HEATING RATES

DEFINITIONS

Prandtl Number

$$\boxed{P_r = \mu \frac{C_p}{K}}$$

where
- C_p = specific heat @ constant pressure
- K = thermal conductivity
- μ = viscosity

$$P_r \propto \frac{\text{frictional dissipation}}{\text{thermal conduction}}$$

$P_r \approx 0.715$ for air at standard conditions

Sutherland's Law (empirical)

viscosity depends on temp

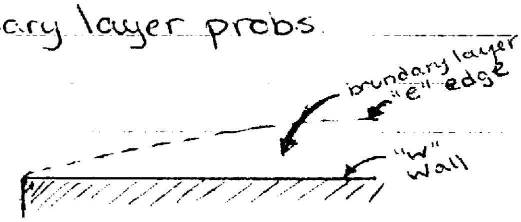
$$\boxed{\frac{\mu}{\mu_{ref}} = \left(\frac{T}{T_{ref}}\right)^{3/2} \frac{T_{ref}+S}{T+S}}$$

for air:
- $\mu_{ref} = 1.789 \times 10^{-5}$ Kg/m·s
- $T_{ref} = 288$ K
- $S = 110$ K
- good up to several thousand degrees

Stanton Number - applies to boundary layer probs

"e" means edge of boundary layer
"w" means wall

$$\boxed{S_T = \frac{\dot{q}_w}{\rho_e V_e (H_{aw} - H_w)}}$$



H = enthalpy
$= C_p T$ — for perfect gas

H_w = enthalpy @ the wall

H_{aw} = enthalpy @ the wall if it were adiabatic

$\rightarrow$ for H_{aw}, $\left.\frac{\partial T}{\partial z}\right|_w = 0$

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Approximating H_{aw}

$$H_{aw} = H_e + r\frac{u_e^2}{2}$$, $r \equiv$ recovery factor

$$H_o = H_e + \frac{u_e^2}{2}$$ ← TOTAL ENTHALPY at edge of boundary layer

so... $$H_{aw} = H_e + r(H_o - H_e)$$

for incompressible flow, $r \approx \sqrt{P_r}$
$= 0.845$ for std. air

r decreases only 2.4% from $M=0$ to $M=16$
→ fairly constant!

Reynold's Analogy - pg 81 in Hankey

$$\frac{S_T}{C_f} = \frac{1}{2} P_r^{-2/3}$$, $C_f \equiv$ skin friction coeff.

since $P_r \approx 1$, $\boxed{S_T \approx \frac{C_f}{2}}$ this is the Reynold's Analogy

Empirical Correlation for C_f:

$$\boxed{\frac{C_f}{2} = \frac{A}{Re^n}}$$

	A	n
LAMINAR FLOW	0.332	0.5
TURBULENT FLOW	0.0296	0.2

transition btwn laminar & turbulent @ ~ $Re = 10^6$

$$\rightarrow \boxed{S_T \approx \frac{A}{Re^n} P_r^{-2/3}}$$

STANTON NUMBER

FRACTION OF HEAT INTO WAKE

$$\dot{q}_A = \rho A V H_s \quad \text{(total enthalpy)}$$

thermal energy flow rate in free stream

$$\dot{q}_w = \dot{q}_B = S_T \rho A V (H_{aw} - H_w)$$

$$= \left[k \frac{\partial T}{\partial y}\right]_{wall}$$

amount absorbed by vehicle (EQN 4.3, pg 81)

usually, $H_{aw} >> H_w$

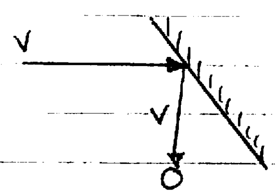
$H_{aw} \approx H_s$

$$\dot{q}_B = S_T \rho A V H_s$$

$$\frac{\dot{q}_B}{\dot{q}_A} = S_T \approx 10^{-3}$$

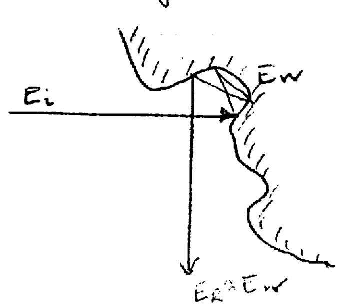
→ 0.1% of heating goes into vehicle!
most of the energy goes into the atmosphere

