Re-Entry Aerodynamics

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which can be used to determine the ratio of $\bar{v}/a$ from Eq. (1.27). This leads to

$$\bar{v}/a = \sqrt{3/\gamma} = 1.35 \tag{1.29}$$

Therefore, the average molecular speed is approximately equal to the speed of sound.

Collision Frequency

To obtain the transport properties for the transport of mass, momentum, and energy, the collision frequency of the molecules is required. It is primarily through collisions that these basic quantities may be transported.

First, only collisions between molecules moving in the z direction will be considered, as depicted in Fig. 1.6. Any molecule moving inside the cylinder of radius σ at velocity $+2\bar{v}$ will collide with the stationary molecule. All others moving in the $+z$ direction will never collide with this molecule, as illustrated in Fig. 1.7. The number of molecules in this cylinder at any one time is

$$n(+z) = \frac{1}{6} N \frac{\pi \sigma^2 L}{L^3}$$

The frequency of these collisions is given by

$$f(+z) = \frac{n(+z)}{\Delta t} = \frac{(\pi/6)(N\sigma^2/L^2)}{L/2\bar{v}}$$

Fig. 1.6 **Collisions between molecules moving in the z direction.**

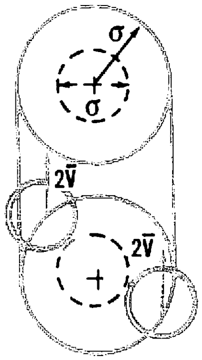

Fig. 1.7 **Molecules moving inside the cylinder of radius σ at velocity $2\bar{v}$.**

or

$$f(+z) = \frac{\pi\rho\sigma^2\bar{v}}{3m} \tag{1.30}$$

Now consider collisions of this particle with molecules moving in the $+x$ direction (see Fig. 1.8). The frequency of these collisions is then given by

$$f(+x) = \frac{n(+x)}{\Delta t} = \frac{(\pi/6)(N\sigma^2/L^2)}{L/\sqrt{2}\,\bar{v}}$$

or

$$f(+x) = \frac{\sqrt{2}\,\pi\rho\sigma^2\bar{v}}{6m} \tag{1.31}$$

From symmetry $f(+x) = f(-x) = f(+y) = f(-y)$; therefore, the total collision frequency f is the sum of collision frequencies in all six directions,

$$f = \Sigma f_i = f(-z) + f(+z) + 4f(+x)$$

$$f = 0 + \frac{\pi\rho}{3m}\sigma^2\bar{v} + \frac{4\sqrt{2}}{6m}\pi\rho\sigma^2\bar{v}$$

(Note that no molecules can collide with particles moving in a $-z$ direction since all travel at the same speed.) Hence,

$$f = \frac{\pi}{3}(1 + 2\sqrt{2})\frac{\rho\sigma^2\bar{v}}{m} \tag{1.32}$$

Of interest is the mean free path λ defined as the average distance traveled between collisions. The mean free path is also related to the mean molecular velocity and frequency of collision through the expression

$$\bar{v} = \lambda f \tag{1.33}$$

Hence, substituting Eq. (1.32) into Eq. (1.33)

$$\lambda = \frac{\bar{v}}{f} = \frac{m/\sigma^2}{(\pi/3)(1 + 2\sqrt{2})\rho} \tag{1.34}$$

Fig. 1.8 Molecules moving in the $+x$ direction.

or

$$\lambda = C_1/\rho \tag{1.35}$$

where

$$C_1 = 3m/\pi\sigma^2(1 + 2\sqrt{2}). \tag{1.36}$$

This shows that the mean free path is inversely proportional to density. Examples of the actual values for the mean free path of air are the following: at sea level, $\lambda = 2.2 \times 10^{-7}$ ft; at 335,000 ft altitude, $\lambda = 1.0$ ft.

Transport Phenomena

Next, the transport properties of diffusivity, viscosity, and thermal conductivity (D, μ, k) will be evaluated.[5] The methods for handling these three phenomena are so similar that they are most conveniently discussed as a group; they are often referred to under the title of *transport phenomena.* Table 1.5 lists the important features for the transport coefficients.

The exchange of momentum will be examined first. Consider a flow with a velocity gradient in the z direction of magnitude $\partial u/\partial z$ in which particles are moving across the field A in the xy plane as shown in Fig. 1.9.

