# Re-Entry Aerodynamics

**Wilbur L. Hankey**
Wright State University
Dayton, Ohio

**AIAA EDUCATION SERIES**
J. S. Przemieniecki
Series Editor-in-Chief
Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

Published by
American Institute of Aeronautics and Astronautics Inc.,
370 L'Enfant Promenade SW, Washington, DC 20024

# 4 RE-ENTRY HEATING

## 4.1 SIMPLIFIED ANALYSIS

In any engineering problem, one requires an approximate method for obtaining rapid estimates.[1] These estimates are used to evaluate the seriousness of the problem and identify the most critical condition. If the situation is warranted, more sophisticated methods can be employed and possibly followed up by model tests to confirm the analysis. In this section, approximate heating methods are developed. In later sections, more exact methods will be derived.

### *Reynolds' Analogy*

The most powerful method for obtaining simple heating estimates is to use Reynolds' analogy. This theory takes advantage of the similarity between shear stress and the heating rate. The analogy is exact for only one flow condition (i.e., zero pressure gradient, $Pr = 1$); however, useful trends can be obtained for other cases. Also, constants can be adjusted to improve the accuracy by comparison with experiment.

The equation for shear stress is

$$\tau_w = \left(\mu \frac{\partial u}{\partial y}\right)_w \equiv \frac{1}{2}\rho_e V_e^2 C_f \tag{4.1}$$

where

$$C_f = C_f(Re) \tag{4.2}$$

is the friction coefficient depending on the Reynolds number $Re$. The heating rate has a similar equation representing the transport of energy as opposed to the transport of momentum in the shear stress equation.

$$\dot{q}_w = \left[k \frac{\partial T}{\partial y}\right]_w \equiv St \rho_e V_e (H_{aw} - H_w) \tag{4.3}$$

Dividing Eq. (4.1) by Eq. (4.3) results in

$$\frac{\tau_w}{\dot{q}_w} = \frac{\frac{1}{2}\rho_e V_e^2 C_f}{St\rho_e V_e(H_{aw} - H_w)} = \frac{\mu\dfrac{\partial u}{\partial y}}{k\dfrac{\partial T}{\partial y}} \cong \frac{\mu\dfrac{V_e}{\delta}}{\dfrac{k}{C_p}\left[\dfrac{H_{aw} - H_w}{\delta}\right]} \tag{4.4}$$

Hence,

$$\frac{C_f}{2} = \frac{\mu C_p}{k} St \tag{4.5}$$

Since the Prandtl number $Pr$ is approximately equal to unity, i.e.,

$$Pr = \mu C_p/k \approx 1 \tag{4.6}$$

then

$$St = C_f/2 \tag{4.7}$$

which is referred to as the Reynolds analogy.

Much information is available on skin friction that can be exploited to obtain heating rate estimates. See Fig. 4.1. Empirical correlations for skin-friction coefficients have been developed over the past century.

$$C_f/2 = A/Re^n \tag{4.8}$$

where, for laminar flow, $A = 0.332$ and $n = 0.5$; for turbulent flow, $A = 0.0296$ and $n = 0.2$. Transition occurs at approximately $Re = 10^6$; however, if the design can permit the conservatism, one uses the higher of the two values to obtain the heating rate estimate.

Inserting Eq. (4.8) into the heat-transfer equation produces

$$\dot{q} = St\rho_e V_e(H_{aw} - H_w) = \frac{A}{[\rho_e V_e x/\mu_e]^n}[\rho_e V_{ex}/_{\mu e}]^n \rho_e V_e(H_{aw} - H_w) \tag{4.9}$$

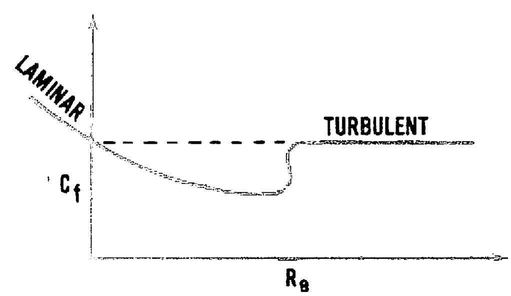


**Fig. 4.1 Heating rate at the stagnation region.**

or

$$\dot{q} = A(\rho_e V_e)^{1-n}[\mu_e/x]^n (H_{aw} - H_w) \tag{4.10}$$

### *Stagnation Region*

The heating rate at the stagnation region[2] will be explored first, since this is known to be critical. Hypersonic approximations will be used to obtain values for $\rho_e V_e$, $\mu_e$, and $H_e$. The stagnation point is shown in Fig. 4.2. The flow crosses a normal shock wave and stagnates. The stagnation conditions can be obtained from Newtonian impact theory,

