Fluids

2/26/04

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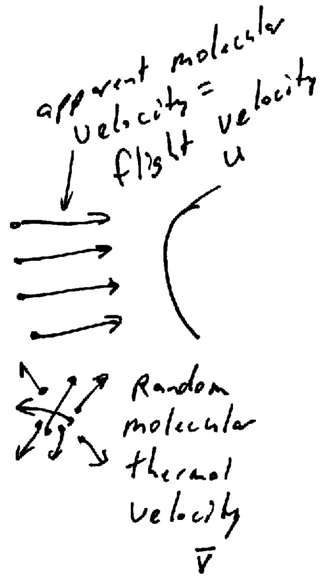
(Some of the first material was covered at the end of Tuesday's lecture)

a = speed of sound in fluid $= \sqrt{\gamma R T}$

γ = ratio of specific heats for gas $= \begin{cases} 1.67 \text{ (monatomic)} \\ 1.4 \text{ (diatomic)} \\ \sim 1.2 \text{ (complex or hot gases)} \end{cases}$

$R = \frac{\mathcal{R}}{\bar{m}}$ $\quad \mathcal{R}$ = universal gas constant $= 8.3144 \ \mathrm{J/mol \cdot {}^\circ K}$

$\bar{m}$ = molecular weight of gas (average for mixtures)

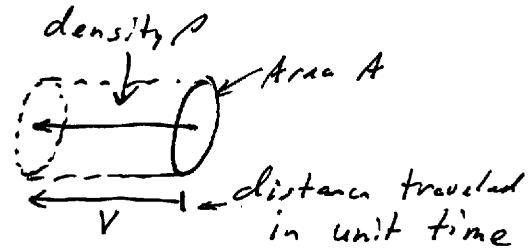


$\gamma = \frac{c_p}{c_v}$ = ratio of specific heats

Mach number $M = \frac{V}{a}$

$$\frac{\text{ordered kinetic energy}}{\text{random kinetic energy}} = \frac{\frac{1}{2} m u^2}{\frac{1}{2} m \bar{v}^2} = \frac{u^2}{3RT} = \frac{\gamma}{3}\frac{u^2}{a^2} = \frac{\gamma}{3} M^2$$

$$\frac{\text{inertial force}}{\text{viscous force}} = \frac{\dot{m} V}{\tau A}$$



mass flux of gas encounter $= \dot{m} = \rho A V$

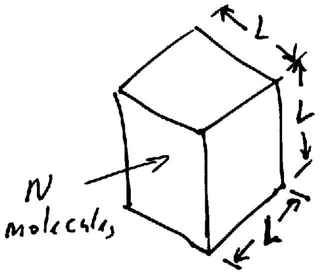
$$\frac{\text{inertial}}{\text{viscous}} = \frac{\rho A V^2}{\mu \frac{V}{L} A} = \frac{\rho V L}{\mu} = R_e \Leftarrow \text{Reynold's number}$$

L = characteristic length of spacecraft

Knudsen number $K = \frac{\lambda}{L}$ $\Leftarrow$ relative likelihood of molecule colliding with another molecule or with spacecraft

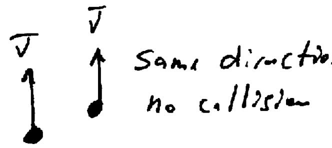
λ = molecular mean free path

Mean Free Path

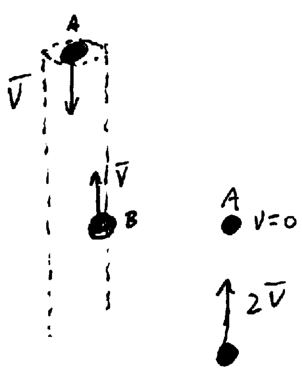


Control region – cubical with volume L^3
contains N molecules

Assume: molecules are identical rigid spheres
- diameter σ
- mass m
- all have same velocity $\bar{v}$
- $N/6$ molecules travelling in each direction ($\pm x, \pm y, \pm z$)



