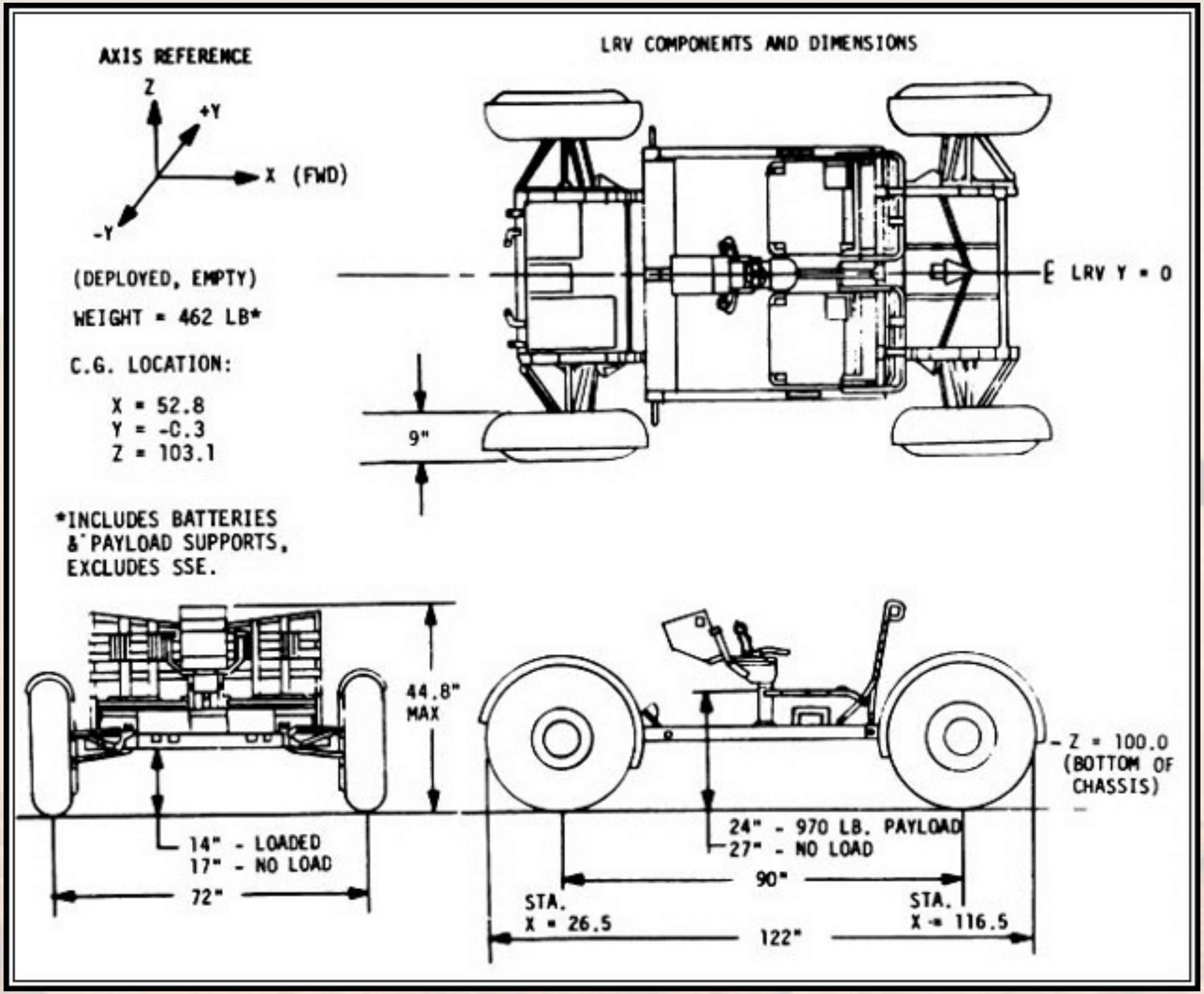
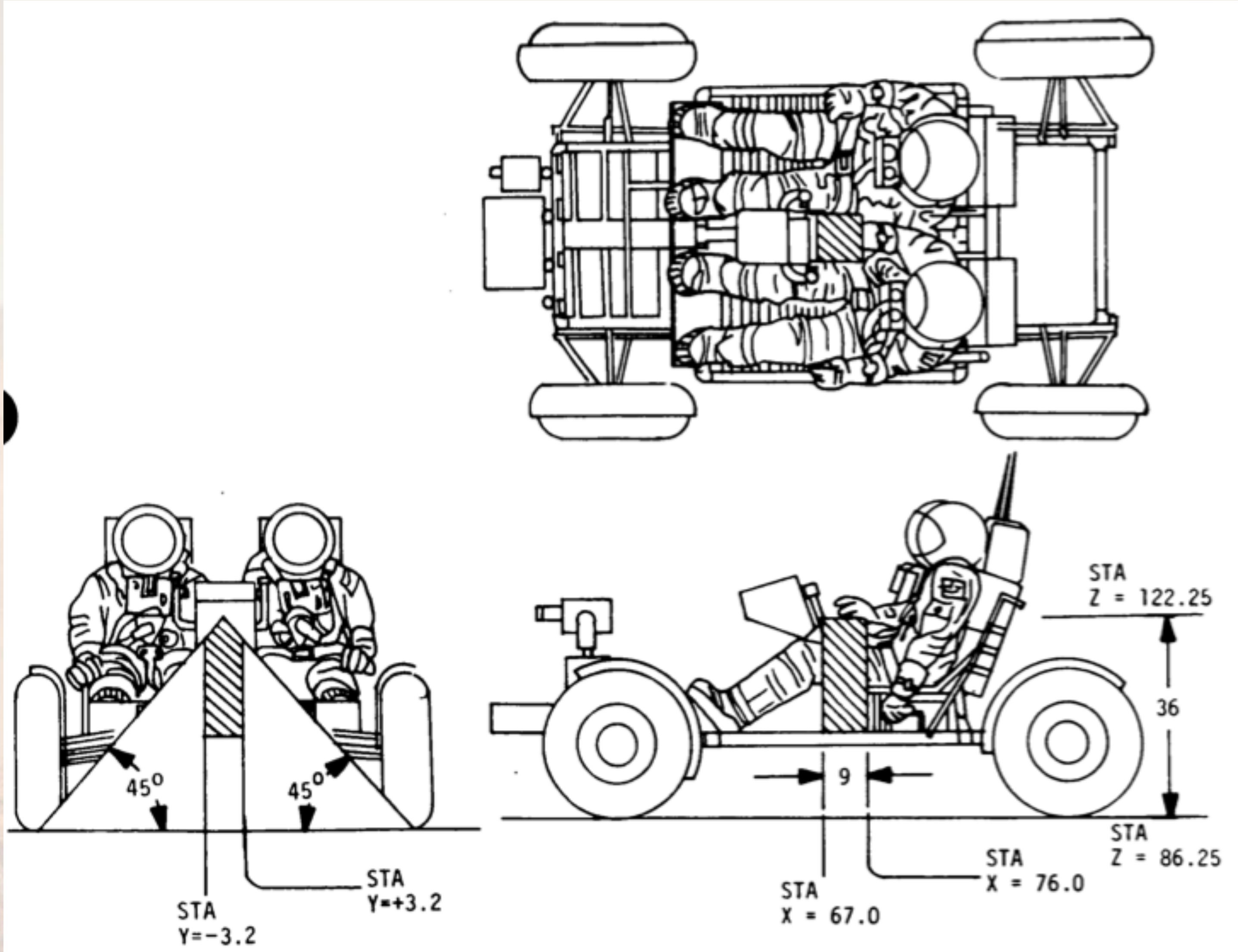
Terramechanics V

- Note - I haven't posted the slides from Tuesday because there were a number of typos (and outright mistakes) that were (almost) all corrected on Thursday. This set of slides are the corrected slides from both Tuesday and Thursday in the appropriate order to make a full set of slides on terramechanics.
- I know this is late (and I apologize for that), so if you would like extra time on the problem set, take it with no penalty. I'd rather you understand and do it right than worry about grades at this point.

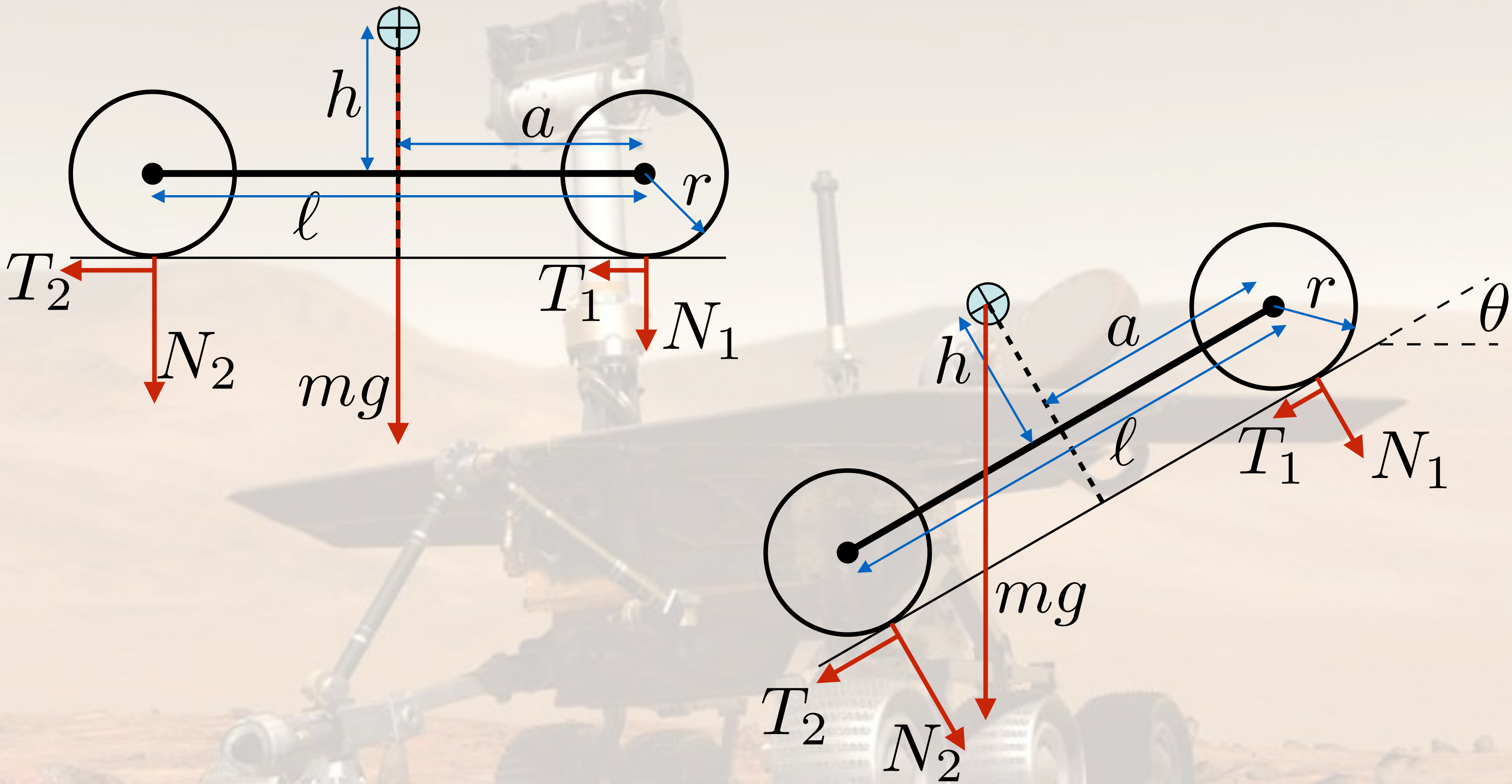
LRV Three-View and Dimensions



Location of LRV Center of Gravity



Rover Climbing/Descending Slope



Static Equilibrium Conditions

$\sum$ Forces $\perp$ to surface

$$N_1 + N_2 = mg\cos\theta$$

$\sum$ Forces $\parallel$ to surface

$$T_1 + T_2 = mg\sin\theta$$

$\sum$ Torques about rear axle

$$T_1 r + T_2 r + N_1 \ell = mg\left[(\ell - a)\cos\theta - h\sin\theta\right]$$

Friction forces proportional to force into surface

$$\frac{T_1}{N_1} = \frac{T_2}{N_2}$$

Four equations, four unknowns

Static Equilibrium Solutions

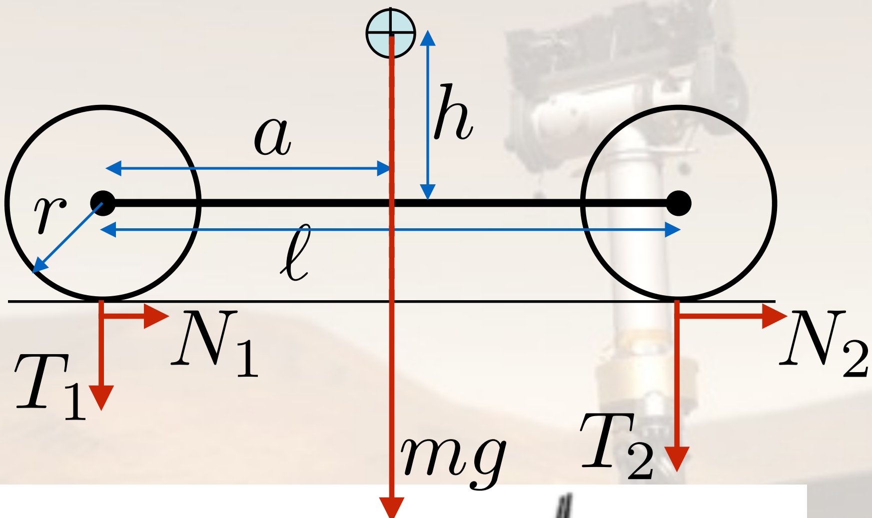
$$N_1 = mg\left[\left(1 - \frac{a}{\ell}\right)\cos\theta - \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta\right]$$

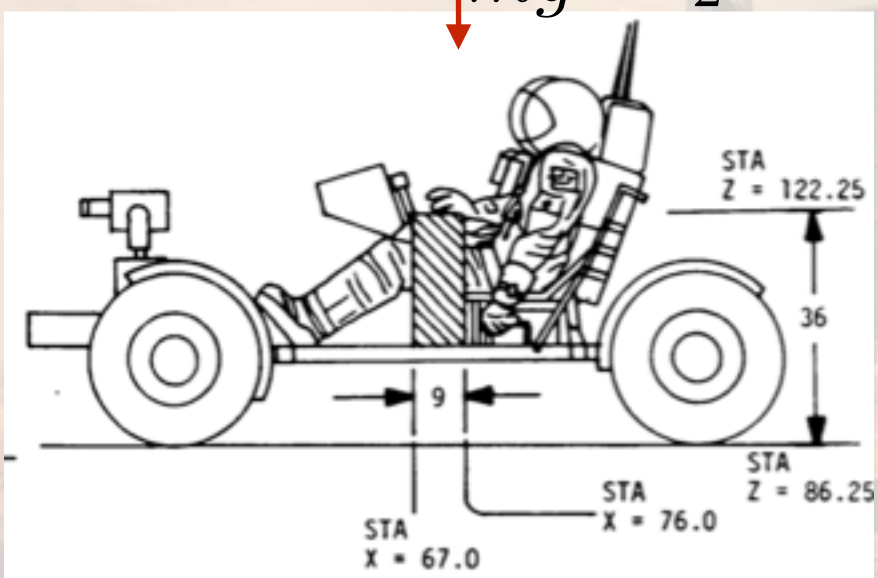
$$N_2 = mg\left[\frac{a}{\ell}\cos\theta + \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta\right]$$

$$T_2 = \frac{N_2}{N_1 + N_2} mg\sin\theta$$

$$T_1 = \frac{N_1}{N_1 + N_2} mg\sin\theta$$

Loaded LRV Weight Distribution





$$m = 690\ kg$$

$$mg = 1120\ N$$

$$\ell = 228.6\ cm$$

$$r = 40.6\ cm$$

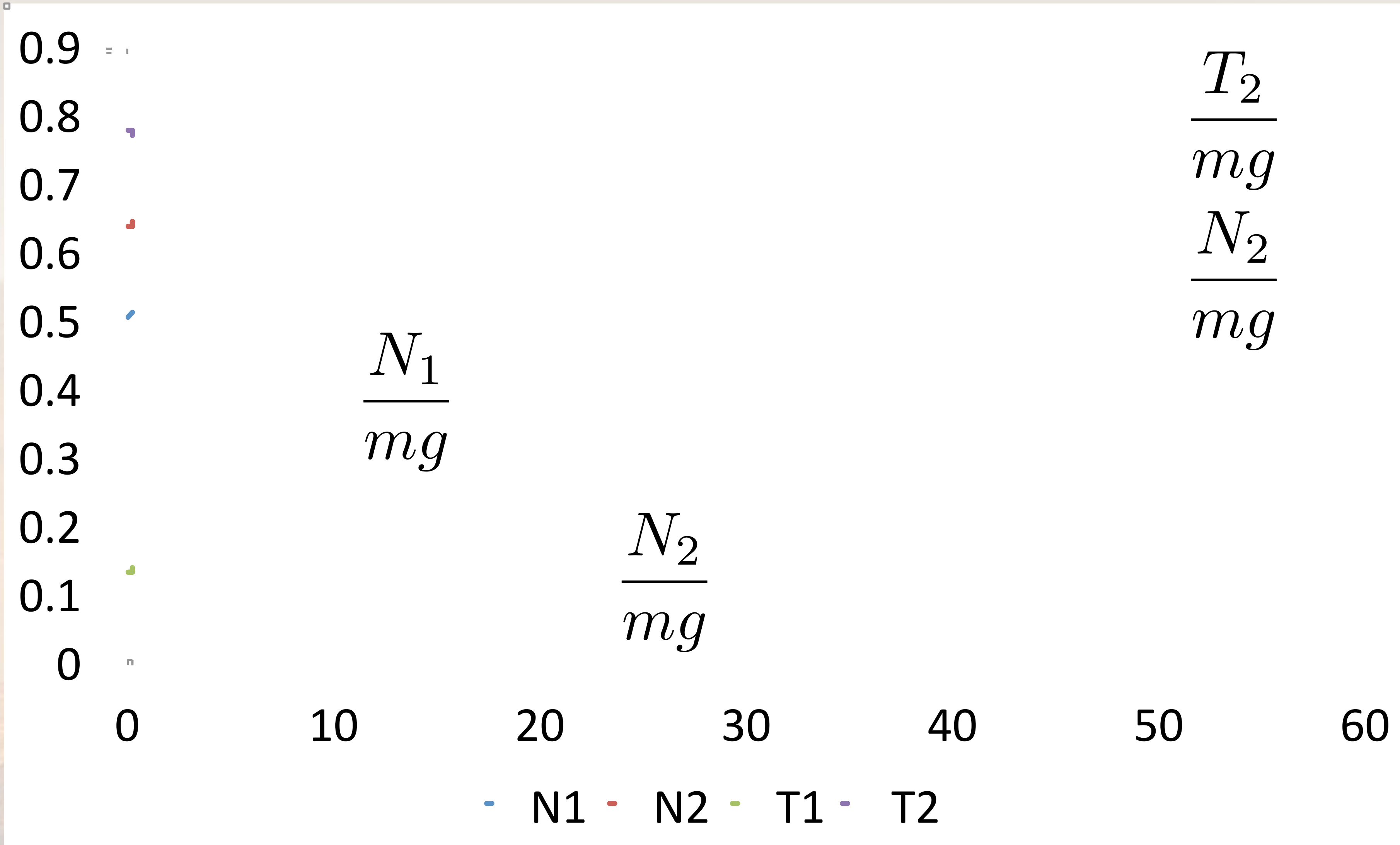
$$a = 114.3\ cm$$

$$h = 50.8\ cm$$

$$N_1 = N_2 = 279.8\ N$$

$$b = 22.9\ cm$$

Normal and Shear Wheel Force w/Slope



Longitudinal Dynamic Conditions

$\sum$ Forces $\perp$ to surface

$$N_1 + N_2 = mg\cos\theta$$

$\sum$ Forces $\parallel$ to surface

$$T_1 + T_2 = mg\sin\theta + ma_x$$

$\sum$ Torques about rear axle

$$T_1 r + T_2 r + N_1 \ell = mg\left[(\ell - a)\cos\theta - h\sin\theta\right]$$

Friction forces proportional to force into surface

$$\frac{T_1}{N_1} = \frac{T_2}{N_2}$$

Four equations, four unknowns

Longitudinal Dynamic Solutions

$$N_1 = mg\left[\left(1-\frac{a}{\ell}\right)\cos\theta - \left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta - \frac{a_x}{g}\right]$$

$$N_2 = mg\left[\frac{a}{\ell}\cos\theta + \left(\frac{h}{\ell}+\frac{r}{\ell}\right)\sin\theta + \frac{a_x}{g}\right]$$

$$T_2 = \frac{N_2}{N_1+N_2}\left(mg\sin\theta + ma_x\right)$$

$$T_1 = \frac{N_1}{N_1+N_2}\left(mg\sin\theta + ma_x\right)$$

Overturn Limits

$$\left(1 - \frac{a}{\ell}\right)\cos\theta_{limit} - \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta_{limit} = \frac{a_x}{g}$$