$$P_s \cong \rho_\infty V_\infty^2 \tag{4.11}$$

$$H_s \cong \frac{V_\infty^2}{2} \tag{4.12}$$

$$\rho_s \cong \frac{\gamma P_s}{(\gamma - 1) H_s} = \frac{2\gamma}{\gamma - 1}\rho_\infty \tag{4.13}$$

The flow model used away from the stagnation point assumes that the streamline external to the boundary layer is isentropic. This is certainly true for any streamline external to the viscous region. Although large entropy gradients occur normal to the streamlines (since the flow passes through a curved shock wave), entropy is constant along streamlines in inviscid steady flow.

The following isentropic relations are used:

$$\rho_e/\rho_s = (P_e/P_s)^{1/\gamma} \tag{4.14}$$

$$h_e/H_s = (P_e/P_s)^{(\gamma-1)/\gamma} \tag{4.15}$$

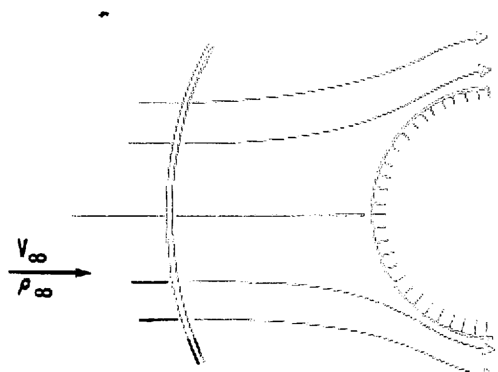


**Fig. 4.2 Skin friction vs Reynolds number in regions of laminar and turbulent flows.**

$$V_e^2/2 = H_s[1 - (h_e/H_s)] \tag{4.16}$$

or

$$V_e/V_\infty = \left[1 - (P_e/P_s)^{(\gamma-1)/\gamma}\right]^{\frac{1}{2}} \tag{4.17}$$

All properties along the streamline can be related to the pressure ratio and stagnation property values. This is analogous to the flow through a wind tunnel in which all local values can be related to *one* local quantity plus the stagnation section values.

A relationship called Sutherland's law is required for viscosity,

$$\mu = \mu_0 T^{3/2}/T + S \tag{4.18}$$

where $S = 198.6°\text{R} \ll T_s$ and $\mu_0 = 2.27 \times 10^{-8}$ lb · s/ft$^2\sqrt{°R}$. Hypersonically,

$$\mu \approx \mu_0 T^{\frac{1}{2}} = \left(\mu_0/C_p^{\frac{1}{2}}\right) h^{\frac{1}{2}} \equiv C_\mu h^{\frac{1}{2}} \tag{4.19}$$

where $C_\mu = \mu_0/\sqrt{C_p}$.

We can now insert these values into the heating rate equation and find a relationship for the stagnation point heat transfer. In addition, we assume $H_{aw} = H_s \gg H_w$. Hence,

$$\rho_e = \left(\frac{2\gamma}{\gamma - 1}\rho_\infty\right)\left(\frac{P_e}{P_s}\right)^{1/\gamma} \tag{4.20}$$

$$V_e = V_\infty\left[1 - \left(\frac{P_e}{P_s}\right)^{(\gamma-1)/\gamma}\right]^{\frac{1}{2}} \tag{4.21}$$

$$\mu e = \frac{C\mu}{\sqrt{2}} V_\infty\left(\frac{P_e}{P_s}\right)^{(\gamma-1)/2\gamma} \tag{4.22}$$