FREE MOLECULAR FLOW



at equilibrium
= no heating

on a larger scale...



more accurately
(rough surface, molecule bounces a few times before leaving vehicle)

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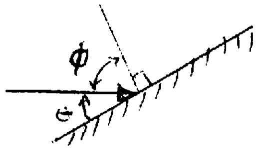
Newtonian Flow: $C_p = 2\sin^2\theta = 2\cos^2\phi$



$\phi \equiv$ complement of θ

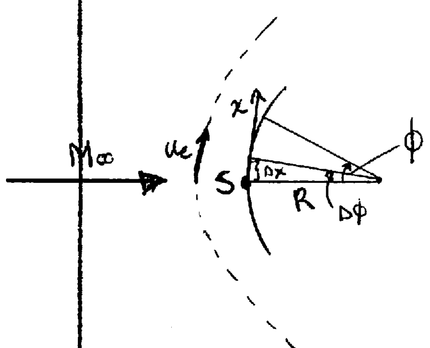
$$\frac{P_e - P_\infty}{q_\infty} = 2\cos^2\phi$$

$$P_e = 2q_\infty\cos^2\phi + P_\infty$$

$$\frac{dP_e}{dx} = -4q_\infty\cos\phi\sin\phi\,\frac{d\phi}{dx}$$

$$\boxed{\frac{du_e}{dx} = \frac{4q_\infty}{\rho_e u_e}\cos\phi\sin\phi\,\frac{d\phi}{dx}}$$ everywhere on the body

<u>Now Evaluate Heating Rate at STAGNATION POINT, S</u>



$\approx$ incompressible flow around S

$$u_e = \left(\frac{du_e}{dx}\right)_s \Delta x$$

at S: $\cos\Delta\phi \approx 1$

$$\sin\Delta\phi \approx \Delta\phi = \frac{\Delta x}{R}$$

$$\frac{\Delta\phi}{\Delta x} = \frac{1}{R}$$

$$C_p = 2\cos^2\Delta\phi \approx 2$$

$$q_\infty = \frac{1}{2}(P_e - P_\infty)$$

$$\left(\frac{du_e}{dx}\right)^2 = \frac{2(P_e - P_\infty)}{\rho_e\,\Delta x}\left(\frac{\Delta x}{R}\right)\left(\frac{1}{R}\right) \Rightarrow \boxed{\frac{du_e}{dx} = \frac{1}{R}\sqrt{\frac{2(P_e-P_\infty)}{\rho_e}}}$$

$\Rightarrow \boxed{\dot{q}_w \propto \frac{1}{\sqrt{R}}}$, BLUNT BODIES STAY COOLER THAN SHARP ONES

Got to here 2/24/98

APPROXIMATIONS FOR HEATING RATES

(these will get you within 10% or so)

Simple Set of Equations for HYPERSONIC CONTINUUM FLOW

$$\boxed{\dot{q}_w = \rho_\infty^{\ N} V_\infty^{\ M} C}$$

q_w in W/cm^2
ρ_∞ in kg/m^3
V_∞ in m/s
R in m

Stagnation Point:
$M = 3$
$N = 0.5$
$C = 1.83 \times 10^{-8} R^{-1/2} \left(1 - \frac{H_w}{H_o}\right)$

Laminar Flow on Flat Plate:
$M = 3.2$
$N = 0.5$
$C = 2.53 \times 10^{-9} (\cos\phi)^{1/2} (\sin\phi)\, x^{-1/2} \left(1 - \frac{H_w}{H_o}\right)$

where x = distance along surface in m

Turbulent Flow on Flat Plate:

CASE I: $V_\infty \leq 3962$ m/s
$M = 3.37$
$N = 0.8$
$C = 3.9 \times 10^{-8} (\cos\phi)^{1.78} (\sin\phi)^{1.6}\, x_T^{-1/5} \left(\frac{T_w}{556}\right)^{-1/4} \left(1 - 1.1\frac{H_w}{H_o}\right)$

CASE II: $V_\infty > 3962$ m/s
$M = 3.7$
$N = 0.8$
$C = 3.2 \times 10^{-9} (\cos\phi)^{2.08} (\sin\phi)^{1.6}\, x_T^{-1/5} \left(1 - 1.1\frac{H_w}{H_o}\right)$

where x_T = distance measured in the turbulent part of the boundary layer