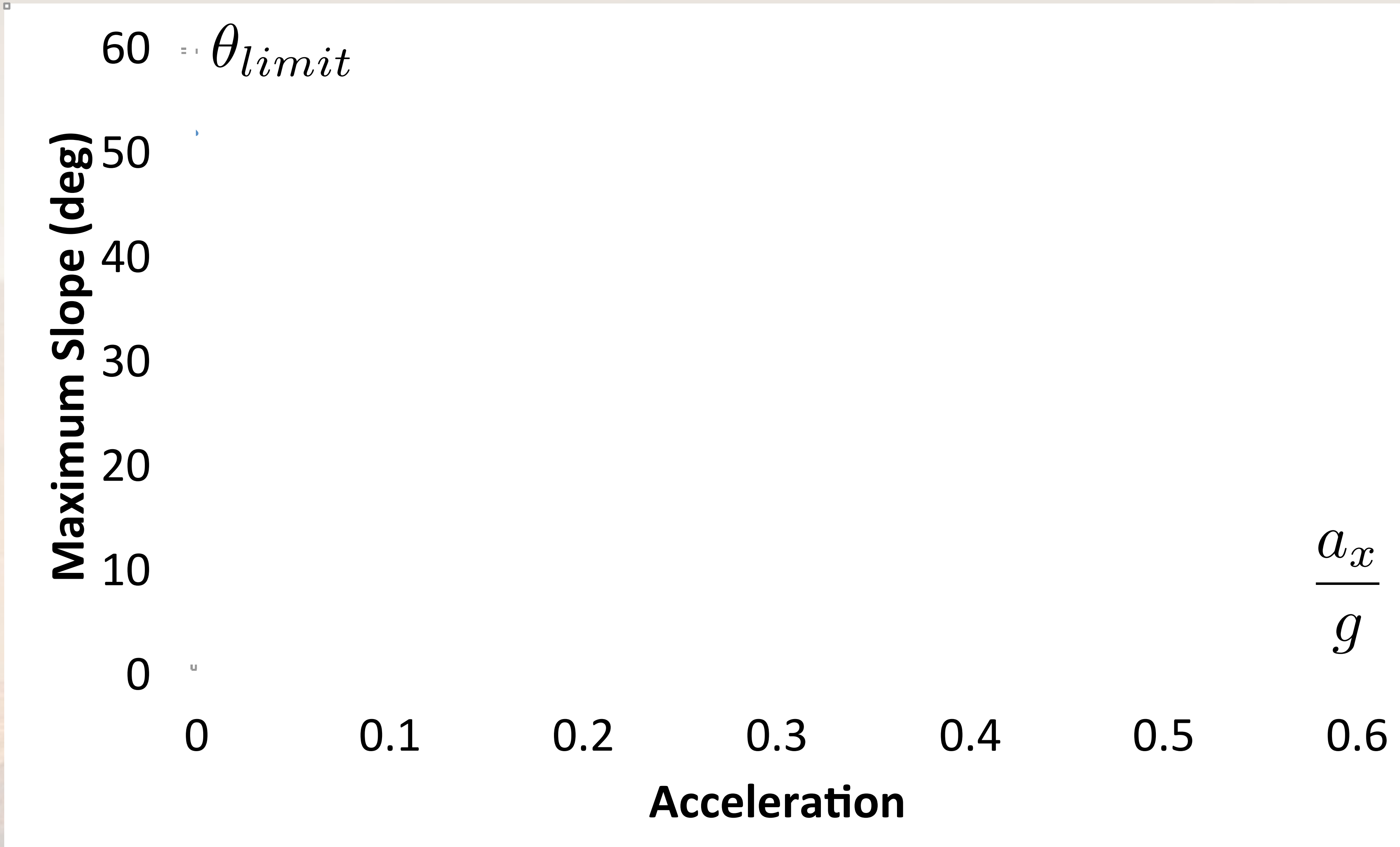
For the static case $(a_x = 0)$

$$\left(1 - \frac{a}{\ell}\right)\cos\theta_{limit} = \left(\frac{h}{\ell} + \frac{r}{\ell}\right)\sin\theta_{limit}$$

$$\tan\theta_{limit} = \frac{\left(1 - \frac{a}{\ell}\right)}{\left(\frac{h}{\ell} + \frac{r}{\ell}\right)} = \frac{\ell - a}{h + r}$$

Limiting accel on flat ground $a_{x,limit} = g\left(1 - \frac{a}{\ell}\right)$

Limiting Slope Under Acceleration



Compression Resistance (Lunar Soil)

$$R_c = \frac{1}{n+1} \left(k_c + b k_\phi\right)^{\frac{-1}{2n+1}} \left(\frac{3W_w}{(3-n)\sqrt{d}} \right)^{\frac{2(n+1)}{2n+1}}$$

$$n = 1$$

$$k_c = 0.14 \ N/cm^2$$

$$k_\phi = 0.827 \ N/cm^3$$

$$R_c = \frac{1}{2} \left(k_c + b k_\phi\right)^{\frac{-1}{3}} \left(\frac{3W_w}{2\sqrt{d}} \right)^{\frac{4}{3}}$$

$$R_c = 20.89 \ N/wheel = 83.57 \ N \ (total)$$

Lunar Regolith Soil Properties

Symbol	Description	Value	Used in
n	*Exponent of sinkage*	*1.0*	*EVERYTHING*
k	*Cohesive modulus of soil deformation*	*1400 N/m*	*Rc, Rb, H*
k	*Frictional modulus of soil deformation*	*820,000 N/m*	*Rc, Rb, H*
ϑ	*Slope angle*		*Rg*
Cf	*Coefficient of friction*	*0.05*	*Rr*
γ	*Soil density*	*1.6 g/cm*	*Rb*
Co	*Cohesive strength of soil*	*0.017 N/cm*	*H*
ɸ	*Angle of internal friction*	*30-40 degrees*	*Rb*
K	*Coefficient of soil Slip*	*1.8 cm*	*H*
s	*slip*	*0.02 – 0.05*	*H*

Equations for Compression Resistance

$$z = \left[\frac{3W_w}{(3-n)bk\sqrt{d}} \right]^{\frac{2}{2n+1}}$$

(Corrected 141117)

$$W_w = \text{weight on wheel}$$

$$d = \text{wheel diameter}$$

$$R_c = \left(\frac{bk}{n+1} \right) z^{n+1}$$

$$R_c = \text{compression resistance (per wheel)}$$

Correction - Page 25 of L03

LRV Sinkage in Lunar Regolith

$$z = \left[\frac{3W_w}{2(k_c + bk_\phi)\sqrt{d}} \right]^{\frac{2}{3}}$$

$$z = 1.812\ cm$$

Back wheel is a tandem wheel

$$W_w = \frac{2}{3}(k_c + bk_\phi)\sqrt{d}z^{\frac{2}{3}}$$

Tandem Wheels, n=1

$$N_i = \frac{2}{3}(k_c + bk_\phi)\sqrt{d}\sqrt{z_i - z_{i-1}}(2z_i - z_{i-1})$$

We already calculated $z_1 = 1.812\ cm$

$$N_2 = \frac{2}{3}(k_c + bk_\phi)\sqrt{d}\sqrt{z_2 - z_1}(2z_2 - z_1$$

Solve for z_2 (numerically would be easiest)

$$z_2 = 2.443\ cm$$

This is the total depth for wheel 2

Compaction Resistance, n=1

$$R_c = \frac{1}{2}(k_c + bk_\phi)z^2$$

$$R_c = 56.92\ N\ \text{(per side)} \Longrightarrow$$

$$R_c = 113.8\ N$$

Or, You Could Cheat…

Approximation formulas for n=1

Two wheels in tandem

$$R_c = \frac{1.7}{2}(k_c + bk_\phi)z^2$$

Three wheels in tandem

$$R_c = \frac{2.3}{2}(k_c + bk_\phi)z^2$$

z is the sinkage depth of the front wheel in both cases

$R_g,\ R_f$ are straightforward

Bulldozing Resistance

General case:

All angles in radians!

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin \alpha \cos \phi} \left(2zcK_c + \gamma z^2 K_\gamma\right) + \frac{\ell_o^3 \gamma}{3} \left(\frac{\pi}{2} - \phi\right) + c\ell_o^2 \left[1 + \tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)\right]$$

$$\ell_o = z \tan^2\left(\frac{\pi}{4} - \frac{\phi}{2}\right)$$

$\ell_o \equiv$ soil disruption depth, and is *not* the same as contact length ℓ

For tracked vehicles, only the first term applies:

$$R_b = \frac{b \sin(\alpha + \phi)}{2 \sin \alpha \cos \phi} \left(2zcK_c + \gamma z^2 K_\gamma\right)$$

Bulldozing Resistance (Rb)

Symbol	Description	Value
φ	*Angle of internal friction*	*30-40 degrees for Lunar Regolith*
γ	*Soil density*	~~*1.6 gm*~~ *0.002595 N/cm*
Co	*Cohesive strength of soil*	*0.017*
Lo *(degrees)*	*Distance of rupture*	$lo = ztan^2\left(45° - \frac{\varphi}{2}\right)$
Kc *(degrees)*	*Modulus of cohesion of soil deformation*	$kc = (Nc - \tan(\varphi))(\cos^2(\varphi))$
K *(degrees)*	*Modulus of density of soil deformation*	$K_\gamma = \left(\frac{2N_\gamma}{\tan\varphi} + 1\right)\cos^2(\varphi)$
Nc *(radians)*	*Coefficient of passive earth pressure*	$Nc = \cot(\varphi)\left(\frac{e^{2\left(\frac{3\pi}{4}-\frac{\varphi}{2}\right)\tan(\varphi)}}{2\cos^2\left(\frac{\pi}{4}+\frac{\varphi}{2}\right)} - 1\right)$
α *(degrees)*	*Angle of approach of the wheel*	$\alpha = \cos^{-1}\left(1 - \frac{2z}{D}\right)$

Bulldozing Example (1)

$$\phi = 33^o = 0.576\ rad$$

$$\alpha = \cos^{-1}\left(1 - \frac{2z}{d}\right) = \cos^{-1}\left(1 - \frac{2(1.812)}{81.2}\right) = 17.18^o = 0.2999\ rad$$

$$\ell_o = z_1 \tan^2\left(\frac{\pi}{2} - \frac{\phi}{4}\right) = 0.5341$$

$$N_q = \frac{e^{(1.5\pi - \phi)\tan\phi}}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} = 32.23$$

$$N_c = \frac{N_q - 1}{\tan\phi} = 48.09$$

Corrected 141123

Bulldozing Example (2)

$$N_\gamma = \frac{2(N_q + 1)\tan\phi}{1 + 0.4\sin 4\phi} = 33.27$$

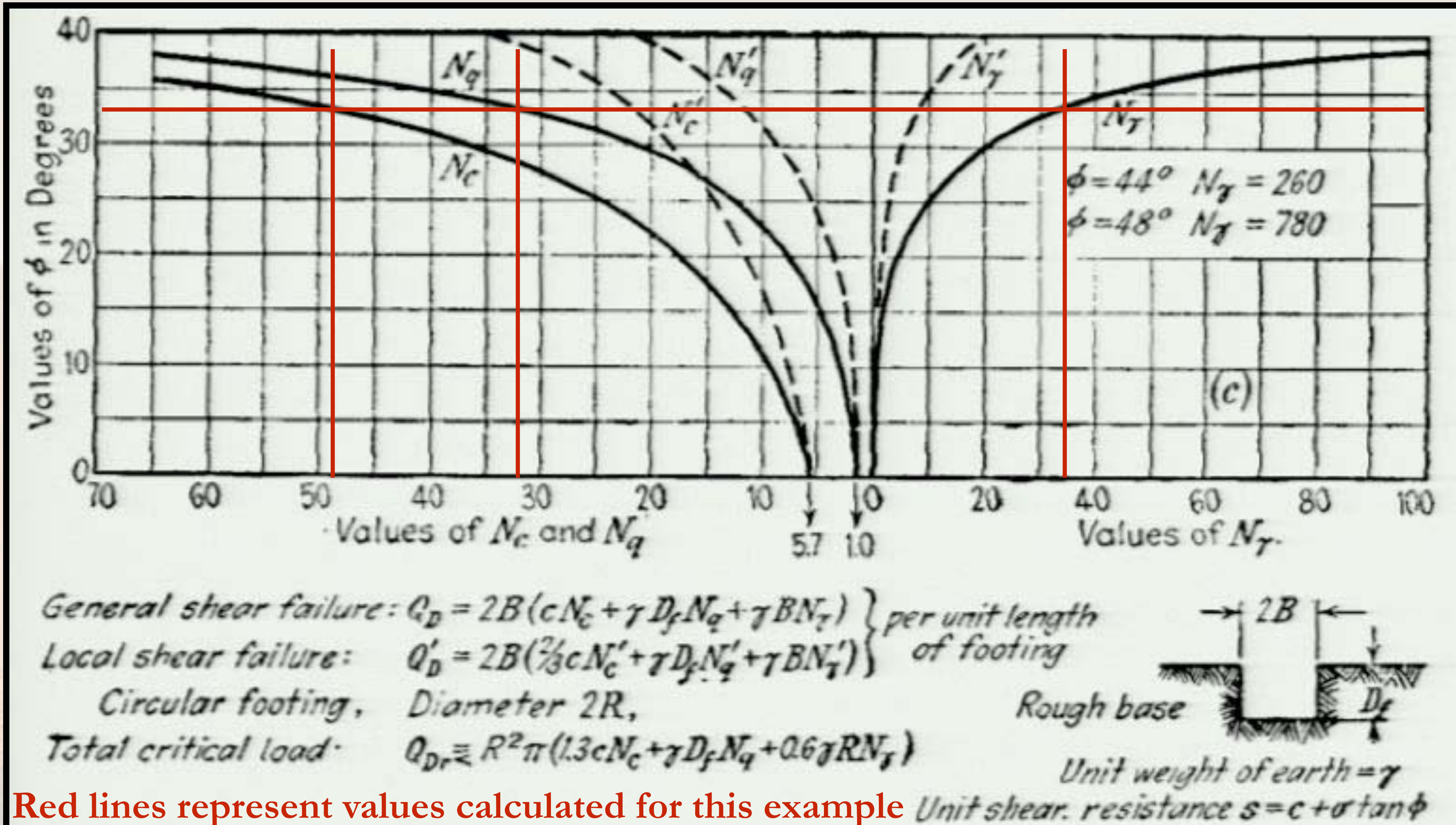
$$K_c = (N_c - \tan\phi)\cos^2\phi = 33.37$$

$$K_\gamma = \left(\frac{2N_\gamma}{\tan\phi} + 1\right)\cos^2\phi = 72.77$$

$$\ell = \frac{d}{2}\cos^{-1}\left(1 - \frac{2z}{d}\right) = 12.18\ cm$$

$$\gamma = 0.002595\ \frac{N}{cm^3};\quad c_o = 0.017\ \frac{N}{cm^2}$$

Terzhagi Parameters



Red lines represent values calculated for this example

Bulldozing Example (3)

$$R_b = \frac{b\sin\left(\alpha+\phi\right)}{2\sin\alpha\cos\phi}\left(2zcK_c+\gamma z^2K_\gamma\right) + \frac{\ell_o^3\gamma}{3}\left(\frac{\pi}{2}-\phi\right)+c\ell_o^2\left[1+\tan\left(\frac{\pi}{4}+\frac{\phi}{2}\right)\right]$$

$$\langle R_b\rangle = cm\left(cm\ \frac{N}{cm^2}+\frac{N}{cm^3}cm^2\right)+cm^3\frac{N}{cm^3}+\frac{N}{cm^2}cm^2$$

$$R_b = 94.98 + 0.000131 + 0.014 = 95.00\ N \text{ per leading wheel}$$

$$R_{b,total} = 190.0\ N$$

Tractive Force per Wheel (No Grousers)

$$H = [AC_b + W_w \tan\phi_b]\left[1 - \frac{K}{\ell}\left(1 - e^{\frac{-s\ell}{K}}\right)\right]$$

$$A = \text{area of contact} = b\ell$$

$$C_b = \text{coefficient of soil/wheel cohesion}$$

$$\phi_b = \text{wheel/soil friction angle}$$

$$s = \text{wheel slip ratio}$$

$$K = \text{coefficient of soil slip}$$

$$\ell = \text{length of contact patch}$$

$$H_{(s=0)} = 251.6\ N$$

Tractive Force per Wheel (With Grousers)

$$H = \left[b\ell C_b \left(1 + \frac{2h}{b}\right) N_g + W \tan\phi_b \left(1 + 0.64\frac{h}{b}\arctan\frac{b}{h}\right)\right]\left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$