Step one – consider collisions between molecules travelling in z direction only



No collision for velocities in same direction

Will collide for opposite velocities, if center of B lies within cylinder with <u>radius</u> of σ from center of A

Can think of this as A = stationary and B has velocity of $2\bar{v}$

number of ($+z$) molecules $= \dfrac{N}{6}$

fraction within impact zone $= \dfrac{\pi\sigma^2 L}{L^3}$

number of ($+z$) collisions $\quad n(+z) = \dfrac{N}{6}\dfrac{\pi\sigma^2}{L^2}$

frequency of ($+z$) collisions

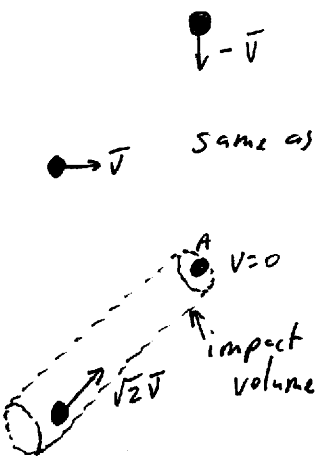
$$f(+z) = \frac{n(+z)}{\Delta t} = \frac{(N/6)\frac{\pi\sigma^2}{L^2}}{L/2\bar{v}} = \frac{N}{6}\frac{2\bar{v}\pi\sigma^2}{L^3}$$

time required to travel length of cylinder

$m \equiv$ mass of molecule $\Rightarrow$ total gas mass $= Nm$

density $\rho = \dfrac{Nm}{L^3}$ so $\dfrac{N}{L^3} = \dfrac{\rho}{m}$

$$\boxed{f(+z) = \frac{\pi \rho \sigma^2 \bar{v}}{3m}}$$



Now consider impact between $(-z)$ and $(+x)$ molecules

Impact cylinder volume $= \pi \sigma^2 L$ (average)

Time to traverse $= \dfrac{L}{\sqrt{2}\bar{v}}$

$$f(+x) = \frac{n(+x)}{\Delta t} = \frac{N}{6} \frac{\pi \sigma^2 / L^2}{L/\sqrt{2}\bar{v}} = \frac{\sqrt{2}\pi \rho \sigma^2 \bar{v}}{6m}$$

By symmetry,

$$\boxed{f(+x) = f(-x) = f(+y) = f(-y) = \frac{\sqrt{2}}{6} \frac{\pi \rho \sigma^2 v}{m}}$$

and, for completeness, $\boxed{f(-z) = 0}$

Total frequency of collisions

$$f = 0 + \frac{\pi \rho}{3m} \sigma^2 \bar{v} + \frac{4\sqrt{2}}{6m} \pi \rho \sigma^2 \bar{v}$$

$$\boxed{f = \frac{\pi}{3} (1 + 2\sqrt{2}) \frac{\rho \sigma^2 \bar{v}}{m}}$$

Mean free path $= \lambda \equiv$ average distance between collisions

$$\lambda = \frac{\bar{v}}{f} = \frac{m/\sigma^2}{(\pi/3)(1+2\sqrt{2})\rho} = \frac{C}{\rho} = \lambda \Rightarrow \boxed{\lambda \propto \frac{1}{\rho}}$$

At sea level, $\lambda = 6.7 \times 10^{-8}$ m
at 100 km, $\lambda = 30$ cm ~ 1 ft

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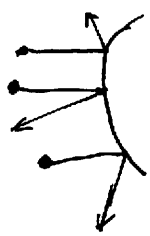
Flow regimes:

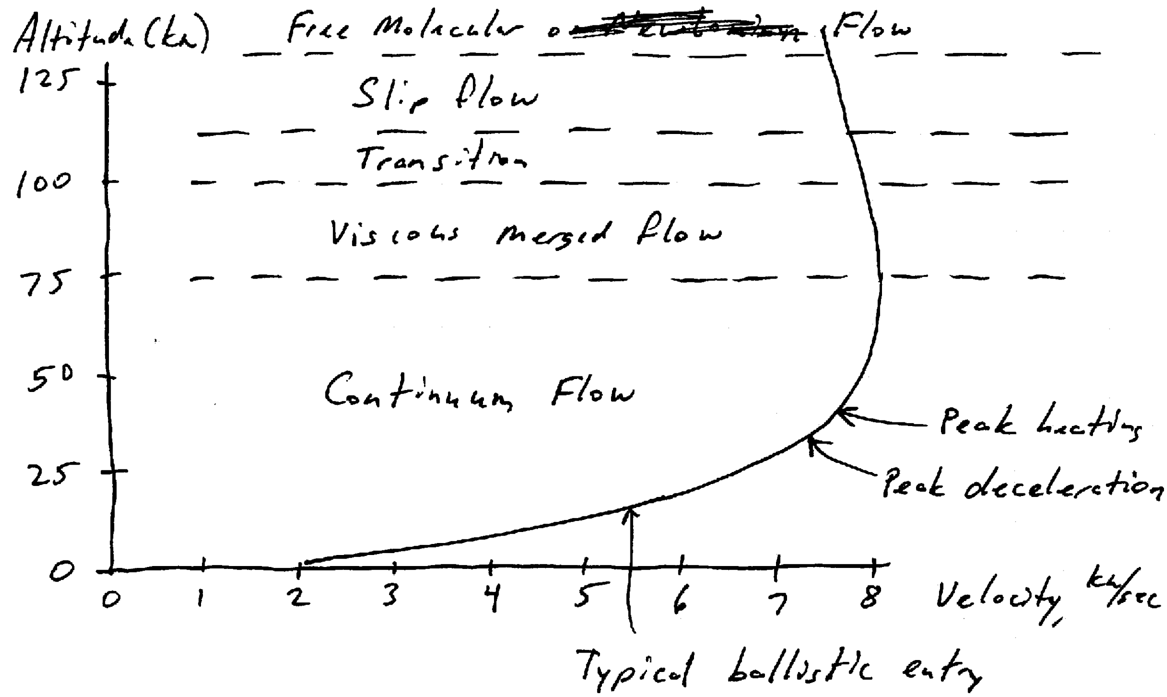
- Free molecular flow $\lambda \gg L$



  - Molecules only interact with spacecraft, not with each other
  - "Ping-pong ball" analysis - "Newtonian flow"

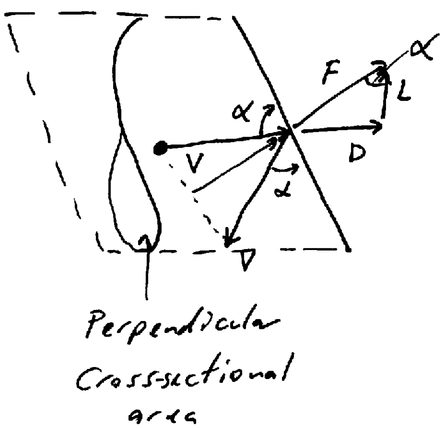
- Slip flow
  - Some noticeable interactions between molecules
  - Beginnings of boundary layer formation
- Transition flow
  - Very complicated + chaotic
- Viscous merged layer
  - start of both shocks and boundary layer
- Continuum flow
  - "classical" aerodynamics

→ hypersonic	$M > 5$
supersonic	$1.3 < M < 5$
transonic	$0.8 < M < 1.3$
subsonic	$0.3 < M < 0.8$
incompressible	$M < 0.3$



Free Molecular flow

- specular reflection (angle in = angle out)