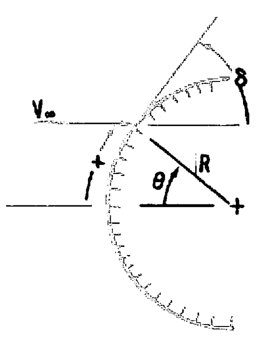


**Fig. 4.3 Circular cross section showing stagnation point heat transfer.**

$$H_{aw} - H_w = V_\infty^2/2 \tag{4.23}$$

$$\dot{q} = K \frac{\rho_\infty^{1-n} V_\infty^3}{R^n} F(x) \tag{4.24}$$

where

$$K = \frac{A[2\gamma/(\gamma-1)]^{1-n} C\mu^n}{J 2^{1+n/2}} \tag{4.25}$$

$$F(x) = \left(\frac{R}{x}\right)^n \left(\frac{P_e}{P_s}\right)^{(2-3n+n\gamma)/2\gamma} \left[1 - \left(\frac{P_e}{P_s}\right)^{(\gamma-1)/\gamma}\right]^{(1-n)/2} \tag{4.26}$$

$R$ = nose radius, ft

$\rho_\infty$ = density, slug/ft$^3$

$V_\infty$ = velocity, ft/s

$J$ = 778 ft · lb/Btu

$\dot{q}$ = Btu/ft$^2$ · s

Given the pressure distribution, the heat transfer in the stagnation region can now be determined. A circular cross section of the region is shown in Fig. 4.3. The following Newtonian flow relationships can be used to obtain the pressure:

$$\delta + \theta = 90 \text{ deg} \tag{4.27}$$

$$C_p = 2\sin^2\delta \tag{4.28}$$

Hence,

$$C_p = 2\cos^2\theta \tag{4.29}$$

or

$$P_e/P_s = \cos^2\theta \tag{4.30}$$

$$x/R = \theta \tag{4.31}$$

Therefore,

$$F(x) = (\cos\theta)^{(2-3n+n\gamma)/\gamma} \frac{[1-\cos^{2(\gamma-1)/\gamma}\theta]^{(1-n)/2}}{\theta^n} \tag{4.32}$$

This distribution, along with the simple relationships derived previously, can provide excellent simple prediction methods (note that $K$ has been modified based upon experimental data), such as

$$\dot{q}_{\text{max,lam}} = 21\sqrt{\frac{\rho_\infty}{R}}\left(\frac{V_\infty}{1000}\right)^3\left(1 - \frac{H_w}{H_s}\right) \tag{4.33}$$

$$\dot{q}_{\text{max,turb}} = \frac{4}{x^{0.2}}\left(\frac{\rho_\infty}{\rho_{sl}}\right)^{0.8}\left(\frac{V_\infty}{1000}\right)^3\left(1 - \frac{H_w}{H_s}\right) \tag{4.34}$$

where $\dot{q}$ is in units of Btu/ft$^2$s, $\rho$ of slug/ft$^3$, $R$ and $x$ of ft, and $V_\infty$ of ft/s.

## 4.2 COMPONENT HEATING

### *Wing Leading-Edge Heating*

After the nose stagnation region has been analyzed, the next most critical condition to be considered is the wing leading edge. The stagnation point heating rate on a cylinder normal to the flow may be related to the heating of a sphere by a coordinate transformation. As will be shown in a later section, this turns out to result in a very simple relationship,

$$\dot{q}_{\text{cylinder}} = \dot{q}_{\text{sphere}}/(2)^n \tag{4.35}$$

The wing leading edge is seldom normal to the flow, however, and geometric considerations for wing sweep and angle of attack must be applied.

It is well known that sweeping the leading edge of the wing of a hypervelocity vehicle will reduce the convective heat input to the wing. The correction for sweep is usually obtained by relating the heating for a swept leading edge to an unswept one. Recall that stagnation heating was derived to be

$$\dot{q} = A(\rho_s u_s)^{1-n}\left(\frac{\mu_s}{x}\right)^n (H_e - H_w) \tag{4.36}$$

Performing a ratio of the heating between a swept and a unswept cylinder produces

$$\frac{\dot{q}(\Lambda)}{\dot{q}(\Lambda = 0)} = \left[\frac{\rho_\Lambda u_\Lambda}{\rho_{\text{ref}} u_{\text{ref}}}\right]^{1-n}\left[\frac{\mu_\Lambda}{\mu_{\text{ref}}}\right]^n \tag{4.37}$$

The stagnation line conditions on a swept cylinder are (see Fig. 4.4),

$$P_s = \rho_\infty V_n^2 = \rho_\infty V^2 \cos^2\Lambda_e \tag{4.38}$$