$A =$ area of contact $\cong b\ell$

$C_b =$ soil/wheel cohesion $= 0.017\ N/cm^2$

$\phi_b =$ wheel/soil friction angle $= 35°$

$s =$ wheel slip ratio (typ. 0.02-0.05)

$K =$ coefficient of soil slip $= 1.8$ cm

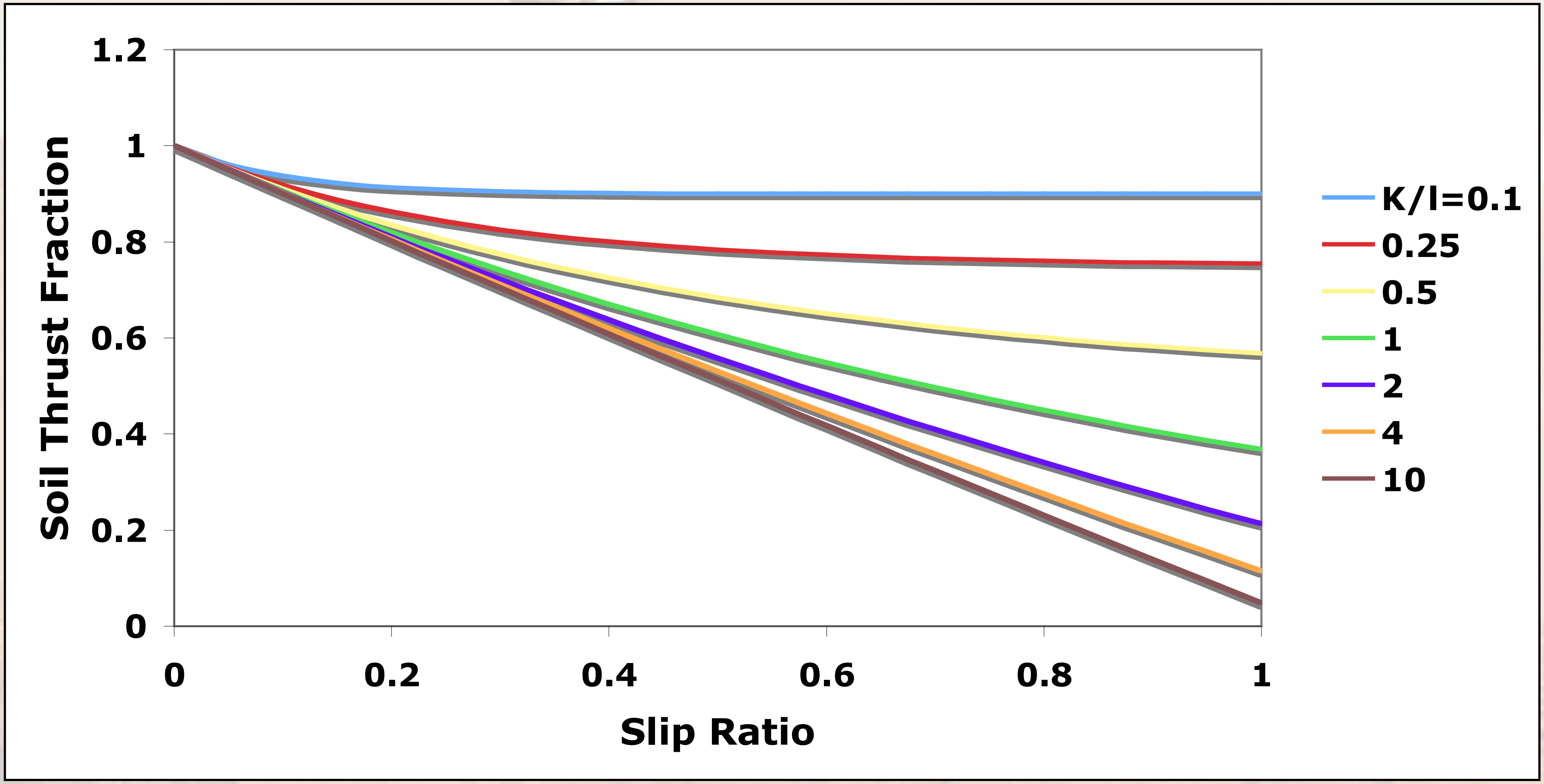
$\ell =$ length of contact patch $= \frac{D}{2}\cos^{-1}\left(1 - \frac{2z}{D}\right)$

$h =$ height of grouser

All values typical for lunar soil

Effect of Soil Thrust Fraction

$$\text{Soil Thrust Fraction} \quad \left[1 - \frac{K}{\ell}\left(1 - e^{-\frac{s\ell}{K}}\right)\right]$$



Basic Equation of Vehicle Propulsion

$$DP = H - (R_c + R_b + R_g + R_r)$$

- DP: Drawbar pull (residual drive force)
- H: Maximum tractive force of wheels
- R_c: Compaction resistance
- R_b: Bulldozing resistance
- R_g: Gravitational resistance
- R_r: Rolling resistance (internal)

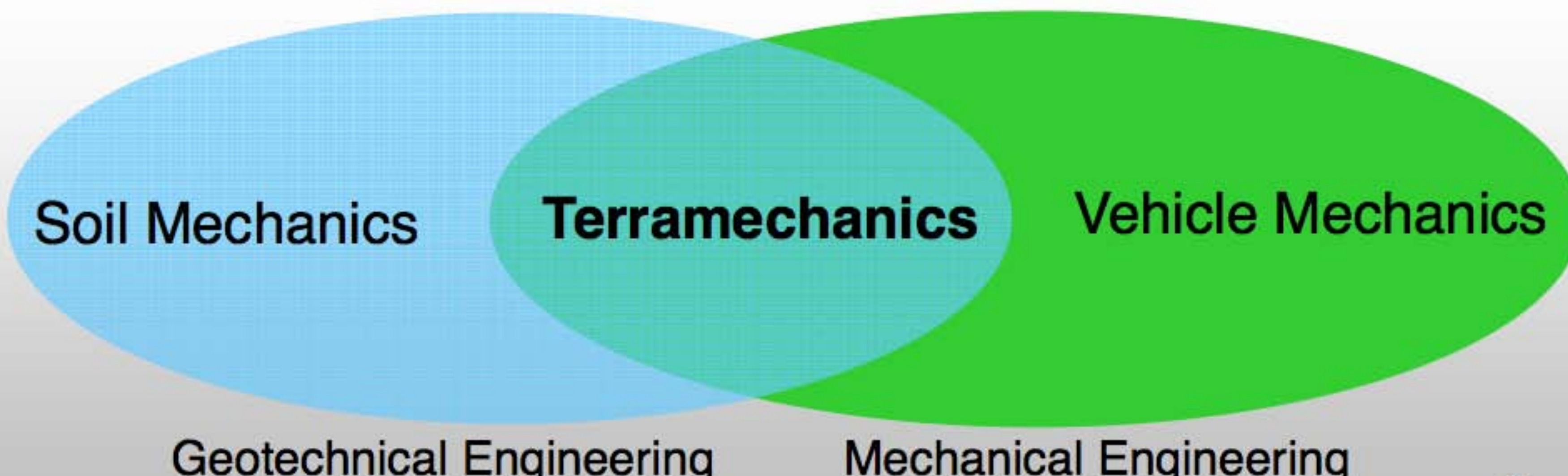
Surface Interaction Modeling

Engineering Methods

Karl Iagnemma, Ph.D.
Massachusetts Institute of Technology

Terramechanics

- Terramechanics
  - Engineering science that studies the interaction between vehicles and (deformable) terrain
- Soil mechanics and vehicle mechanics
- Analysis of wheeled, tracked, legged systems



An Engineer's Job

- Design vehicle for robust mobility on Mars surface
  - Wheels, tracks, legs?
    - Number, diameter and width?
    - Required nominal torque?
    - Required peak power?
  - Obstacle crossing performance?
    - Suspension configuration?
  - Steering mechanism?
- **How to address in a principled, systematic fashion?**

NASA's Mars Science Laboratory (MSL)
Design/Test Model (DTM) in the sandy Mars Yard at JPL

An Engineer's Reality

- How to model this scenario?
  - High sinkage
  - High slip ratio
  - Material transport effects
    - Clogged grousers
  - Variables of interest
    - Soil properties
    - Soil state
    - Wheel load
    - Wheel geometric properties
    - Wheel linear and angular velocity

Terramechanics

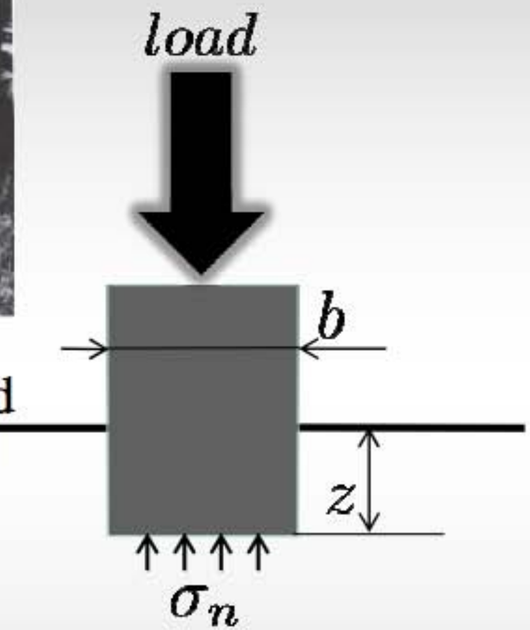
- Limitations of terramechanics modeling
  - Attempt to model all soil types with single set of relations
    - Frictional soils, crusty materials, clay
  - Assumption of homogeneity
  - Attempt to apply (semi)-empirical models in predictive manner
  - Little consideration of off-nominal operation
  - Difficulty in employing quasi-static models for dynamic simulation
- Assertion: General approach remains valid
  - Not all limitations are fundamental
- Goals
  - Understand limits of applicability of terramechanics
  - Identify areas requiring new research

Pressure-Sinkage

- Pressure-sinkage relationship for geomaterials

$$\sigma = kz^n$$

  - σ is normal pressure
  - k is empirical constant
  - z is sinkage from free surface
- Bekker proposed semi-empirical formulation



Cohesion-dependent soil coefficient (k_c)

Friction-dependent soil coefficient (k_ϕ)

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

Sinkage exponent (n)

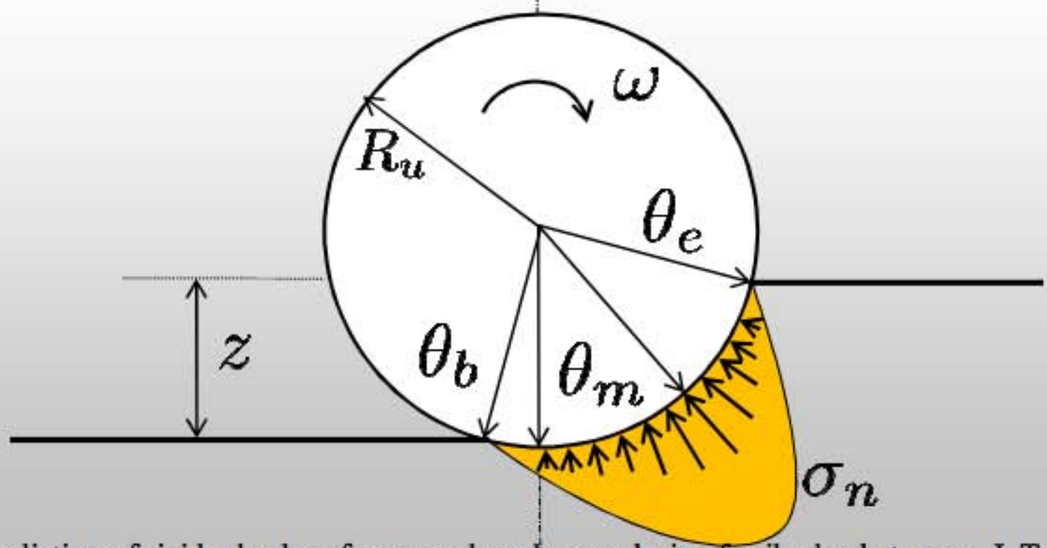
M. G. Bekker. *Theory of Land Locomotion*. Ann Arbor, University of Michigan Press, 1950.

Pressure-Sinkage for Wheels

- Can compute normal stress for wheels along terrain interface

$$\sigma_{nf} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos(\theta) - \cos(\theta_e)\right)\right]^n \qquad \theta_m < \theta \leq \theta_e$$

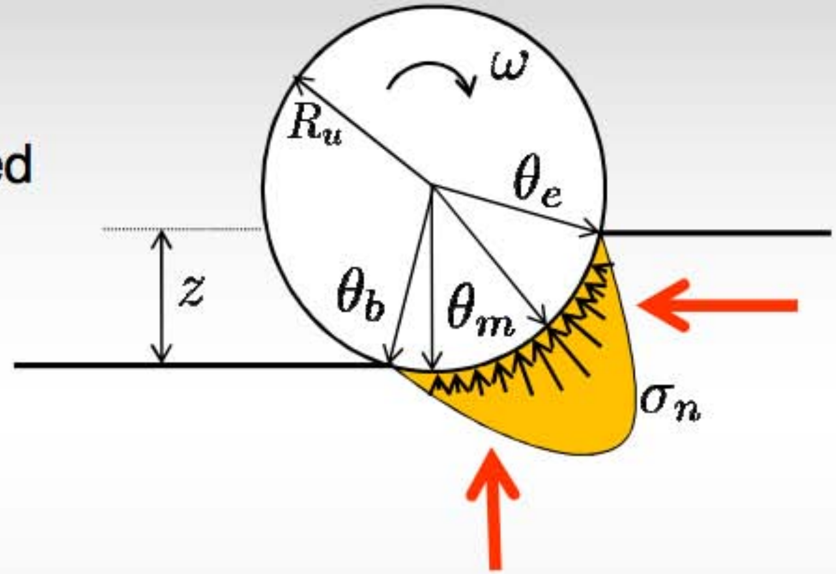
$$\sigma_{nr} = \left(\frac{k_c}{b} + k_\phi\right)\left[R\left(\cos\left(\theta_e - \left(\frac{\theta - \theta_r}{\theta_N - \theta_r}\right)(\theta_e - \theta_N)\right) - \cos(\theta_e)\right)\right]^n \qquad \theta_b \leq \theta \leq \theta_m$$

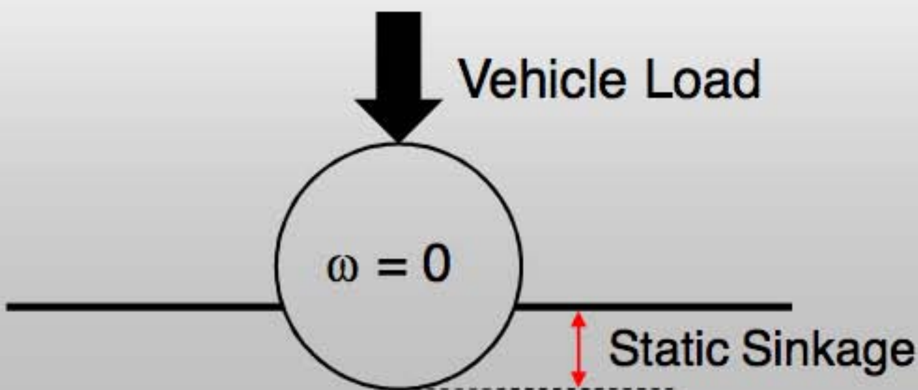


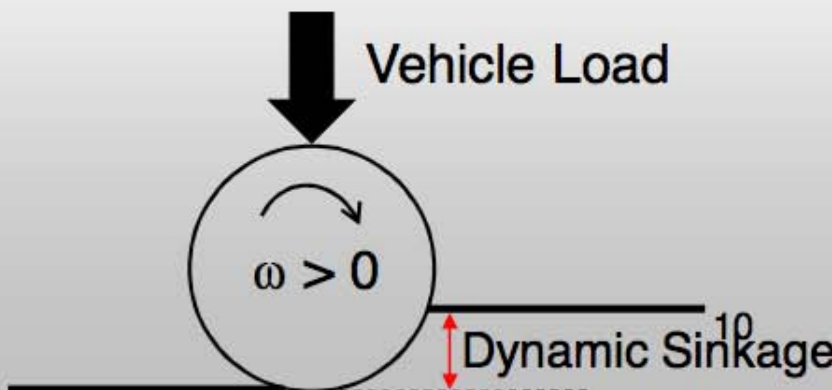
J. Y. Wong and A. R. Reece. Prediction of rigid wheel performance based on analysis of soil-wheel stresses. J. Terramechanics,1967

Pressure-Sinkage for Wheels

- Sinkage plays critical role in mobility
  - Increased sinkage causes increased motion resistance
  - Energy lost in terrain compaction
- Sinkage can be divided in two components
  - Static sinkage
  - Dynamic sinkage (or slip-sinkage)