The exchange of momentum as the particles pass through the field represents the impulse, i.e.,

$$\text{Impulse} = \Delta\text{Momentum}$$

$$F\Delta t = M(U_b - U_a) \tag{1.37}$$

$$\tau L^2 \frac{L}{\bar{v}} = \tfrac{1}{6}Nm\left[\left(U_0 + \lambda\frac{\partial u}{\partial z}\right) - \left(U_0 - \lambda\frac{\partial u}{\partial z}\right)\right]$$

or

$$\tau = \frac{\lambda}{3}\frac{Nm}{L^3}\bar{v}\frac{\partial u}{\partial z} = \frac{\rho\bar{v}\lambda}{3}\frac{\partial u}{\partial z} \equiv \mu\frac{\partial u}{\partial z} \tag{1.38}$$

Table 1.5 Transport Properties

Conservation Relationship	Phenomenon	Gradient	Equation	Result
Mass (of species)	Diffusion D	Concentration $\frac{\partial C_i}{\partial z}$	$\dot{m}_i = \rho D_{ij}\frac{\partial C_i}{\partial z}$	Mass flux $\dot{m}_i$
Momentum	Viscosity μ	Velocity $\frac{\partial u}{\partial z}$	$\tau = \mu\frac{\partial u}{\partial z}$	Shear stress τ
Energy	Thermal conductivity k	Temperature $\frac{\partial T}{\partial z}$	$\dot{q} = k\frac{\partial T}{\partial z}$	Heat transfer $\dot{q}$

1.4 DIMENSIONAL ANALYSIS

By knowing only the units of the parameters involved in a physical process, useful information can be deduced by forming grouping of these parameters as *dimensionless* ratios. This technique is called "dimensional analysis" and is a very powerful engineering *tool*. It is an important part of the judgment process necessary to make engineering decisions and help eliminate errors.

Following is a list of such dimensionless ratios, important in fluid mechanics, which bear the names of early investigators:

$$M = V/a = \text{Mach number} \tag{1.46}$$

$$Re = \rho VL/\mu = \text{Reynolds number} \tag{1.47}$$

$$Pr = \mu C_p/K = \text{Prandtl number} \tag{1.48}$$

$$Le = \rho DC_p/K = \text{Lewis number} \tag{1.49}$$

$$Nu = h^*L/K = \text{Nusselt number} \tag{1.50}$$

$$Kn = \lambda/L = \text{Knudsen number} \tag{1.51}$$

where h^* is the heat-transfer coefficient.

The ratios of different forms of energy in fluid flows will be summarized next. This provides physical interpretation of the dimensionless numbers.

$$\frac{\text{Ordered energy}}{\text{Random energy}} = \frac{\frac{1}{2}mu^2}{\frac{1}{2}m\bar{v}^2} = \frac{u^2}{3RT} = \frac{\gamma}{3}\frac{u^2}{a^2} = \frac{\gamma}{3}M^2 \tag{1.52}$$

$$\frac{\text{Inertial force}}{\text{Viscous force}} = \frac{\dot{m}v}{\tau A} = \frac{\rho AV^2}{\mu\frac{V}{L}A} = \frac{\rho VL}{\mu} Re \tag{1.53}$$

$$\frac{\text{Viscous dissipation energy}}{\text{Energy conducted}} = \frac{\tau AV}{\dot{q}A} = \frac{\mu V^2/L}{kT/L} = \frac{\mu C_p}{k}\frac{V^2}{C_pT} = Pr\frac{M^2}{\gamma - 1} \tag{1.54}$$

$$\frac{\text{Heat diffused}}{\text{Heat conducted}} = \frac{\dot{m}iH}{\dot{q}A} = \frac{\rho D\frac{\partial C_i}{\partial z}AH}{k\frac{\partial T}{\partial z}A} = \frac{\rho DH}{kT}$$

$$= \frac{\rho DC_p}{k}\left(\frac{H}{C_pT}\right) = Le\left(\frac{H}{C_pT}\right) \tag{1.55}$$

$$\frac{\text{Heat convected}}{\text{Heat conducted}} = \frac{h^*T}{kT/L} = \frac{h^*L}{k} = Nu \tag{1.56}$$

$$\frac{\text{No. of collisions with body}}{\text{No. of collisions with another molecule}} = \frac{f_L}{f_\lambda} = \frac{\bar{v}/L}{\bar{v}/\lambda} = \frac{\lambda}{L} = Kn \tag{1.57}$$