Flat plate of area A at angle of attack α

Perpendicular area intercepted $= A \sin\alpha$

Incoming momentum $\perp$ to plate $= \dot{m} V \sin\alpha$

Outgoing momentum $\perp$ to plate $= -\dot{m} V \sin\alpha$

Total momentum transfer $\rightarrow$ Force on plate

$F = 2\dot{m} V \sin\alpha \qquad \dot{m} = \rho A \sin\alpha \, V$

$F = 2\rho A V^2 \sin^2\alpha$

Resolve into components parallel (drag) and perpendicular (lift) to velocity vector

$L = 2\rho A V^2 \sin^2\alpha \cos\alpha$
$D = 2\rho A V^2 \sin^3\alpha$

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but definitions are $L = \frac{1}{2}\rho V^2 A C_L$ $\quad D = \frac{1}{2}\rho V^2 A C_D$

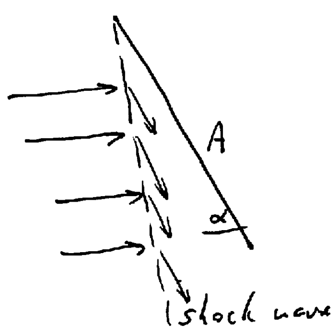
so
$$\boxed{\begin{aligned} C_L &= 4\sin^2\alpha\cos\alpha \\ C_D &= 4\sin^3\alpha \end{aligned}}$$

Flat plate perpendicular to velocity vector in free molecular flow $\Rightarrow C_D = 4$

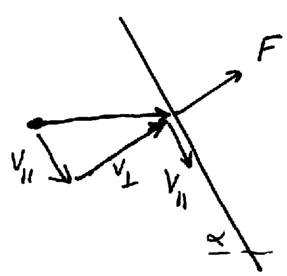
$\Rightarrow$ This analysis is great for calculating drag terms for orbital decay, etc.

<u>NOT</u> for entry!

Continuum flow - hypersonic



- Oblique shocks form from leading edge
- Turn incoming flow parallel to plate
- Same as ping-pong balls, losing all $\perp$ momentum to plate! (no reflection)



$$F = \dot{m} V \sin\alpha = \rho A V^2 \sin^2\alpha$$

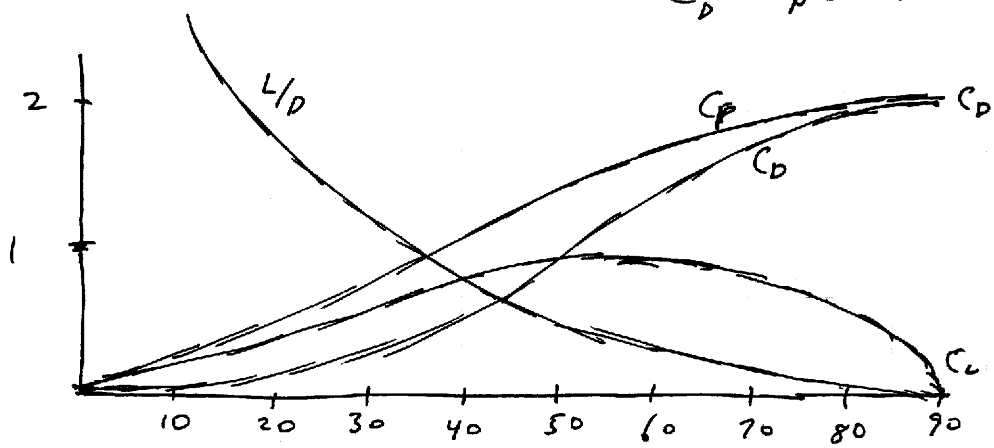
$$\boxed{\begin{aligned} L &= \rho A V^2 \sin^2\alpha\cos\alpha & \Rightarrow C_L &= 2\sin^2\alpha\cos\alpha \\ D &= \rho A V^2 \sin^3\alpha & \Rightarrow C_D &= 2\sin^3\alpha \end{aligned}}$$