Pressure-Sinkage for Wheels

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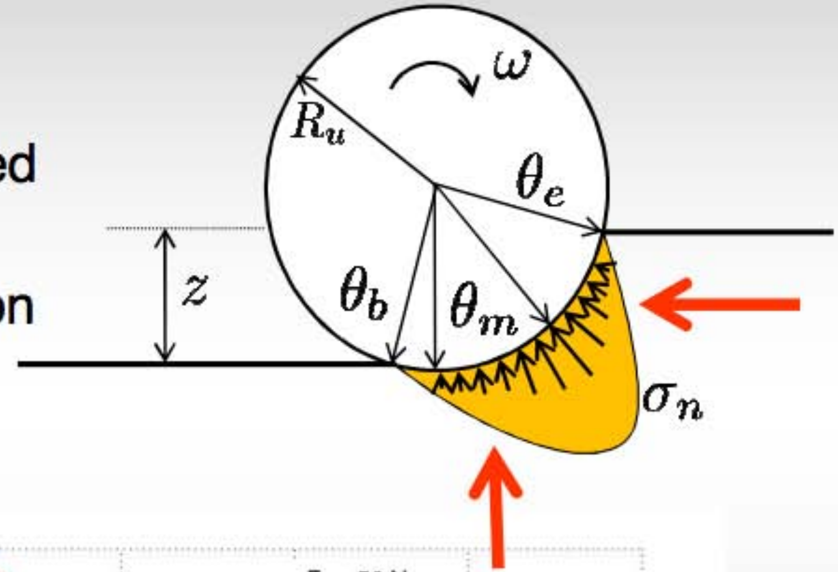
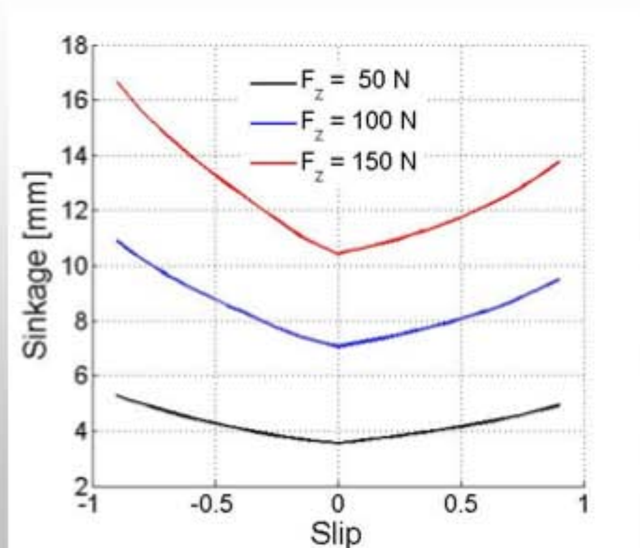
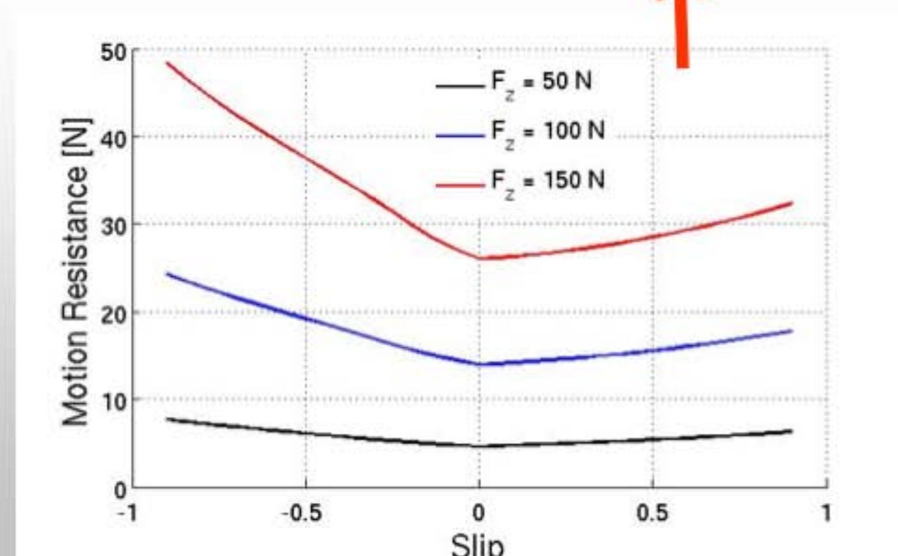






Shearing Properties of Soil

- Motion of a wheel or track causes shearing at the soil interface
  - Resistance forces generated by soil mass
  - Depends on slip, loading conditions

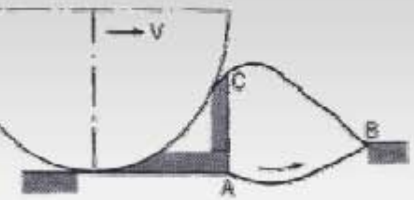


Figure 2.20: Flow patterns and soil wedge formed in front of a locked rigid wheel at 100% skid in sand

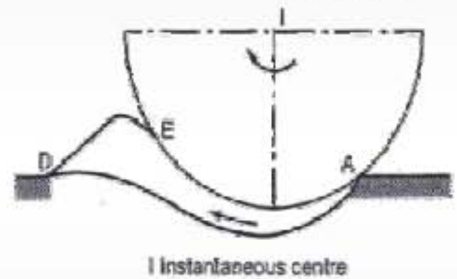
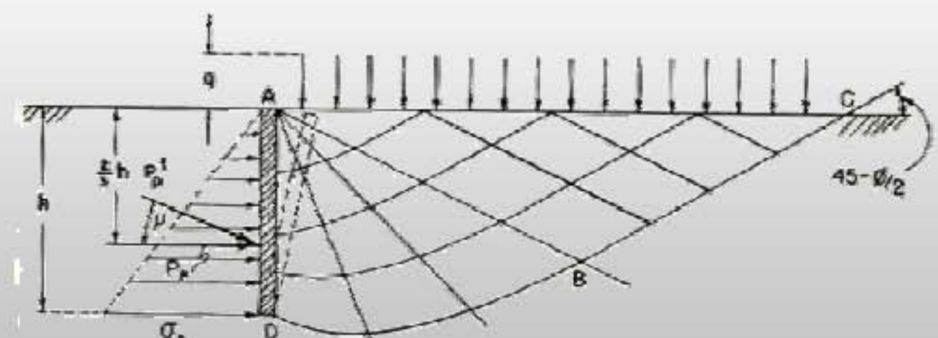


Figure 2.19: Flow patterns beneath a driven rigid wheel at 100% slip in sand

Figure 1.? 2: Soil flow under the action of grousers of a wheel in sand (Reprinted by permission of ISTVS from Wong et al., 1984)

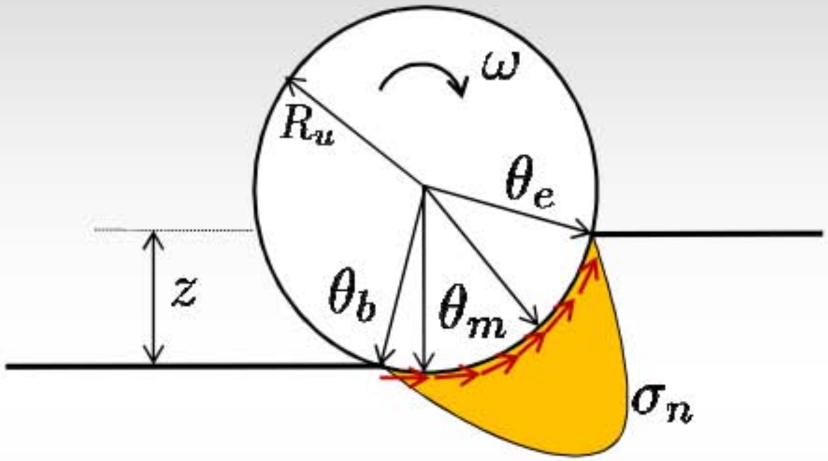


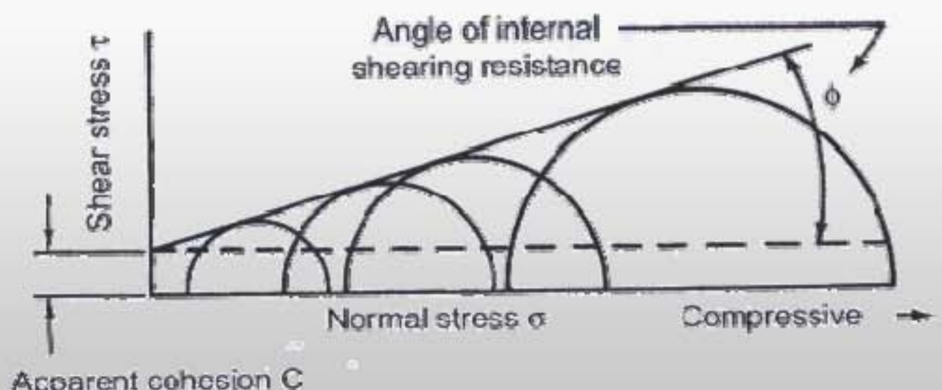
Shearing Properties of Soil

- Shear stress at wheel-soil interface produces traction
- Shear stress is a function of shear displacement
  - Relative motion required to generate traction
    - Non-zero slip ratio
- Soil failure estimated through Mohr-Coulomb failure criterion

$$\tau = c + \sigma \tan \phi$$

  - τ is failure stress
  - c is soil cohesion
  - ϕ is soil internal friction angle





Shearing Properties of Soil

- Can compute shear stress at wheel-terrain interface
  - Janosi-Hanamoto formulation

Soil shear displacement

Limit tangential stress

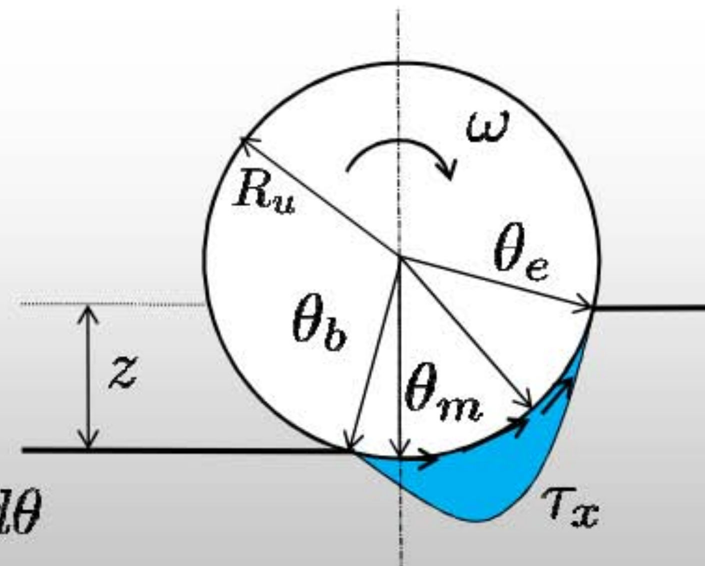
$$\tau_x(\theta) = \tau_{max}\left(1 - e^{\frac{-j_x}{k_x}}\right)$$

Soil shear deformation modulus

$$\tau_{max} = c + \sigma_n(\theta)\tan\phi$$

- Soil shear displacement

$$j_x(\theta) = \int_{\theta_b}^{\theta_e} R_u[1-(1-s_d)\cos(\theta)]\,d\theta$$



Z. Janosi and B. Hanamoto. Analytical determination of drawbar pull as a function of slip for tracked vehicles in deformable soils, Proc. ISTVS

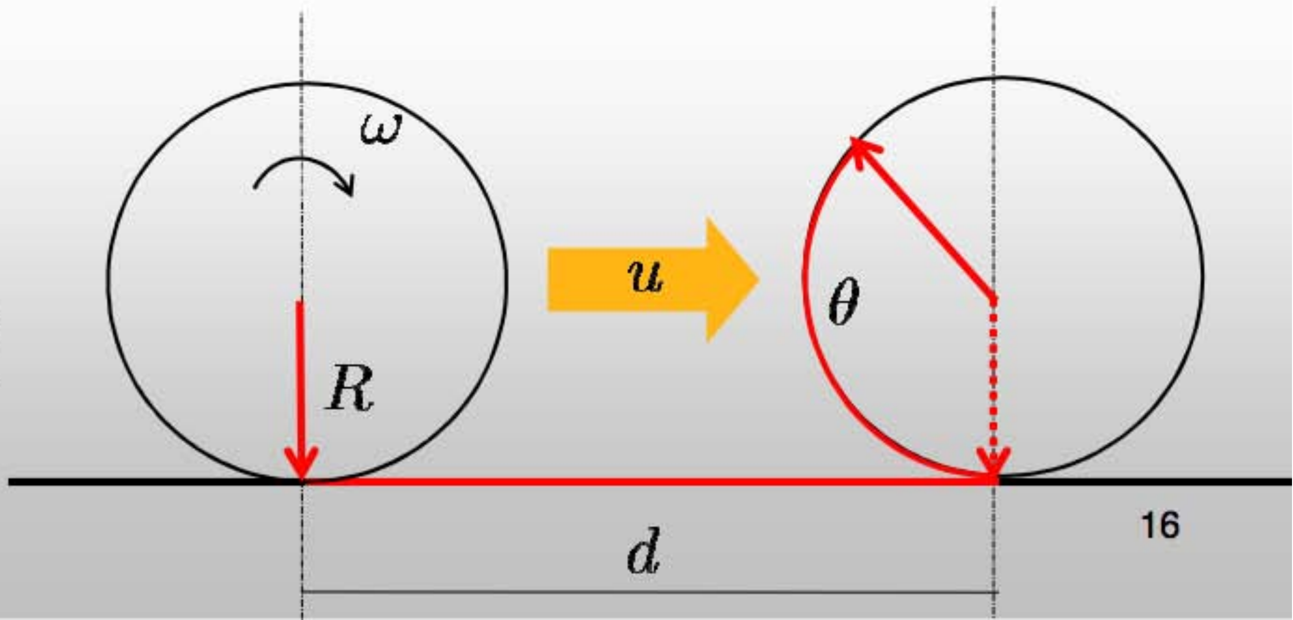
Slip Ratio

- Slip ratio is measure of relative motion between wheel and terrain surface
  - For driven wheel, distance traveled is less than that in free rolling
  - When slip ratio = 1, spinning in place
  - When slip ratio = 0, pure rolling
  - When slip ratio = -1, skidding

$$u \leq \omega R$$

$$d \leq \theta R$$

$$s_d = 1 - \frac{u}{\omega R}$$



Terrain Interaction Forces

- Forces between wheel and terrain can be computed from stress distribution along contact path
- Vertical load

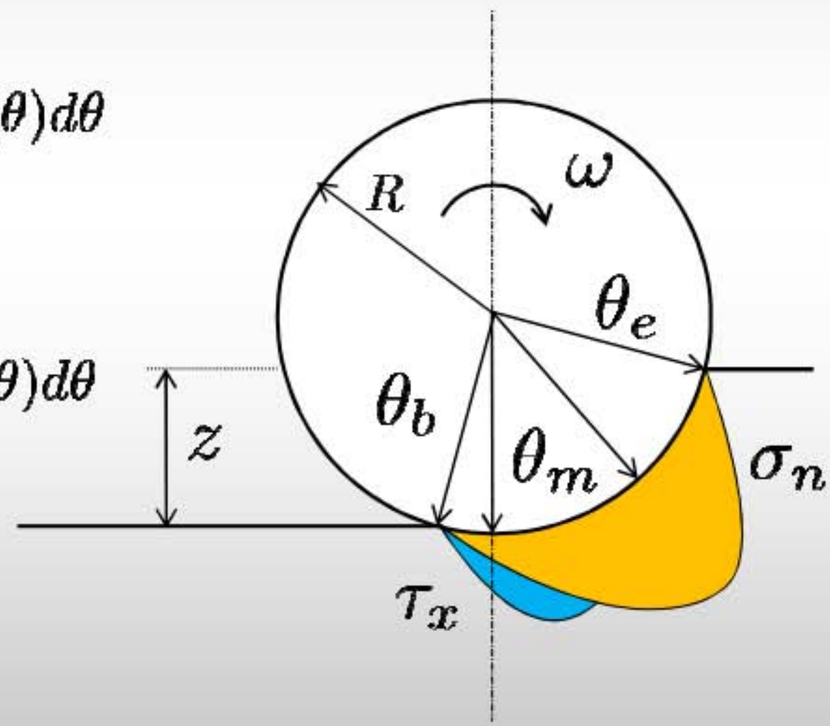
$$W = bR \int_{\theta_b}^{\theta_e} \tau_x(\theta)\sin(\theta) + \sigma_n(\theta)\cos(\theta)d\theta$$

- Longitudinal force

$$F_x = bR \int_{\theta_b}^{\theta_e} \tau_x(\theta)\cos(\theta) - \sigma_n(\theta)\sin(\theta)d\theta$$

- Torque on wheel axle

$$T = bR^2 \int_{\theta_b}^{\theta_e} \tau_x(\theta)d\theta$$



Summary

- Stresses at wheel-terrain interface
  - Decompose into normal and shear stresses
  - Modeled with semi-empirical formulations
  - Integration yields forces acting on vehicle
- Given
  - Terrain properties
  - Slip
  - Loading conditions
- Can compute
  - Sinkage
  - Thrust
  - Required torque

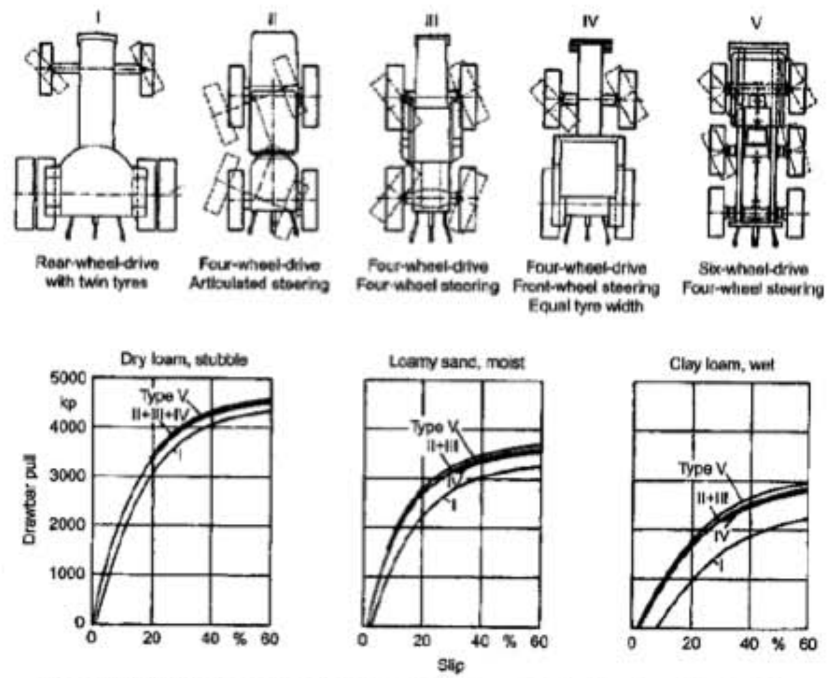


Figure 1.3: Comparison of various configurations for agricultural tractors (Reprinted by permission of *ISTVS* from Sohne, 1976)

Effect of Grousers

- Grousers are small features on wheel surface
  - Designed to improve traction and climbing performance
- Have been modeled through Terzaghi's bearing capacity theory

Pathfinder, MER, and MSL wheels

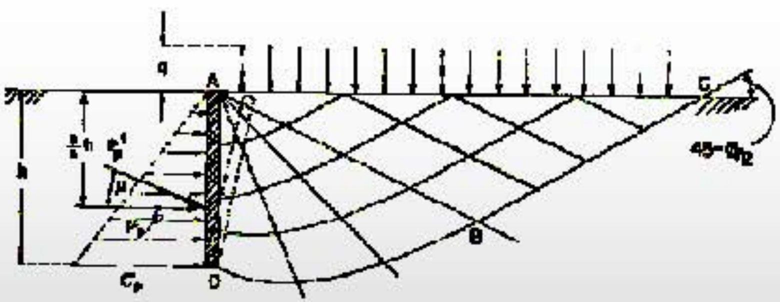


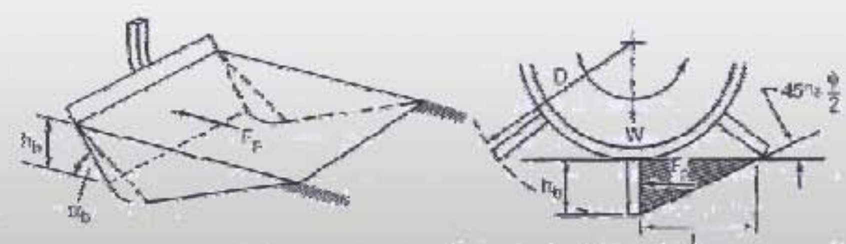
Fig. 69

The value of the force P_p assumed for $\mu = 0$ may be calculated by integrating the pressure σ_p determined by equation (134):

$$P_p = \int_0^h \sigma_p \, dz = \int_0^h (q N_\phi + 2c\sqrt{N_\phi} + \gamma z N_\phi) \, dz$$