1.5 FREE MOLECULE FLOW HEATING

For free molecule flow (FM) $K_n \gg 1$. For this case,[6] a collision between an air molecule and a surface will be examined (see Fig. 1.12). The energy balance for the exchange during the collision is given by

$$Q = E_i - E_r \equiv \alpha(E_i - E_w) \tag{1.58}$$

where

$$\alpha = \frac{E_i - E_r}{E_i - E_w} = \text{thermal accommodation coefficient} \tag{1.59}$$

This coefficient can be determined experimentally. With reference to Fig. 1.13, if $E_r \approx E_w$, then $\alpha \approx 1$. Based upon this value for α the rate of heat transfer can be computed, i.e.,

$$\dot{Q} = \alpha(\dot{E}_i - \dot{E}_w) = \alpha(\dot{m}H_s - \dot{m}H_w) \tag{1.60}$$

$$\dot{q}_A = \alpha\rho AV(H_s - H_w) \tag{1.61}$$

$$\dot{q}_{\text{FM}} = \alpha\rho V(H_s - H_w) \equiv St\rho V(H_s - H_w) \tag{1.62}$$

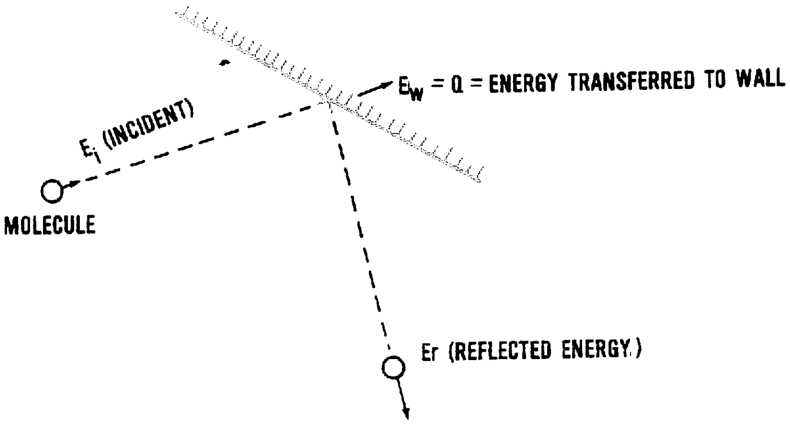

Fig. 1.12 Collision between air molecules and a surface.

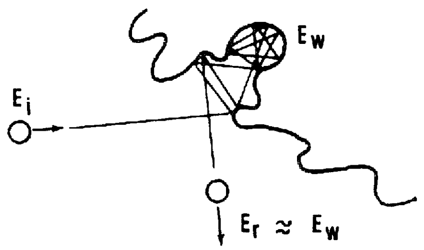

Fig. 1.13 Energy transition between a molecule and the surface.

Hence,

$$St = \alpha \approx 1 \tag{1.63}$$

The transition between free molecule and continuum heating is, for flight cases where $K_n \gg 1$,

$$\dot{q} = \dot{q}_{\mathrm{FM}} \tag{1.64}$$

and, for flight cases where $K_n \ll 1$,

$$\dot{q} = \dot{q}_{\mathrm{continuum}} \tag{1.65}$$

Little is known about the transition from one regime to the other where $K_n \approx 1$. As will be explained later, little information is needed, fortunately, in this region so that an ad hoc interpolation may be used as follows:

$$\dot{q}_{\mathrm{transitional}} = \frac{\dot{q}_{\mathrm{continuum}} + Kn\dot{q}_{\mathrm{FM}}}{1 + Kn} \tag{1.66}$$

This simple relationship insures that both limits are satisfactorily obtained and permits a continuous calculation of heating throughout the transition region.

1.6 SLIP FLOW REGIME

Up to this point, only free molecule flow and the transition region into the continuum regime have been discussed. There is yet another small regime within the transition region called the "slip flow" regime. This region has not been found to be very important in re-entry engineering; however, for completeness, it should be identified.

At the wall, a boundary layer forms with a gradient of magnitude

$$\frac{\partial u}{\partial z} \cong \frac{U_\infty}{\delta} \tag{1.67}$$

where U_∞ is the freestream velocity at infinity and δ an effective height. This gradient can be examined in a manner similar to the free molecular flow analysis for evaluating viscosity.

