_i)}{2R'}$$

$$\Omega'_z = \frac{r\omega_o(1-i_o) + r\omega_i(1-i_i)}{2R'}$$

$$= \frac{r\omega_i[K_s(1-i_o) - 1 - (i_i)]}{B}$$

$$= \frac{r\omega_i\,[K_s(1-i_o) - 1 - i_i]}{B}$$

$\frac{R'}{R}$ always >1 (slip causes larger turning radius)

$\frac{\Omega_z'}{\Omega_z}$ always <1 (slower turn)