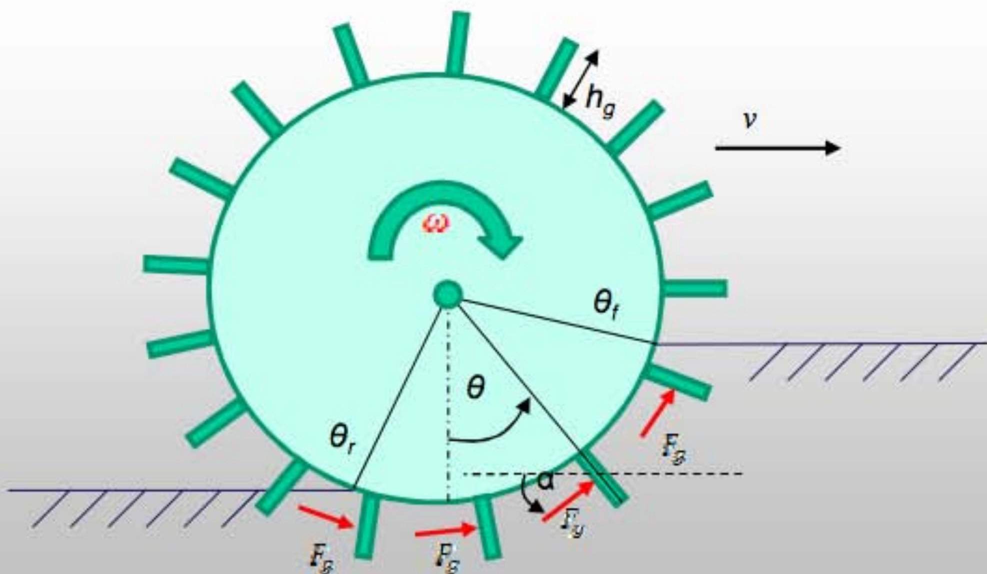
and

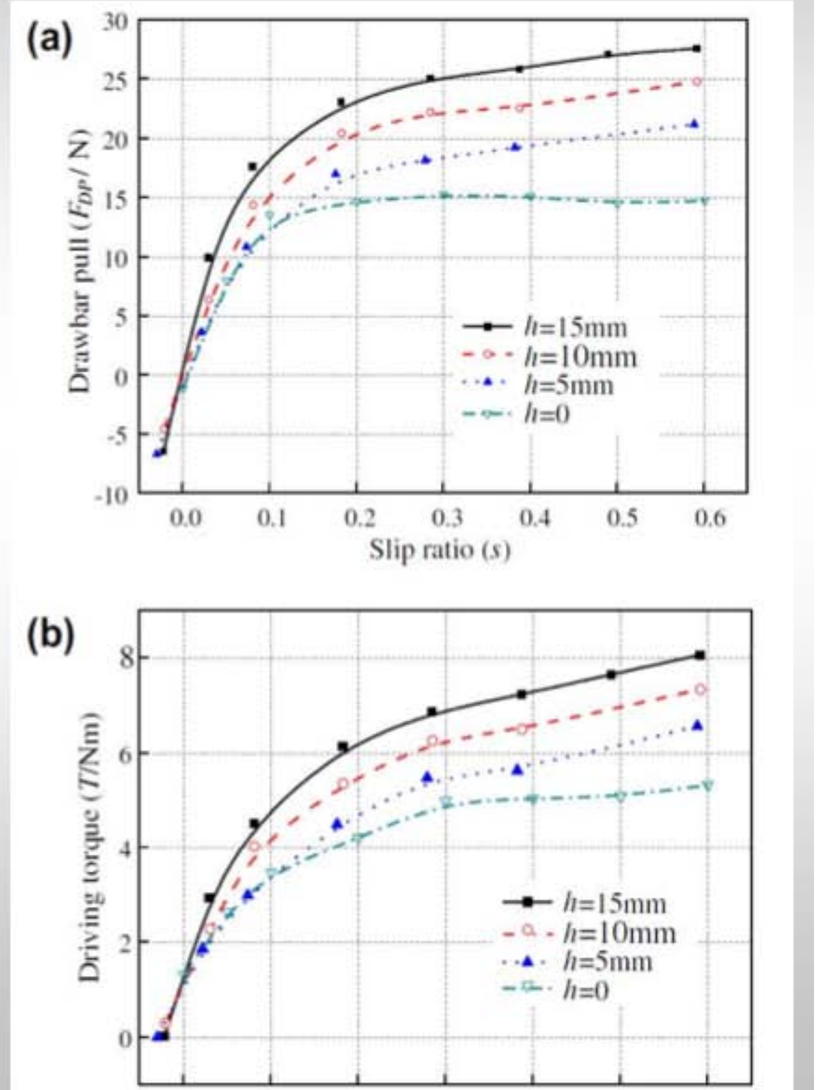
$$P_p = q h N_\phi + 2ch\sqrt{N_\phi} + \tfrac{1}{2}\gamma h^2 N_\phi .$$



Effect of Grousers

- Grouser effect has also been empirically studied
  - Grouser height, spacing, geometry affect torque, traction, turning performance





Ding L. et al. Journal of Terramechanics 48, 2011, 27-45

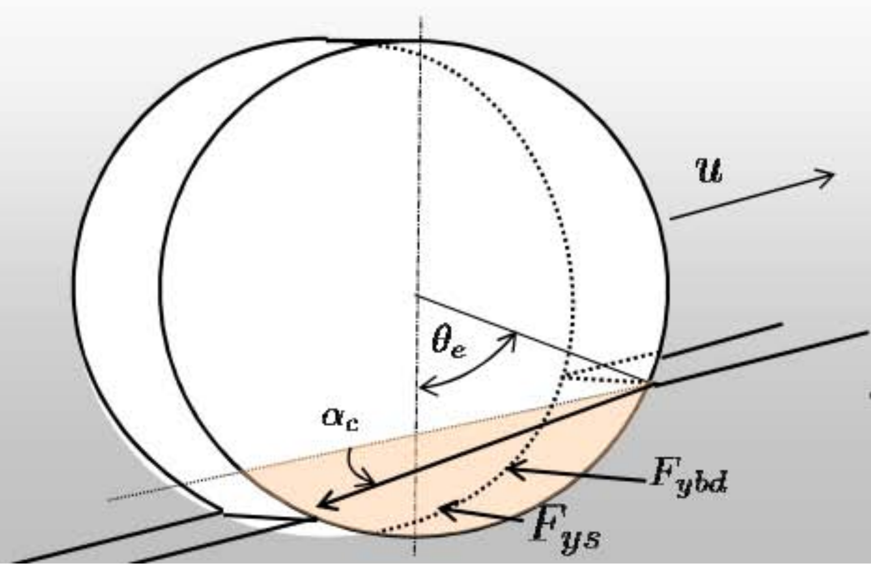
Lateral Forces

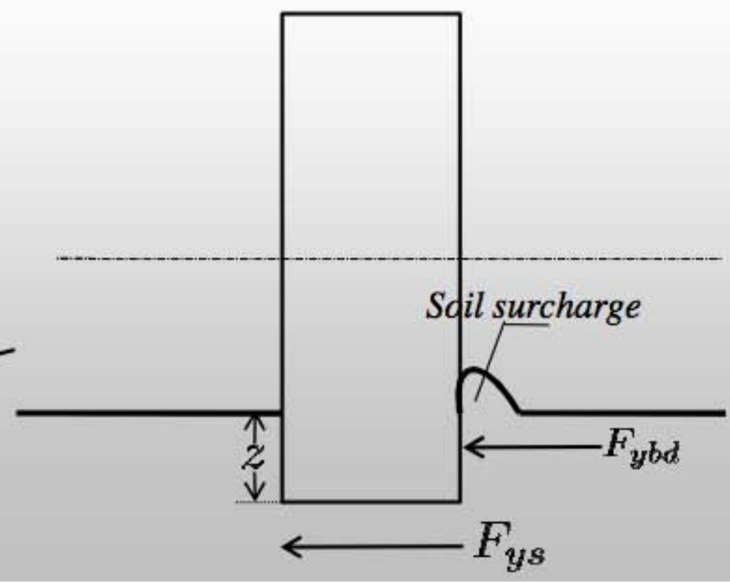
- Lateral forces act on wheel sidewall during turning
  - Forces arise from soil shearing and bulldozing

$$F_y = F_{ys} + F_{ybd}$$

$$F_{ybd} = w \int_{\theta_b}^{\theta_e} (\gamma_s z N_\gamma + c N_c + q N_q) \cos(\delta_f) d\theta$$

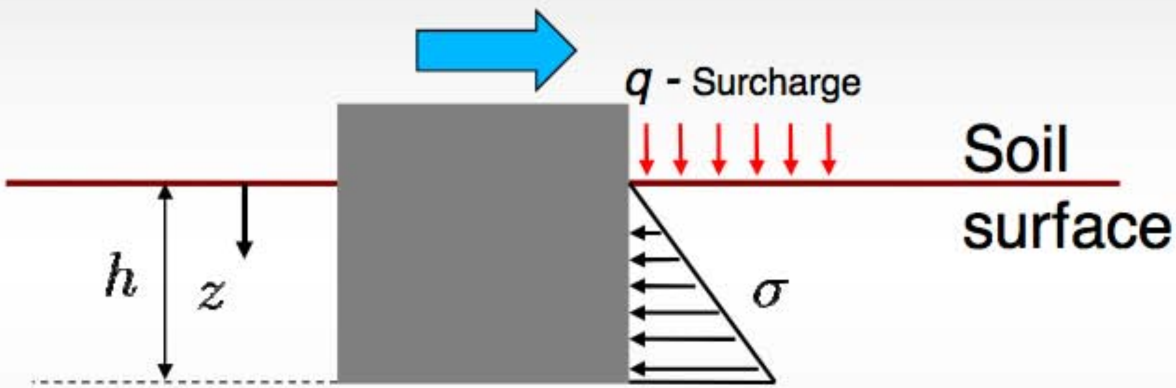
$$F_{ys} = b \int_{\theta_b}^{\theta_e} (c + \sigma_n(\theta) \tan(\phi)) \left(1 - e^{\frac{-j_y}{k_y}}\right) d\theta$$





Lateral Force - Bulldozing

- Like grouser effect, bulldozing is typically modeled through soil bearing capacity analysis



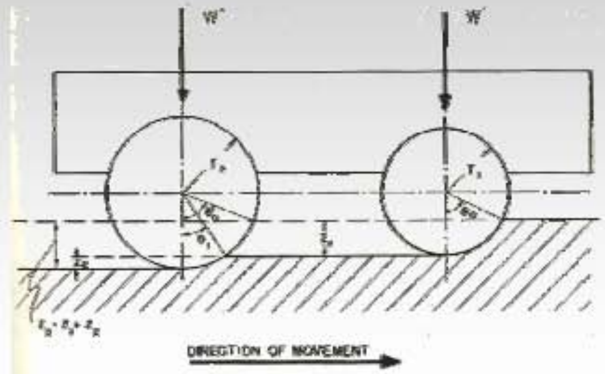
$$\sigma = \gamma z N_\gamma + c N_c + q N_q \ \text{[Pa]}$$

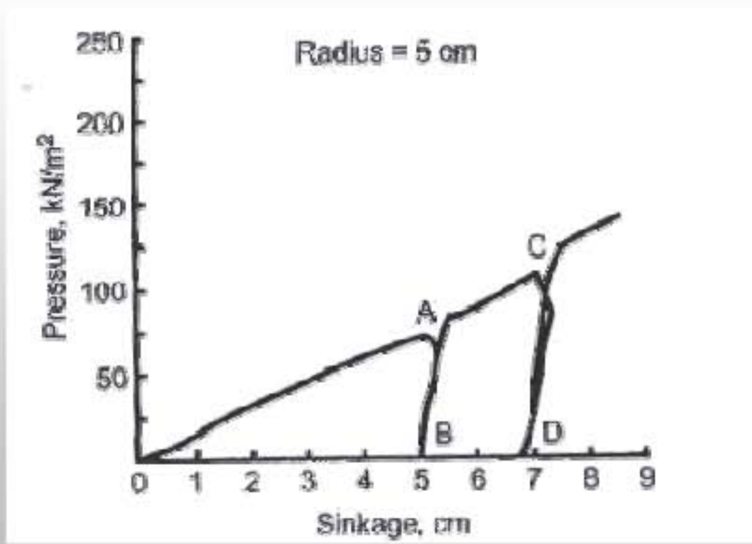
- *N*-factors are function of soil angle of internal friction

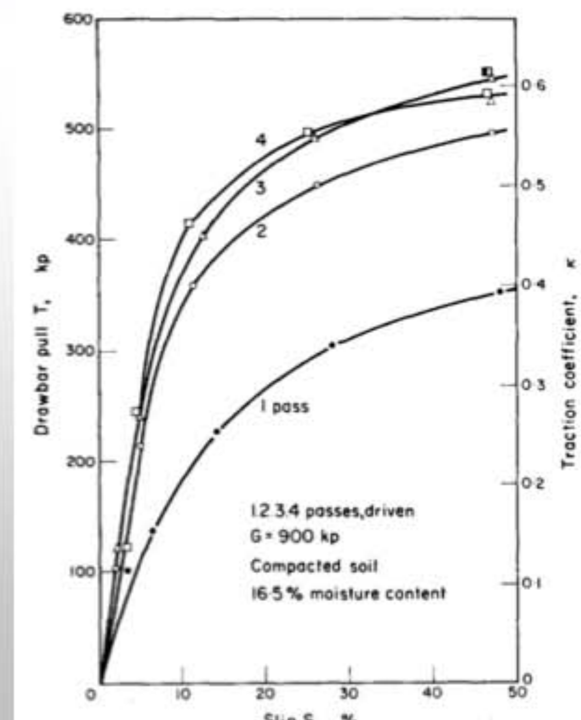
$$N_\gamma = \frac{2(N_q + 1)\tan\phi}{1 + 0.4\sin 4\phi} \qquad N_c = \frac{N_q - 1}{\tan\phi} \qquad N_q = \frac{e^{(1.5\pi - \phi)\tan\phi}}{2\, cos^2(\pi/4 + \phi/2)}$$

Repetitive Loading

- Rover trailing wheels may pass through soil deformed by leading wheels
  - Repetitive loading alters soil behavior
  - Increases compaction (relative density)







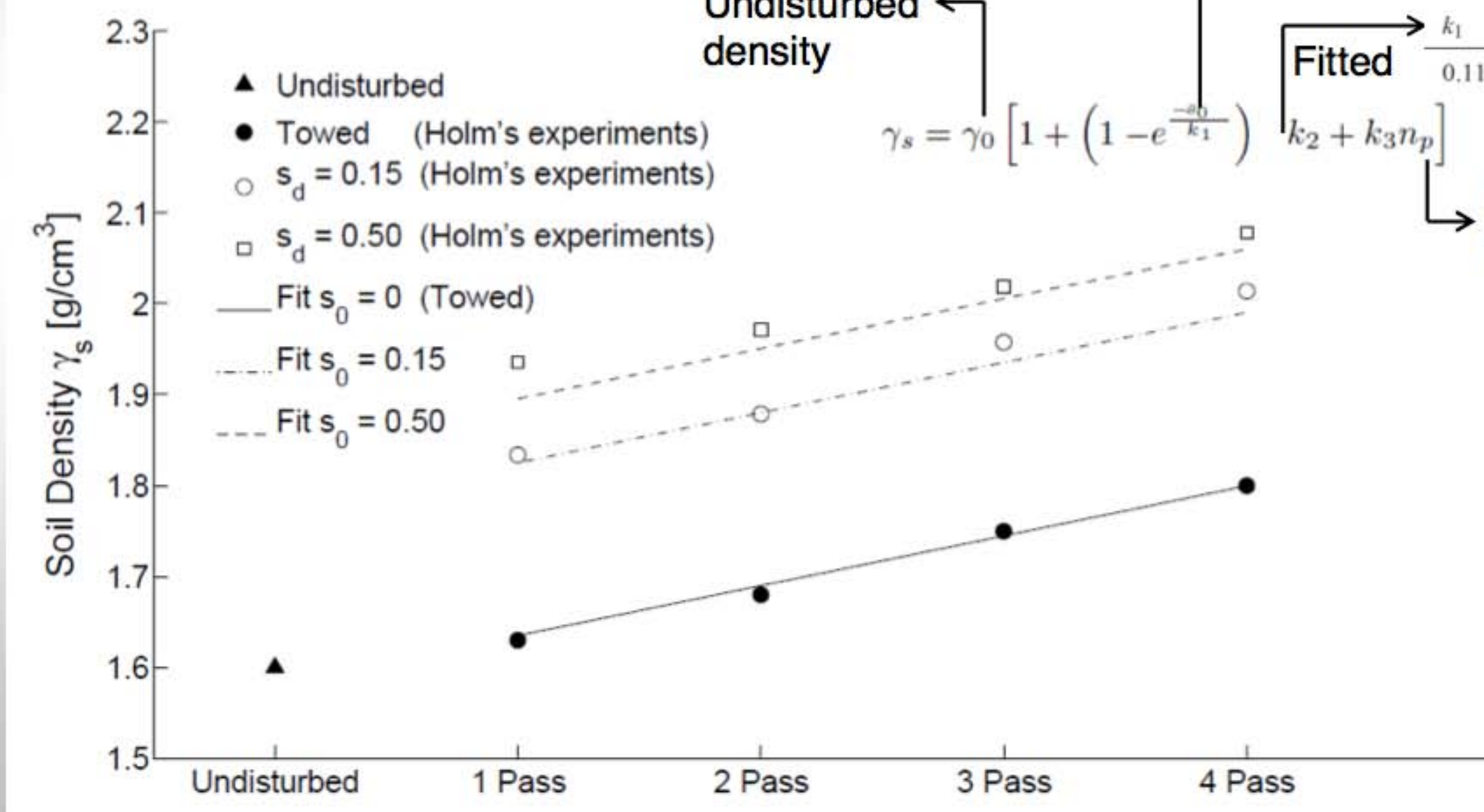
Repetitive Loading

- Multi pass can be modeled by modifying soil parameters according to number and type of passes

$$\gamma_s = \gamma_0 \left[1 + \left(1 - e^{\frac{-s_0}{k_1}}\right)\left(k_2 + k_3 n_p\right)\right]$$