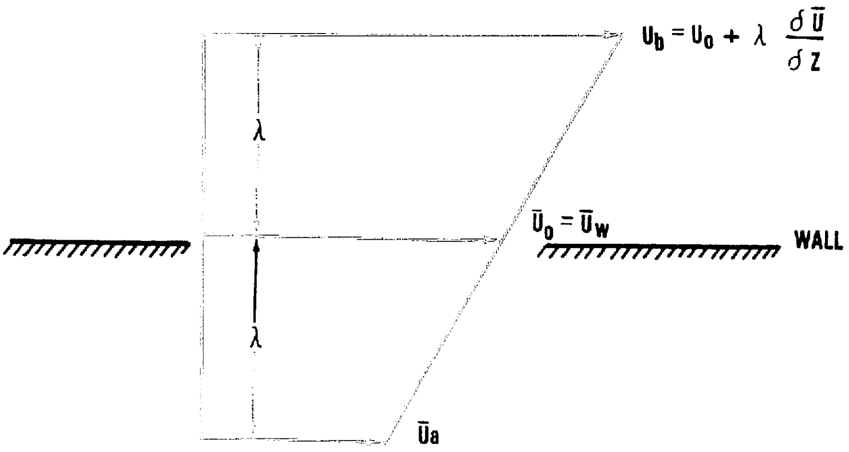

Fig. 1.14 Velocity gradient located a mean free path above and below U_0.

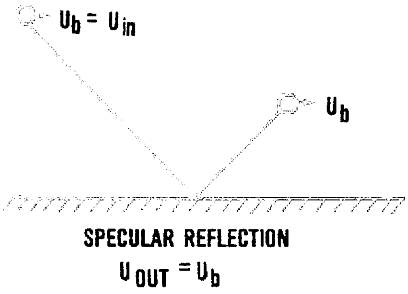

Fig. 1.15 Specular reflection at the wall.

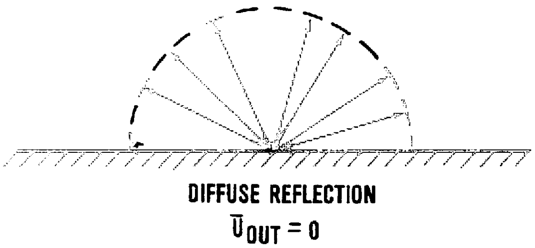

Fig. 1.16 Diffuse reflection occurring after the molecule strikes the wall.

Consider the wall at the location of the velocity U_0 as shown in Fig. 1.14. A molecule coming from a mean free path distance above the wall will have a velocity of U_b since no collisions occur until it strikes the wall. At the wall, two extremes in the reflection velocity are possible, i.e., specular or diffuse, as shown in Figs. 1.15 and 1.16.

Experiments indicate diffuse reflection generally occurs consistent with a "perfectly rough" wall hypothesis. The velocity value at the wall is, therefore, the average of the incoming and outgoing velocities, i.e.,

$$\overline{U}_w = \frac{U_{\text{in}} + U_{\text{out}}}{2} = \frac{\overline{U}_w + \lambda(\partial u/\partial z) + 0}{2} \tag{1.68}$$

Hence

$$\overline{U}_w = \lambda\frac{\partial u}{\partial z} \approx \lambda\frac{U_e}{\delta}$$

or

$$\frac{\overline{U}_w}{U_e} = \frac{\lambda}{\delta} = Kn \tag{1.69}$$

Thus, it is apparent that there is always some slip at the wall, even for the continuum regime. However, for $Kn \ll 1$, we conveniently set $\underline{U}_w = 0$ for boundary-layer theory. For $0.01 < Kn < 1$, a "slip" velocity of $\overline{U}_w/U_\infty = \lambda/\delta$ can be applied as a correction to the boundary-layer method. The domain where this is appropriate is entitled the *slip flow regime*.

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Problems

1.1 Solve $P(h)$ for an adiabatic atmosphere, i.e., $p \sim \rho^\gamma$. What is the temperature "lapse" rate, dT/dh, for this case?

1.2 Compute the radiation equilibrium heat-transfer rates for the materials in Table 1.1. Use an emittance of 0.8.

1.3 Compute the heat-transfer rate in free molecule flow to a flat plate traveling at orbital speed between altitudes from 250,000–400,000 ft.