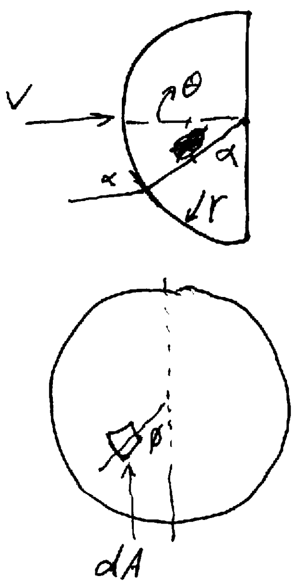
Newtonian Flow (continued)



Drag on a hemisphere

$$dA = r^2 \sin\theta \, d\theta \, d\phi \qquad \alpha = \theta + \frac{\pi}{2}$$

$$\text{Force on } dA = \rho V^2 \sin^2\alpha \, dA$$

$$= \rho V^2 \underbrace{\sin^2\left(\theta + \frac{\pi}{2}\right)}_{\cos^2\theta} r^2 \sin\theta \, d\theta \, d\phi$$

$$= \rho V^2 r^2 \cos^2\theta \sin\theta \, d\theta \, d\phi$$

$$\text{Drag on } dA = (\text{Force on } dA)\sin\alpha = (\text{Force on } dA)\cos\theta$$

$$\text{Drag} = \rho V^2 r^2 \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos^3\theta \sin\theta \, d\theta \, d\phi$$

$$= 2\pi \rho V^2 r^2 \int_0^{\frac{\pi}{2}} \cos^3\theta \sin\theta \, d\theta$$

$$= 2\pi \rho V^2 r^2 \left[\frac{\cos^4\theta}{4} \right]_{\frac{\pi}{2}}^{0}$$

note reversal of bounds to eliminate (-1) term

$$D = 2\pi \rho V^2 r^2 \left(\frac{1}{4}\right)$$

$$= \frac{1}{2} \underbrace{\pi r^2}_{=A} \rho V^2 = \frac{1}{2} \rho V^2 A \, C_D \Rightarrow \boxed{C_D = 1}$$

Can solve this again for lift, but symmetry gives $C_L = 0$

Modified Newtonian Flow

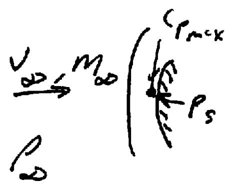
~~Date~~ 3/2/04

1

Newtonian Flow explains behavior after oblique shocks — but most entry vehicles are blunt! → normal (detached shocks) — how do we model that?

Newtonian Flow: $C_p = 2\sin^2\alpha$

Modified Newtonian Flow: $C_p = C_{p_{max}} \sin^2\alpha$



$C_{p_{max}}$ = pressure coefficient behind normal shock

$$C_{p_{max}} = \frac{p_s - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$$

From more ideal gas theory and algebra than I care to go through,

$$\boxed{C_{p_{max}} = \frac{2}{\gamma M^2}\left\{\left[\frac{(\gamma+1)^2 M^2}{4\gamma M^2 - 2(\gamma-1)}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma M^2}{\gamma+1}\right] - 1\right\}}$$

We can rewrite this as

$$C_{p_{max}} = \left[\frac{(\gamma+1)^2}{4\gamma - \frac{2(\gamma-1)}{M^2}}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{2(1-\gamma)}{\gamma(\gamma+1)M^2} + \frac{4}{\gamma+1}\right] - \frac{2}{\gamma M^2}$$

Now let $M \to \infty$

$$C_{p_{max}}\Big|_{\lim M\to\infty} = \left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{\gamma}{\gamma-1}}\left(\frac{4}{\gamma+1}\right)$$

$= 1.839$ for $\gamma = 1.4$

$= 2.0$ for $\gamma = 1.0$

2/18/93

From Anderson, J.D. "Hypersonic and High Temperature Gas Dynamics"

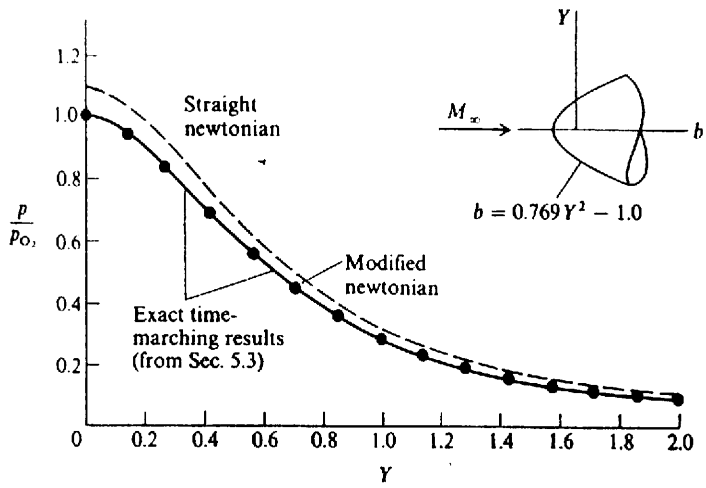


FIGURE 3.8
Surface pressure distribution over a paraboloid at $M_\infty = 8.0$; p_{O_2} is the total pressure behind a normal shock wave at $M_\infty = 8.0$.