Undisturbed density (γ_0); Wheel slip of previous pass (s_0); Number of passes (n_p); Fitted (k_1, k_2, k_3):

k_1	k_2	k_3
0.1178	0.1672	0.0348

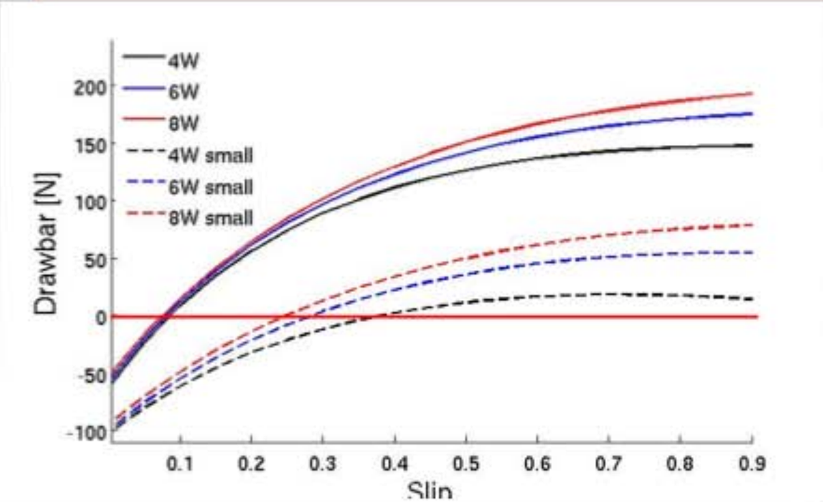


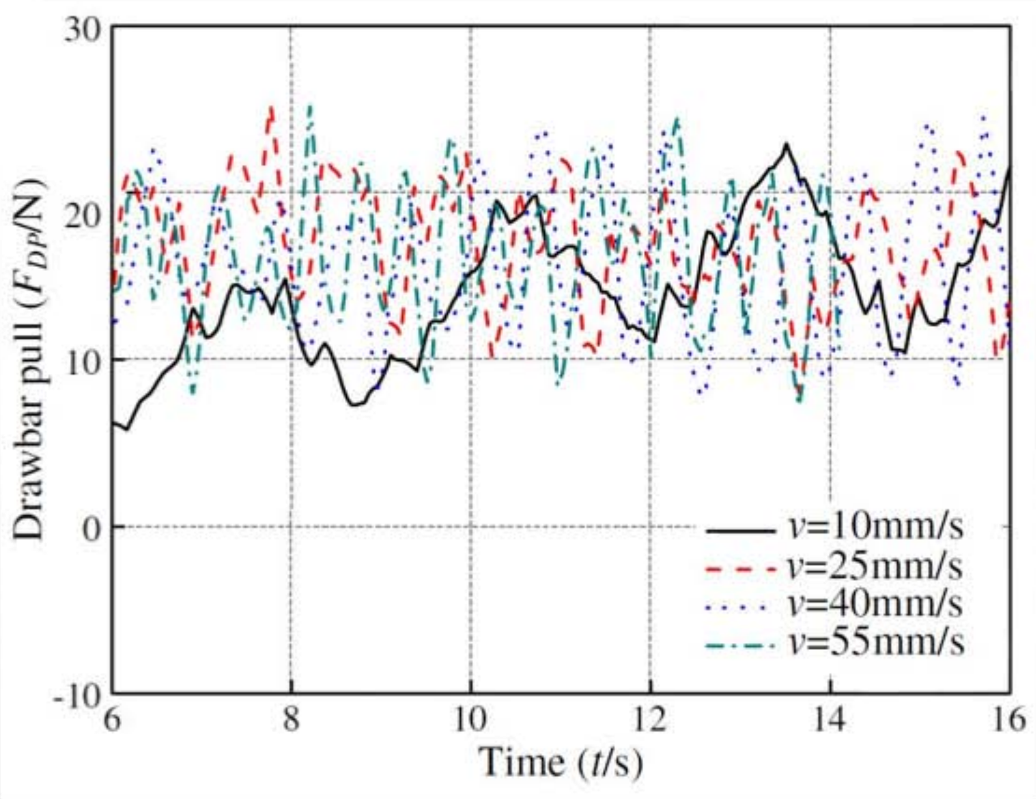
Classical Model Limitations

- Terramechanics developed in context of large vehicles, for design trade space analysis
  - Would like to apply to smaller, lighter systems, for dynamic sim

- Key limitations
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence

Terrain Inhomogeneity (1)





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  - Effects related to slipping and sinking
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    - Rate dependence

Terrain Inhomogeneity

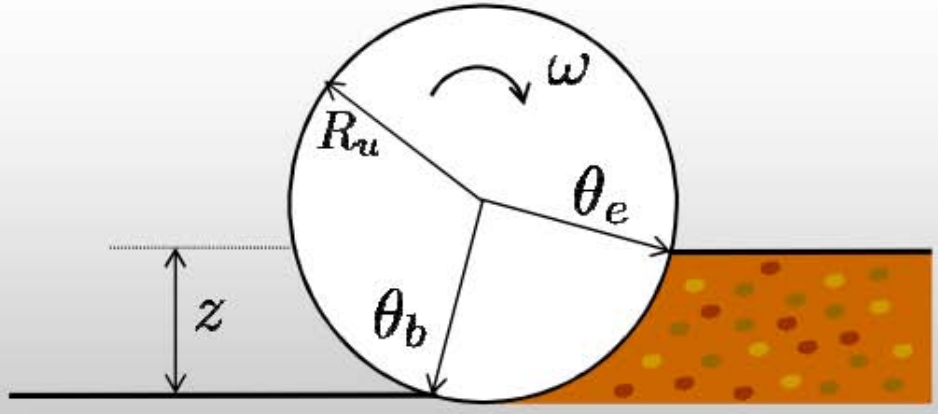
- Pressure-sinkage relation characterizes wide range of terrains with single equation
  - Loose, granular soils, crusty materials, clay

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

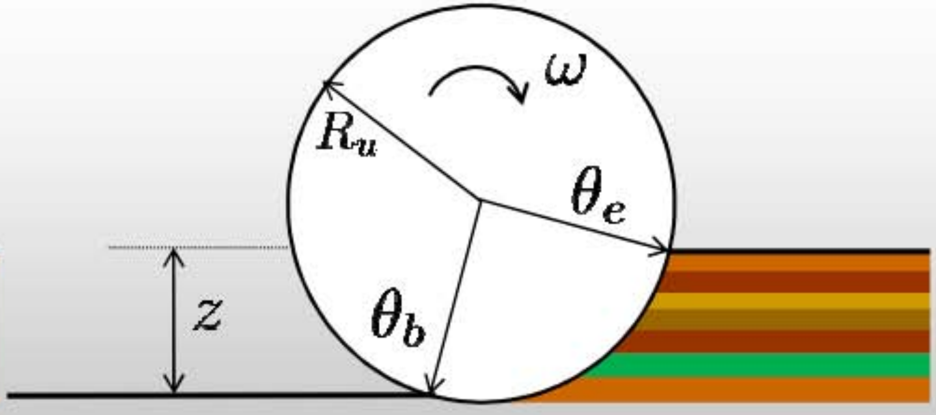
- Observations: significant experimental variation wrt soil condition
  - Layering
  - Relative density
  - Moisture content

Terrain Inhomogeneity (1)

- Bekker theory assumes homogenous soil
  - Soil is often layered, inhomogeneous
- Lack of analytical formulations for pressure-sinkage, shear stress-shear deformation



Inhomogeneous



Layered

Terrain Inhomogeneity (1)

- Pressure-sinkage relations

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

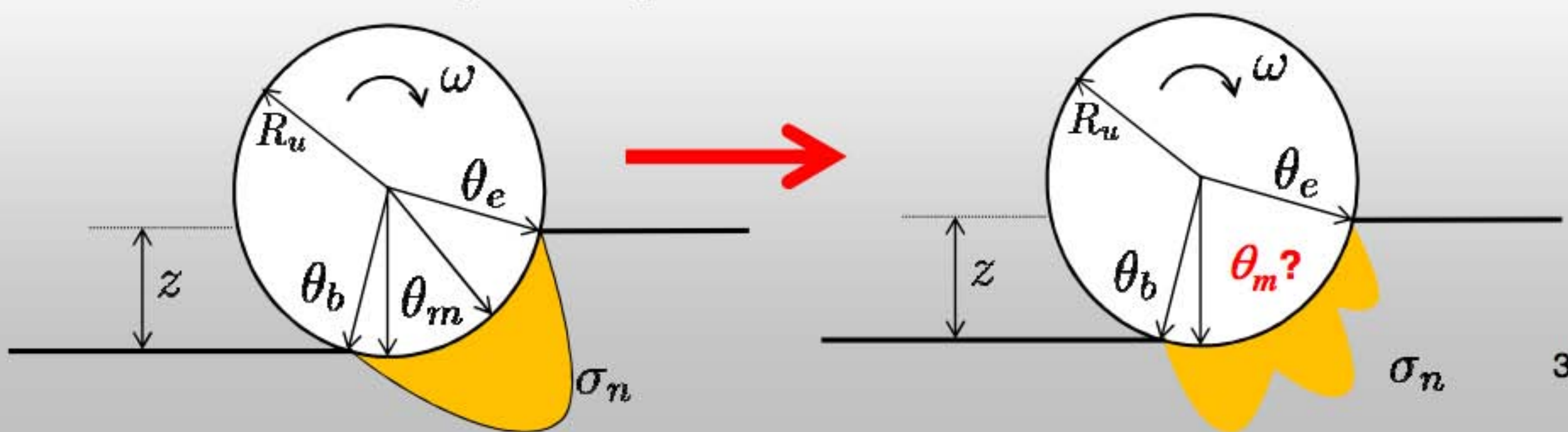
$$\sigma_{nf} = \left(\frac{k_c}{b} + k_\phi\right) \left[R\left(\cos(\theta) - \cos(\theta_e)\right)\right]^n \qquad \theta_m < \theta \leq \theta_e$$

$$\sigma_{nr} = \left(\frac{k_c}{b} + k_\phi\right) \left[R\left(\cos\left(\theta_e - \left(\frac{\theta - \theta_r}{\theta_N - \theta_r}\right)(\theta_e - \theta_N)\right) - \cos(\theta_e)\right)\right]^n \qquad \theta_b \leq \theta \leq \theta_m$$

How to define?

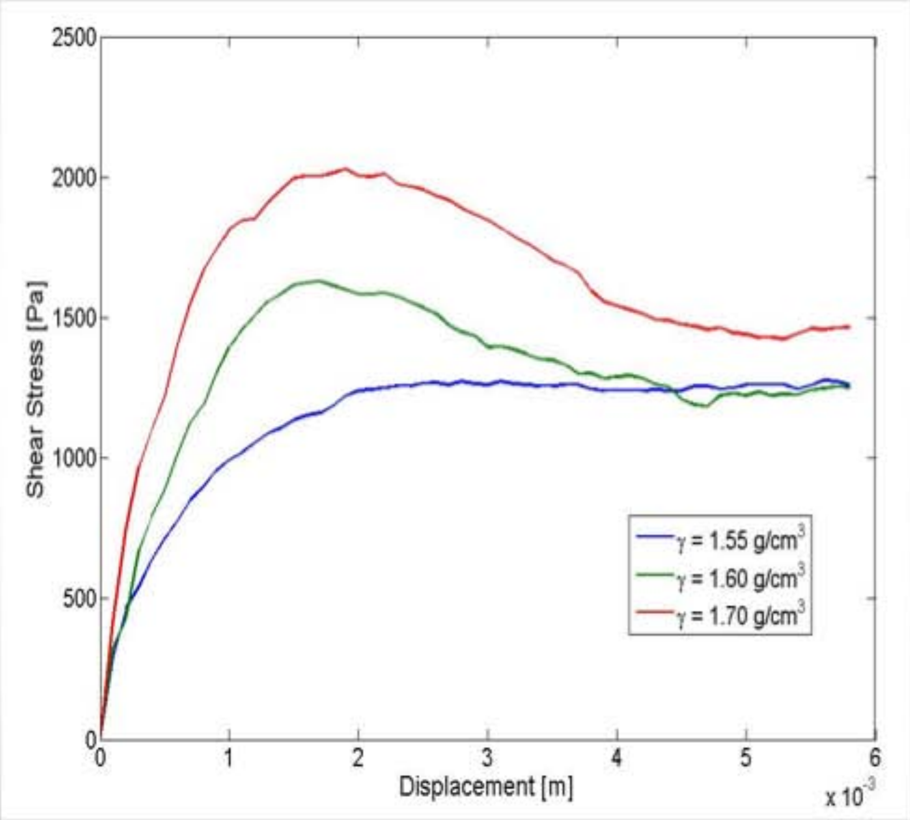
- Shear stress-shear displacement

$$\tau_x(\theta) = \tau_{max}\left(1 - e^{\frac{-jx}{k_x}}\right) \qquad \tau_{max} = c + \sigma_n(\theta)\tan\phi$$



Terrain Inhomogeneity (2)

- Bekker theory (generally) ignores soil state
- Large vehicles tend to compact terrain to dense state upon passage
  - For small rovers, weight is insufficient to compact soil
- Relative density can strongly influence shear stress at interface
  - Strong influence on thrust
  - Strong influence on torque during digging/scooping



Shear box test of MMS

Terrain Inhomogeneity (2)

- Bekker theory (generally) ignores soil state
- Large vehicles tend to compact terrain to dense state upon passage
  - For small rovers, weight is insufficient to compact soil
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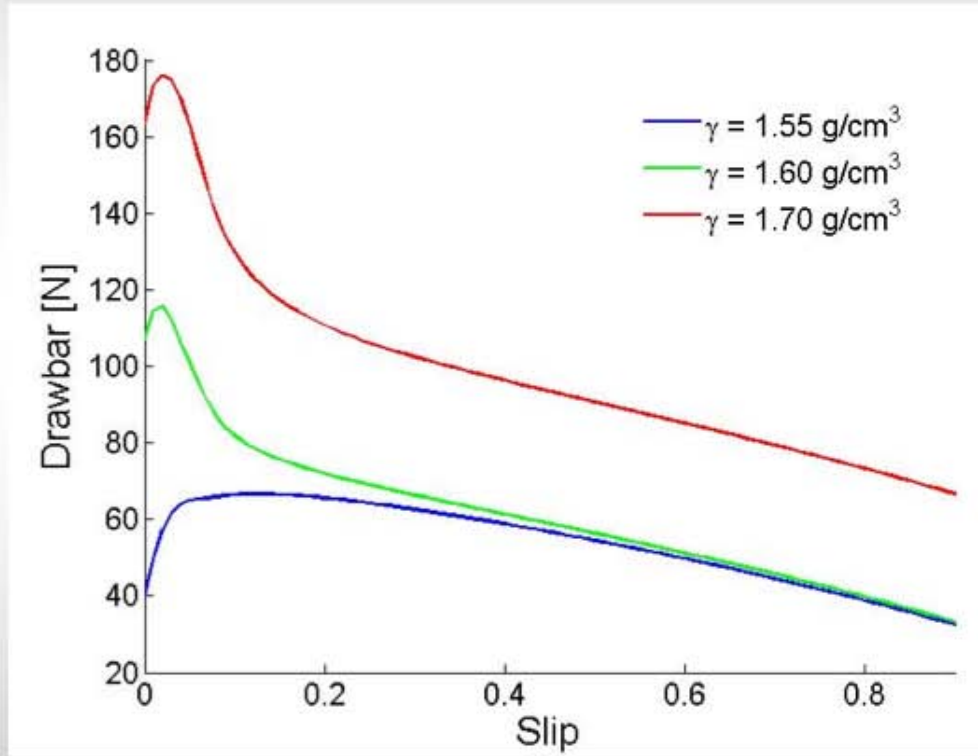

Terrain Inhomogeneity

- Questions (Solutions?)
  - How to compute sinkage in inhomogeneous soil?
    - Express sinkage in integral form (layered)?

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

    - Effective parameters for mixed soils?
  - How to compute failure of layered (crusty) soil?

$$\tau_{max} = c + \sigma_n(\theta) \tan\phi$$

    - Piecewise formulation?
      - Smoothness of stress distribution?
  - How to represent parameters?
    - Intervals? Distributions?
    - State dependent? (For all soils, or only some?)
  - How to represent governing equations?
    - Deterministic? Stochastic?

