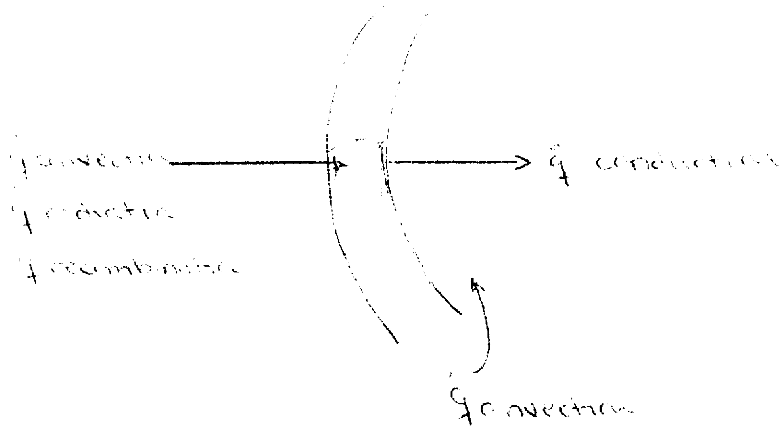
MAE 138T 3/11/93

__Vehicle.__

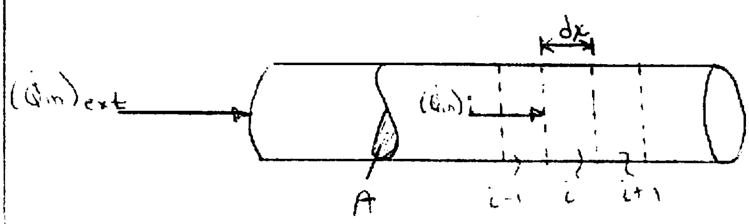


$\dot{Q}_{in}$ goes into the thermal mass of the system

$$\dot{Q}_{in} = \oiint_A \dot{q}\, dA = \frac{d}{dt}\iiint_{Vol} \rho C_v T \, dV$$

ρ = density
C_v = specific heat
T = temperature

__1-D analysis - Thermal Conduction in a Rod__



$$\dot{q} = \underbrace{K}_{\text{thermal conductivity}} \underbrace{\nabla T}_{\text{gradient of Temp}} \xRightarrow{\text{1-D conduction}} \dot{q} = K\frac{dT}{dx}$$

$$(Q_{in})_i = \left(AK\frac{dT}{dx}\right)_{i-1} + \left(AK\frac{dT}{dx}\right)_{i+1} = \left(\rho C_v V \frac{dT}{dt}\right)_i$$

$V = A\Delta x$

AT TIME STEP n

$$AK\,\frac{T_{i-1}^{n} - T_i^{n}}{\Delta x} + AK\,\frac{T_{i+1}^{n} - T_i^{n}}{\Delta x} = \rho C_v A \Delta x \,\frac{T_i^{n+1} - T_i^{n}}{\Delta t}$$

where q's are positive into the differential element

With Algebra: $\boxed{T_i^{n+1} = T_i^n + d\left(T_{i+1}^n - 2T_i^n + T_{i-1}^n\right)}$

where $d = \dfrac{\alpha \Delta t}{\Delta x^2}$ ≡ numerical diffusion number

$\alpha = \dfrac{K}{\rho C_v}$ ≡ thermal diffusivity
high → conduct
low → store heat internally

FINITE DIFFERENCE STABILITY

Von Neumann Stability Analysis (small perturbations)

$\hat{T}(x,t) = \hat{T}(t)e^{i\beta x}$: fourier series expansion for small perturbations as a function of position

$$\hat{T}^{n+1} e^{i\beta x} = \hat{T}^n e^{i\beta x} + d\left(\hat{T}^n e^{i\beta(x+\Delta x)} - 2\hat{T}^n e^{i\beta x} + \hat{T}^n e^{i\beta(x-\Delta x)}\right)$$

$$\frac{\hat{T}^{n+1}}{\hat{T}^n} = 1 + d\left(e^{i\beta\Delta x} - 2 - e^{-i\beta\Delta x}\right) \equiv G$$

why is this?

G is like a GAIN on perturbation.
Want $G < 1$, so perturbation dies out.

[illegible]

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$$G = 1 + d\left(\cos\beta\Delta x + i\sin\beta\Delta x - 2 + \cos(-\beta\Delta x) + i\sin(-\beta\Delta x)\right)$$

$$= 1 + d\left(2\cos\beta\Delta x - 2\right)$$

$$\boxed{G = 1 - 2d\left(1 - \cos\beta\Delta x\right)}$$

for stability, $|G| < 1$, for any β
(for any phase angle $\beta\Delta x$)

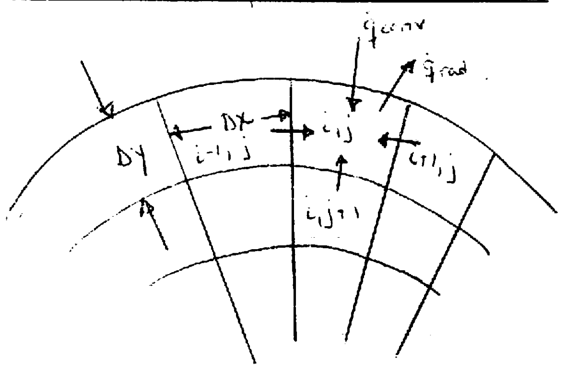
worst case $\rightarrow$ $\cos\beta\Delta x = -1$

$$|1 - 4d| < 1 \Rightarrow \text{need } d < \tfrac{1}{2}$$

$$d = \frac{\alpha\Delta t}{\Delta x^2} \Rightarrow \boxed{\text{need } \Delta t < \frac{\Delta x^2}{2\alpha}}$$

finished here 3/3/98

More Complicated Case:



$$(\dot{Q}_{in})_{ij} = \sum \dot{q}A = \rho C_v V \frac{dT}{dt}$$

$$(\dot{Q}_{in})_{ij} = \Delta x\,\dot{q}_{conv} - \Delta x\,\dot{q}_{rad} + \Delta y\,K\left(\frac{T^n_{i+1,j} - T^n_{i,j}}{\Delta x}\right) + \Delta y\,K\left(\frac{T^n_{i-1,j} - T^n_{i,j}}{\Delta x}\right)$$
$$+ \Delta x\,K\left(\frac{T^n_{i,j+1} - T^n_{i,j}}{\Delta y}\right)$$

$$(\dot{Q}_{in})_{ij} = \rho C_v\,\Delta x\,\Delta y\left(\frac{T_{ij}^{n+1} - T^n_{ij}}{\Delta t}\right)$$

$$T^{n+1}_{ij} = T^n_{ij} + \frac{\alpha\Delta t}{\Delta x^2}\left(T^n_{i+1,j} - 2T^n_{ij} + T^n_{i-1,j}\right) + \frac{\alpha\Delta t}{\Delta y^2}\left(T^n_{i,j+1} - T^n_{ij}\right)$$

$$= \text{[illegible]}\,\frac{\text{[illegible]}}{\Delta y K}\left(T^n_{i,j+1} - T^n_{ij}\right) + \frac{\text{[illegible]}}{\Delta x K}\left(T_B^n - T^n_{ij}\right)$$