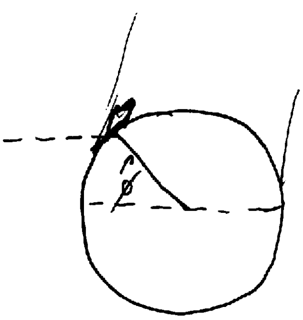
Lift + drag coefficients in continuum (hypersonic) flow $= \frac{1}{2}$ those in free molecular flow

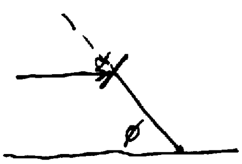
$\Rightarrow$ "Newtonian" analysis good for hypersonics, bad for subsonic

Can also define total force on plate

$$F = \tfrac{1}{2}\rho V^2 A C_p \Rightarrow \quad \begin{aligned} C_p &= 2\sin^2\alpha \\ C_L &= C_p\cos\alpha \\ C_D &= C_p\sin\alpha \end{aligned}$$







What else can we do with this?

Lift, Drag on a crossways cylinder

$dA = r\,d\phi\,dL \qquad \alpha = 90° + \phi = \frac{\pi}{2} + \phi$

$\delta C_p = 2\sin^2\left(\frac{\pi}{2} + \phi\right) = 2\cos^2\phi$

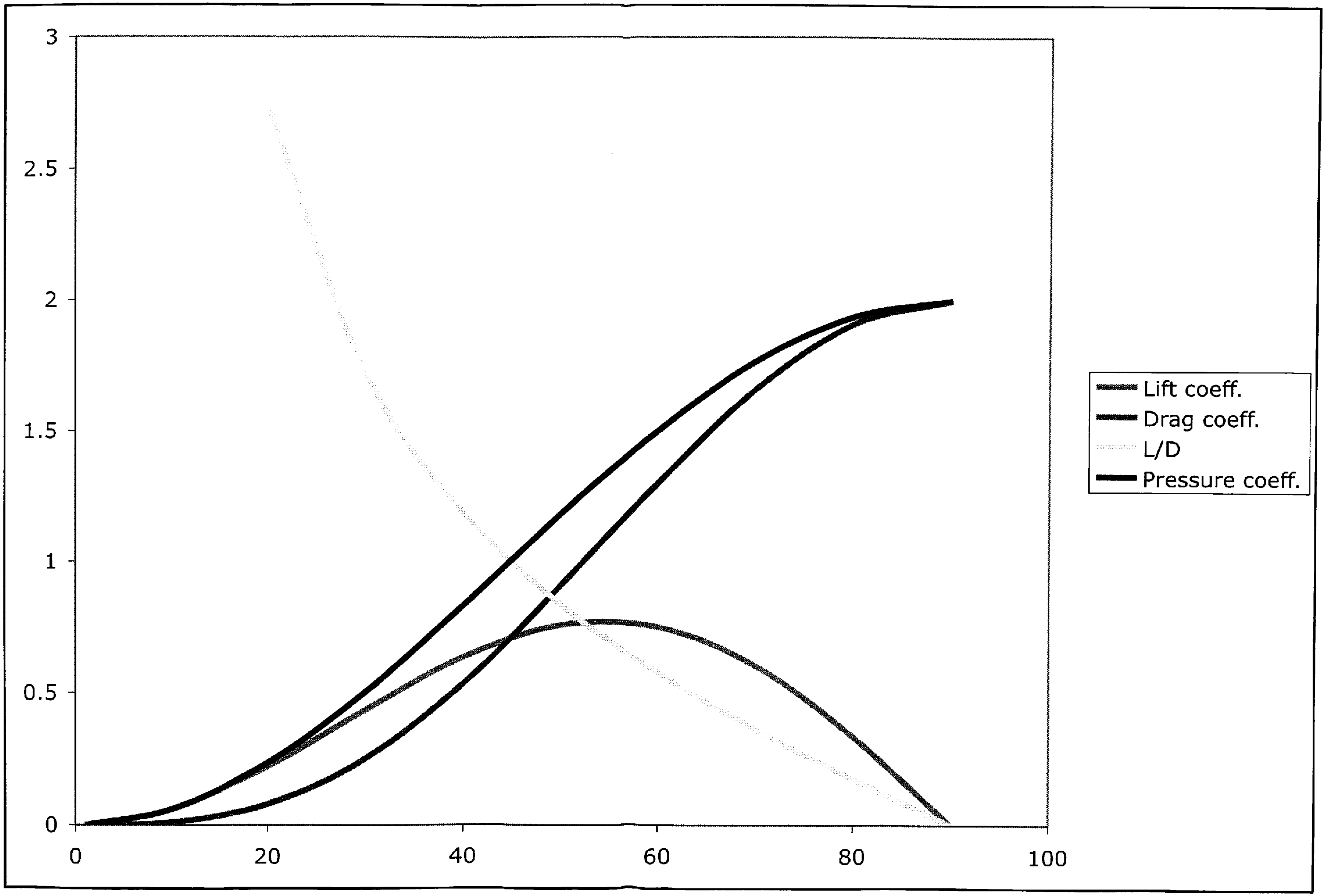
$$F = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^L \tfrac{1}{2}\rho V^2\, \delta C_p\, dA$$