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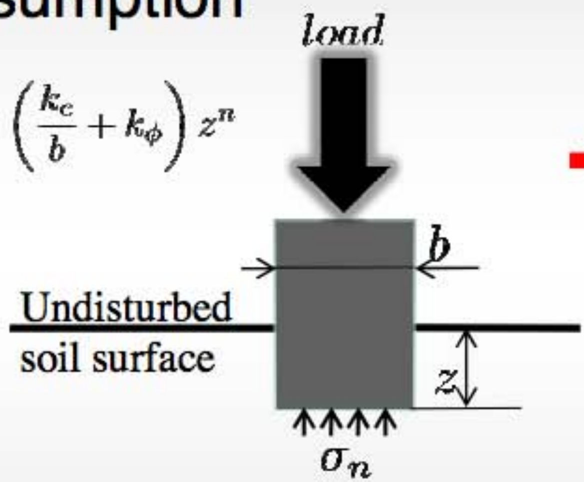
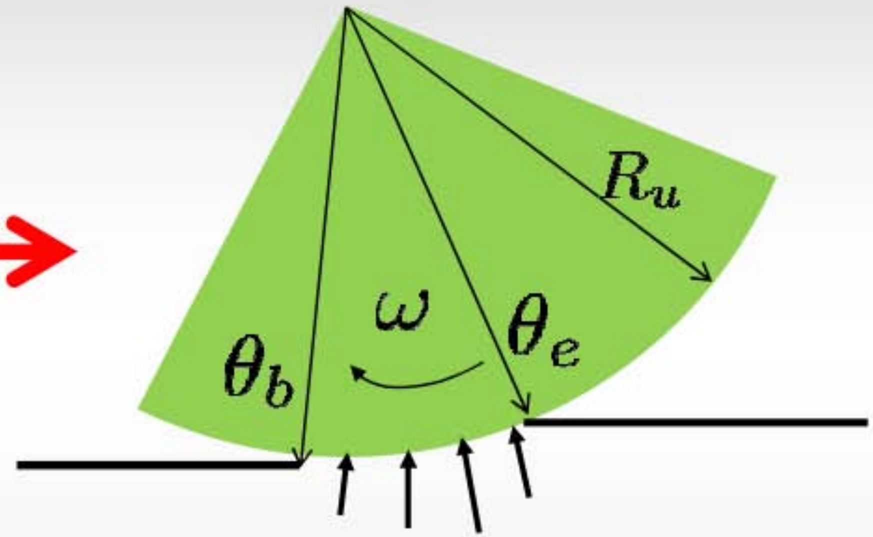
    - Rate dependence

Scale Effects (1)

- Pressure-sinkage relations developed under flat plate assumption

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$



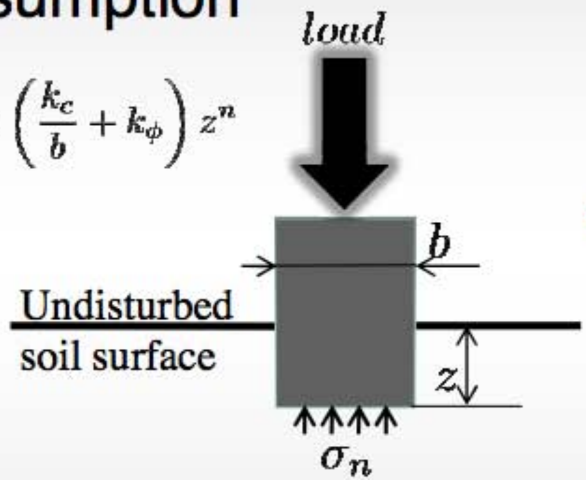


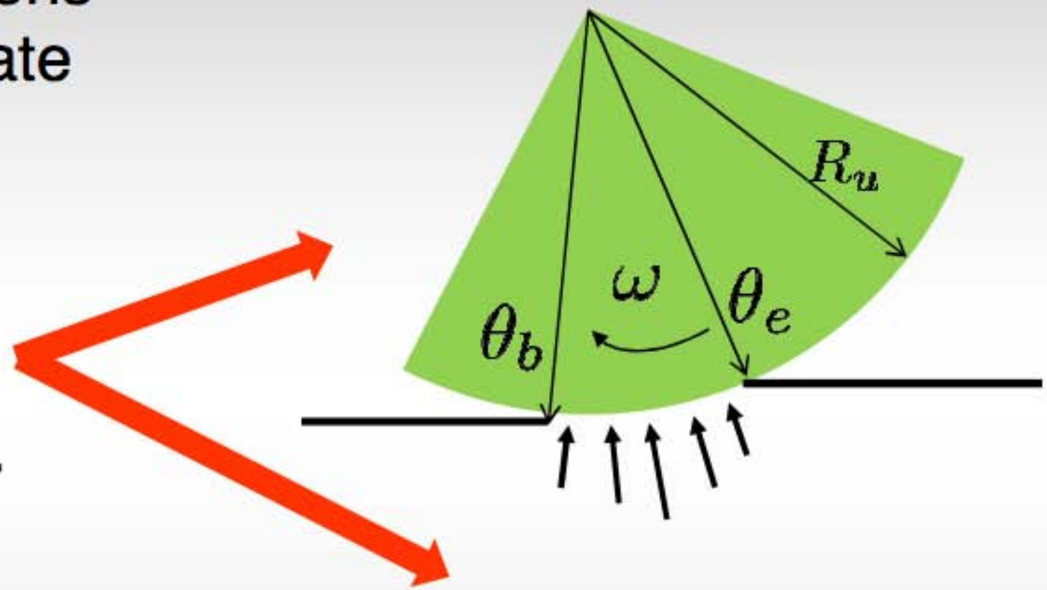
- Reasonable for large vehicles
  - Uniform stress distribution at interface

Scale Effects (1)

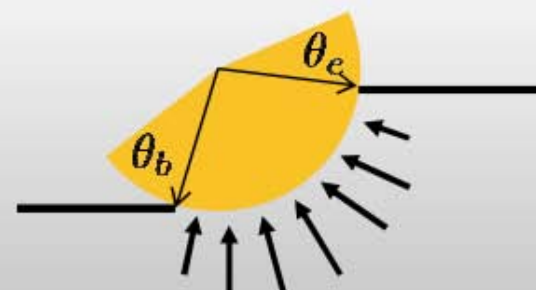
- Pressure-sinkage relations developed under flat plate assumption

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$





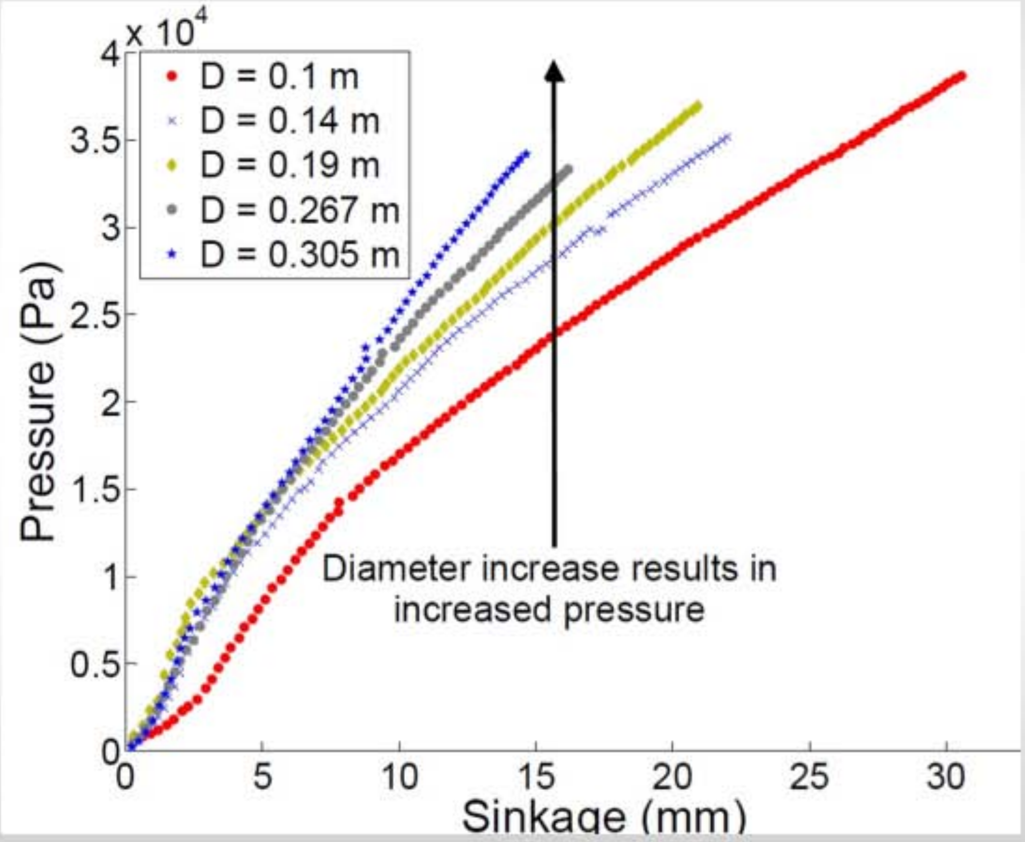
- What about for small vehicle, with high wheel curvatures?
  - Stress distribution at interface non-uniform
  - Component of normal stress balances load



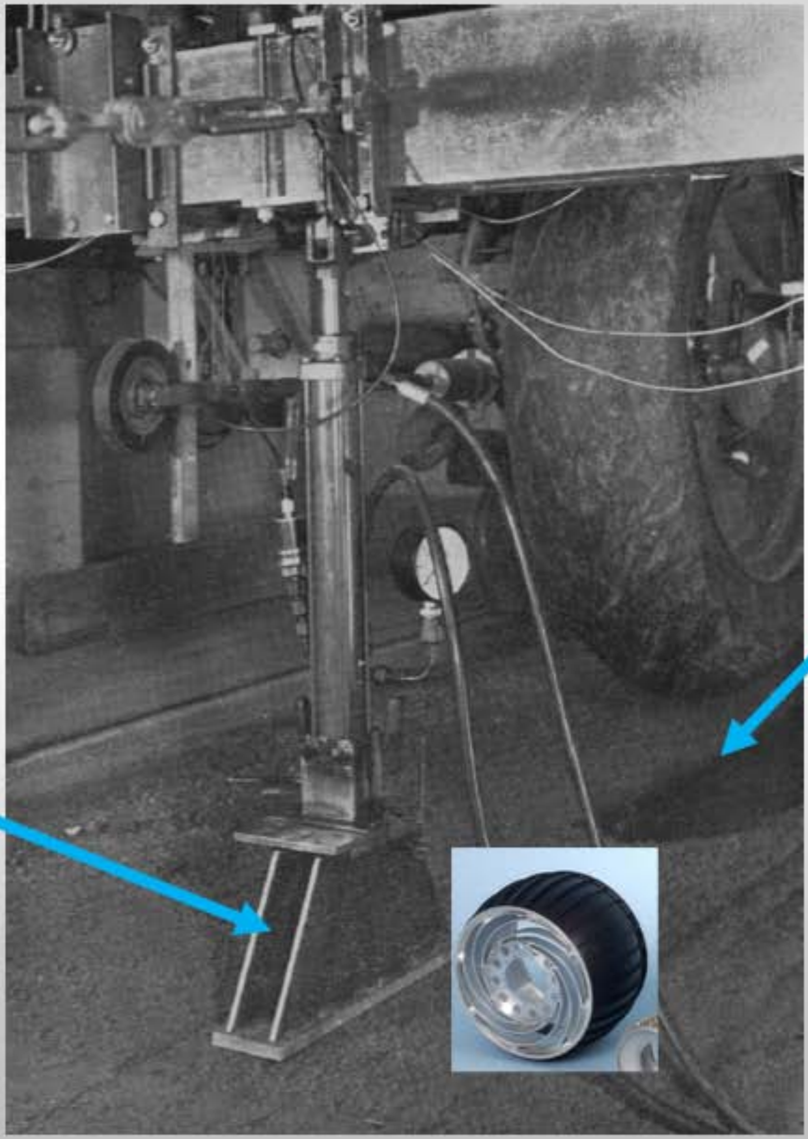
Scale Effects (1)

- Result: Poor prediction of sinkage
- Why is this?
  - Intrinsic parameters not really intrinsic

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$



Scale Effects (1)



Tire imprint

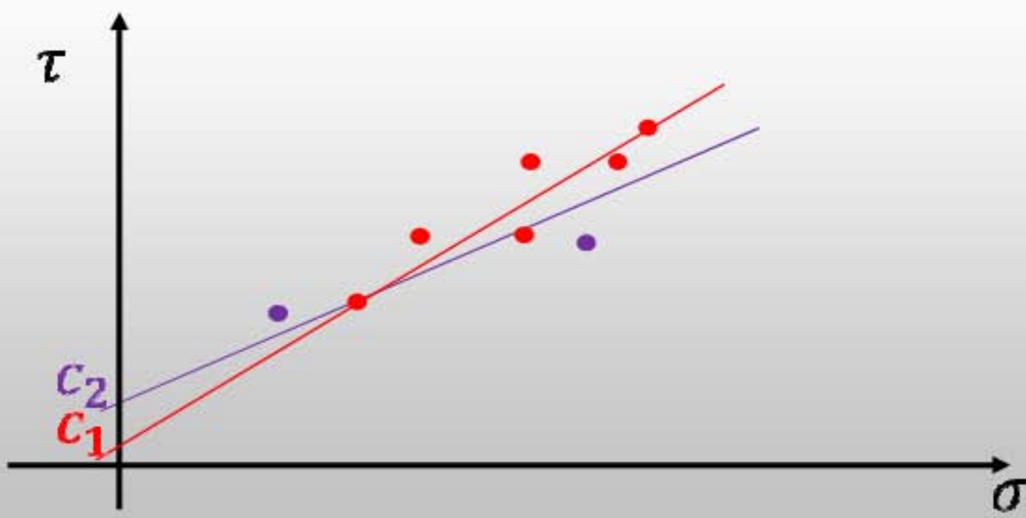
Bevameter plate

Scale Effects (2)

- Soil shear failure is governed by soil cohesion and internal friction angle

$$\tau_{max} = c + \sigma_n(\theta) \tan \phi$$

- Cohesion often measured at high normal stress
  - At low normal loads, effect of cohesion can dominate



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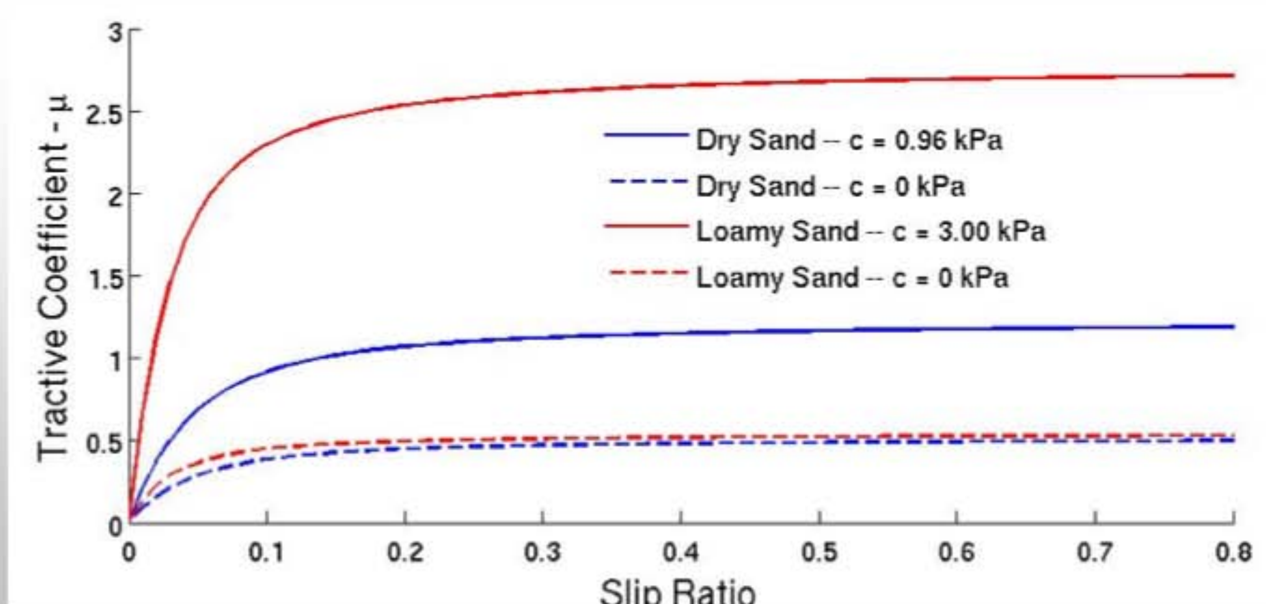
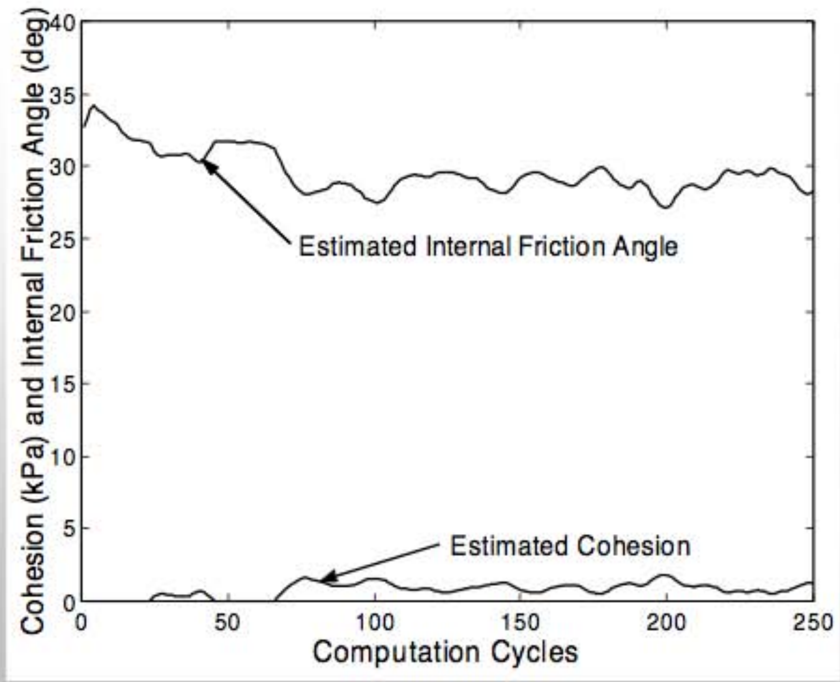


Scale Effects

- Questions (Solutions?)
  - Can we formulate terramechanics relations with intrinsic parameters?
    - Consistent results across scales
  - Can we develop in situ measurement/estimation procedures for parameter estimation?
  - Can we develop lab test devices/procedures for measurement at low normal stress?



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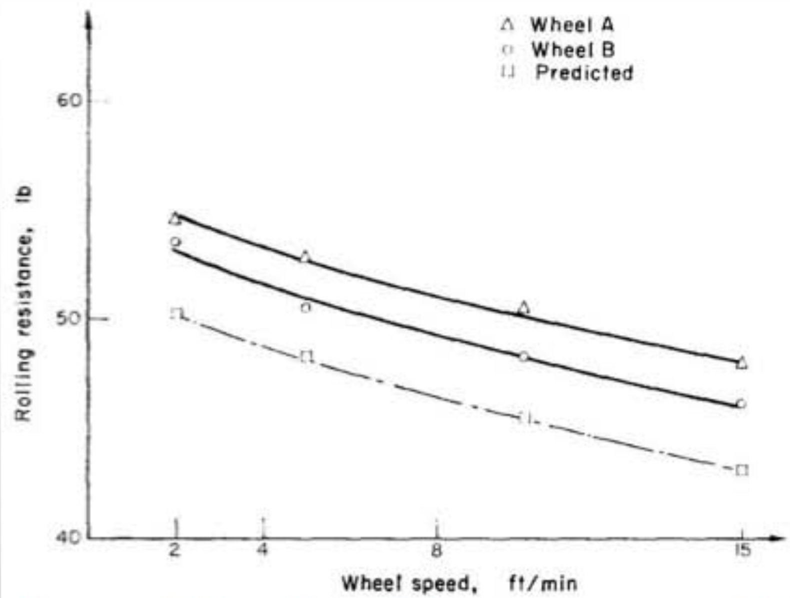
    - Slip ratio definition
    - Rate dependence

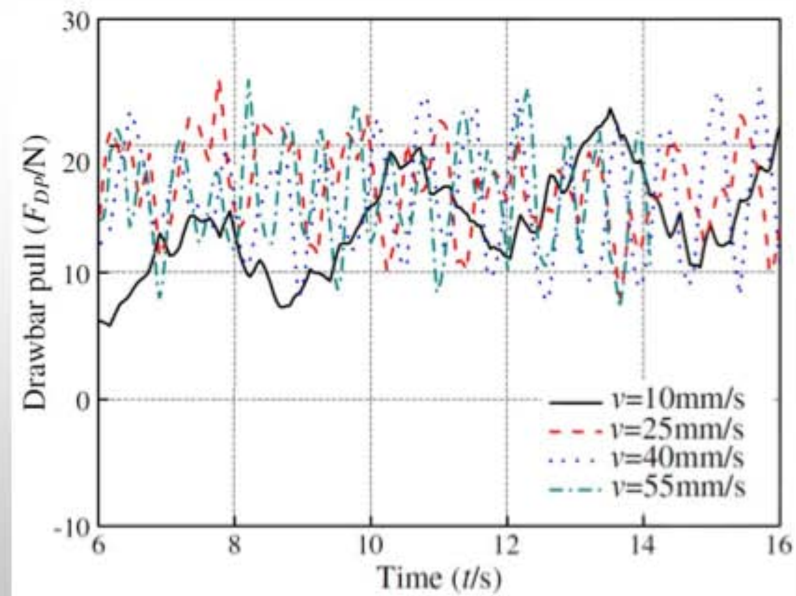
Slipping and Sinking (1)

- Terramechanics models are not rate dependent

- Studies on large wheels show that at higher velocity[1,2]:
  - Sinkage decreases
  - Traction improves

- Experiments[3] on small wheels have suggested little influence

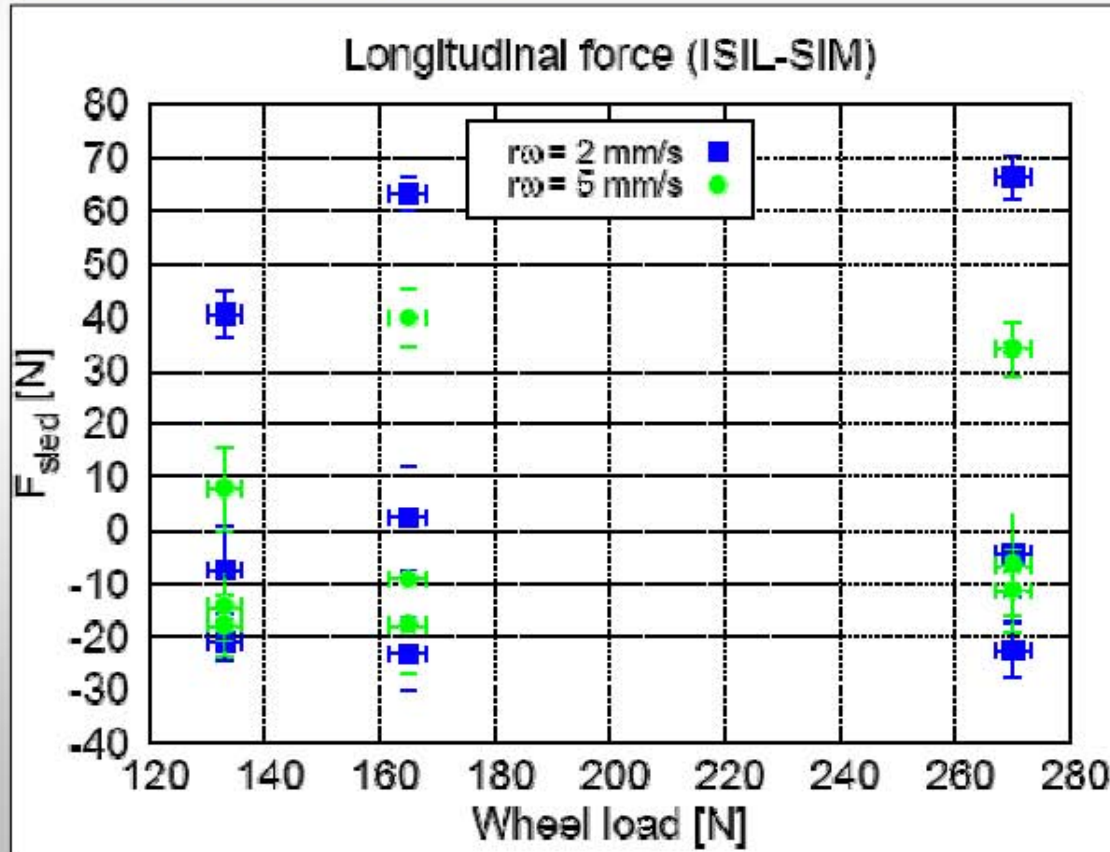




1. Shmulevic I. et al./Journal of Terramechanics 35,1998, 189-207
2. Pope R.G./ Journal of Terramechancis 8(1), 1971, 51-58
3. Ding L. et al./Journal of Terramechanics 48, 2011, 27-45

Slipping and Sinking (1)

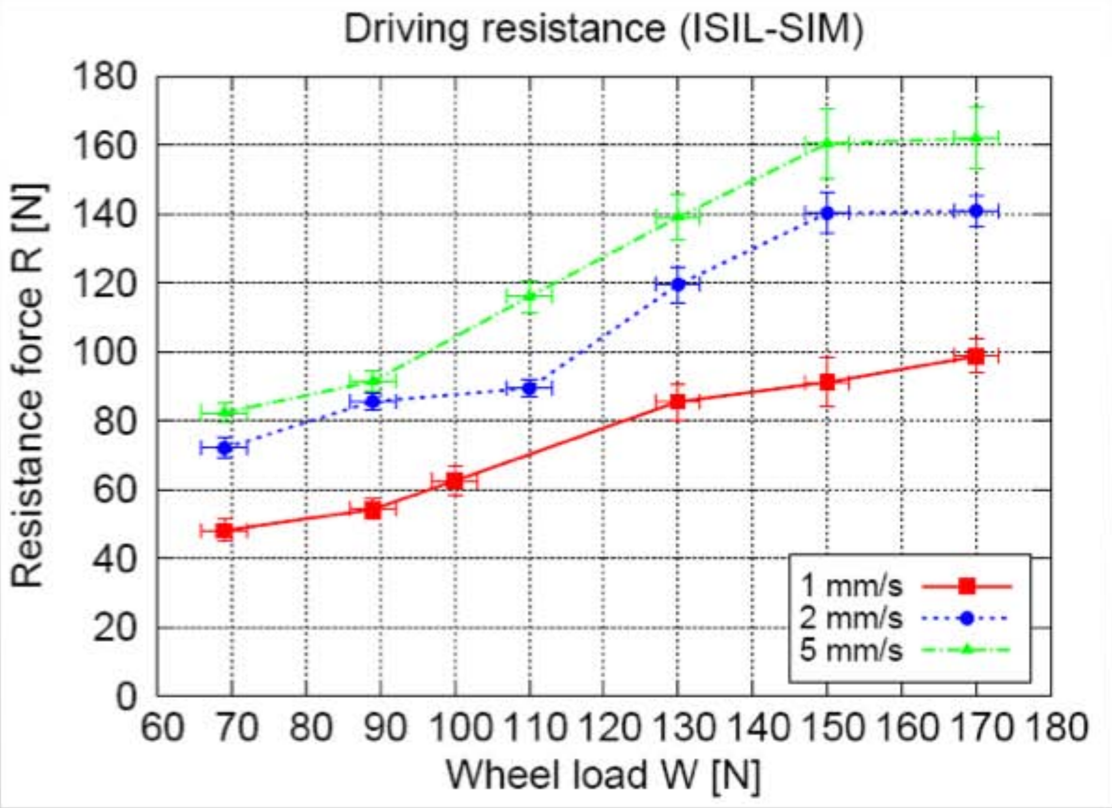
- Experiments with MER wheels have shown significant velocity effect
  - Plot of thrust force vs. vertical wheel load



- Vertical array of same-color data points: slip increasing top to bottom (83 %, 92 %, 98 %)

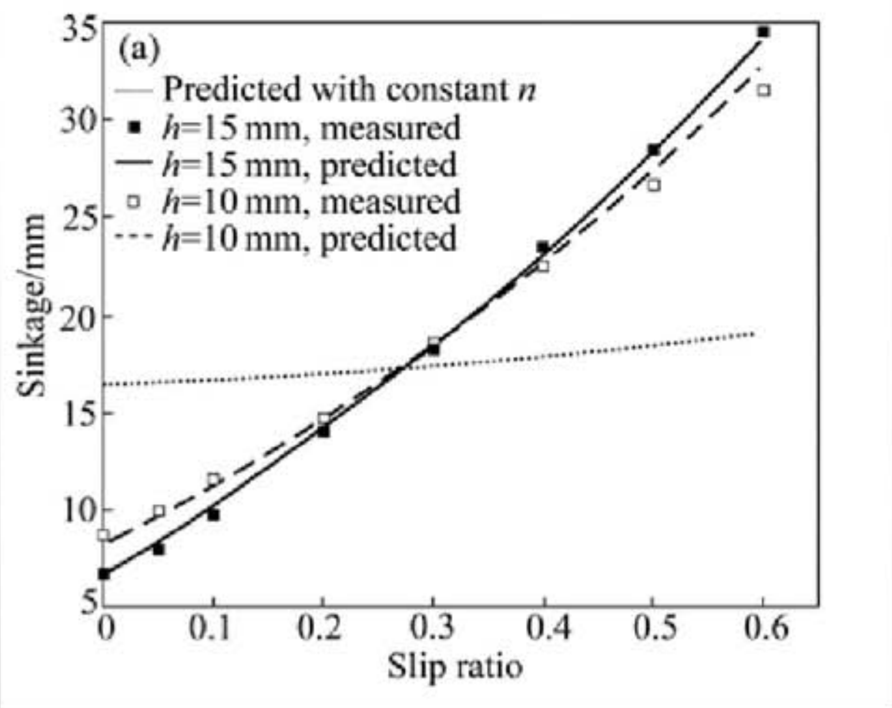
Slipping and Sinking (1)

- Experiments with MER wheels have shown significant velocity effect
  - Resistance from blocked RF wheel vs wheel load and drag velocity



Slipping and Sinking (2)

- Terramechanics theory is not well suited for modeling motion with high slippage
  - No model of material transport
  - No temporal dependence

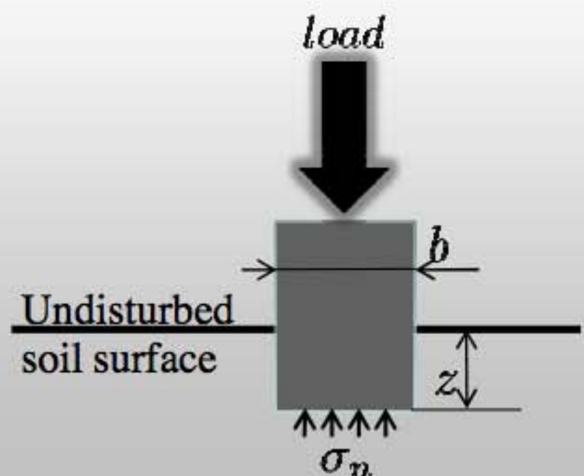


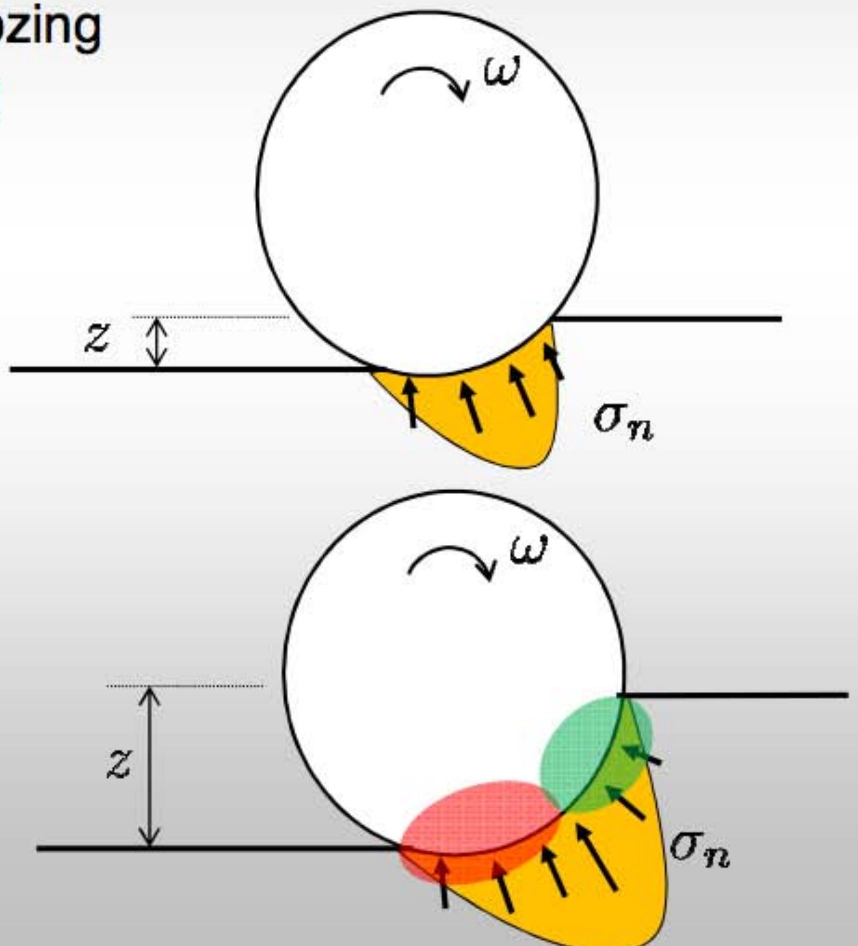
Slipping and Sinking (3)

- Terramechanics theory is not well suited for modeling motion with high sinkage
  - Compaction resistance vs. bulldozing
  - "Flattening" soil vs. "shoving" soil

$$F_x = bR \int_{\theta_b}^{\theta_e} \tau_x(\theta)\cos(\theta) - \sigma_n(\theta)\sin(\theta)d\theta$$

$$\sigma_n = \left(\frac{k_c}{b} + k_\phi\right) z^n$$

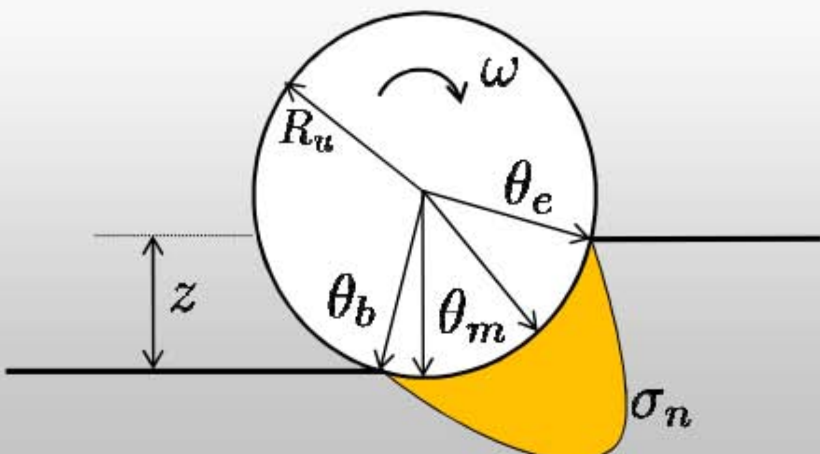




Slipping and Sinking (4)

- Slip ratio defines relative velocity between wheel and soil
  - Dictates shear stress, deformation

$$s_d = 1 - \frac{u}{\omega R}$$



Slipping and Sinking (4)

- Problems with slip ratio
  - Undefined at zero angular velocity
    - Issue for simulation
  - Transition from positive to negative not handled by theory
    - Can occur during free rolling

$$s_d = 1 - \frac{u}{\omega R}$$

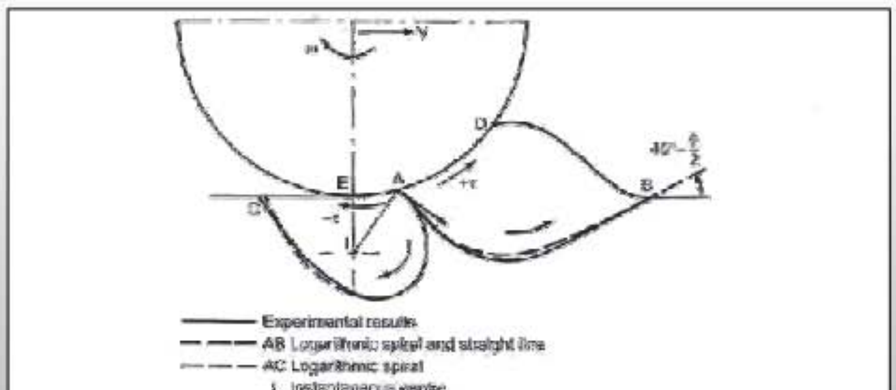


Figure 2.18: Flow patterns and bow wave under the action of a towed rigid wheel in sand

Slipping and Sinking (4)

- Problems with slip ratio
  - Undefined at zero angular velocity
    - Issue for simulation
  - Transition from positive to negative not handled by theory
    - Can occur during free rolling

$$s_d = 1 - \frac{u}{\omega R}$$

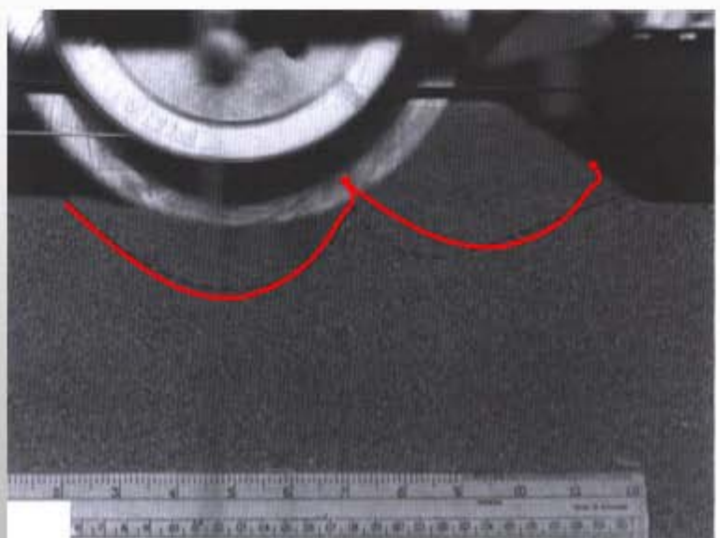


Figure 1.11: Soil flow patterns under a driven rigid wheel in sand

Slipping and Sinking

- Questions (Solutions?)
  - How to model rate dependence?
    - Effect on motion resistance, thrust
    - Momentum formulation of terramechanics relations?
  - How to model temporal effects?
    - Effect on sinkage
    - Model material transport based on grouser geometry?
      - For some soils? All?
  - How to model motion resistance due to high sinkage?
    - Piecewise formulation?
  - "Unified" model of wheel slip?
    - Analysis of particle motion under wheels

Conclusions

- Fundamental limitations of terramechanics modeling
  - Effect of terrain inhomogeneity
    - Soil condition dependence
      - Layering, relative density, moisture content
  - Scale effects
    - Parameter scale dependence (non-intrinsic soil properties)
  - Effects related to slipping and sinking
    - Slip ratio definition
    - Rate dependence
- Issues affect computation, simulation
- Tradeoff between generality and accuracy
- Tradeoff between measurement burden and accuracy

Terramechanics References

- Bekker, M.G, Introduction to Terrain-Vehicle Systems, Ann Arbor, University of Michigan, 1969
- Terzaghi, Theoretical Soil Mechanics, 1943
- Wong, J.Y. Theory of Ground Vehicles 2nd Edition, Canada