$$= \tfrac{1}{2}\rho V^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^L 2\cos^2\phi\, r\,d\phi\,dL$$

$$= \rho V^2 r L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\phi\, d\phi$$

$$= \rho V^2 r L \left. \frac{\phi}{2} - \frac{1}{4}\sin 2\phi \right|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \rho V^2 r L \left(\frac{\pi}{4} + \frac{\pi}{4}\right) = \tfrac{1}{2}\rho V^2 r L \pi = \tfrac{1}{2}\rho V^2 (2rL)\, C_p$$



3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
Lift coeff.
Drag coeff.
L/D
Pressure coeff.

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$$\pi = 2c_p \qquad c_p = \pi/2$$

$$L = \tfrac{1}{2}\rho V^2 (r\,d\phi\,dL)(2\sin^2\alpha\cos\alpha)$$

$$= \rho V^2 r \int_0^L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2\alpha\cos\alpha\,d\phi\,dL$$

$$= \rho V^2 r L \int_{-\frac{\pi}{2}}^{\pi/2} \sin^2(\phi+\tfrac{\pi}{2})\cos(\phi+\tfrac{\pi}{2})\,d\phi$$

$$\cos^2(\phi)[-\sin(\phi)]\,d\phi$$

$$= -\rho V^2 r L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\phi\sin\phi\,d\phi$$

$$u = \cos\phi \qquad du = -\sin\phi\,d\phi$$

$$= \rho V^2 r L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} u^2\,du = \frac{\rho V^2 r L}{3} \cos^3\phi \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = 0$$

$$D = \rho V^2 r L \int_{-\frac{\pi}{2}}^{\pi/2} \sin^3\alpha\,d\phi = \rho V^2 r L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3(\tfrac{\pi}{2}+\phi)\,d\phi$$

$$= \rho V^2 r L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^3\phi\,d\phi = \rho V^2 r L$$

$$= \rho V^2 r L \quad \frac{3}{4}\sin\phi + \frac{1}{12}\sin(3\phi) \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \rho V^2 r L \left[\frac{3}{4}(1+1) + \frac{1}{12}(-1-1)\right]$$

$$\frac{6}{4} - \frac{1}{6} = \frac{18}{12} - \frac{2}{12} = \frac{16}{12} = \frac{4}{3}$$

$$\frac{4}{3}\rho V^2 r L = \frac{1}{2}\rho V^2 (2rL) C_D \Rightarrow \boxed{C_D = 4/3}$$