Reliability, Redundancy, and Resiliency

- Review of probability theory
- Component reliability
- Confidence
- Redundancy
- Reliability diagrams
- Intercorrelated Failures
- System resiliency
- Resiliency in fixed fleets

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Review of Probability

- Probability that *A* occurs

$$0 \le P(A) \le 1$$

- Probability that *A* does not occur

$$P(\overline{A})$$

- Sum of all probable outcomes

$$P(A) + P(\overline{A}) = 1$$

Review of Probability

- Probability of both A and B occurring

$$P(A) \cap P(B) = P(A)P(B)$$

- Probability of either A or B occurring

$$P(A) \cup P(B) = 1 - P(\overline{A})P(\overline{B})$$

$$= 1 - [1 - P(A)][1 - P(B)]$$

$$= P(A) + P(B) - P(A)P(B)$$

Expected Value Theory

- Probability of an outcome does not determine value of the outcome
- Combine probabilities and values to determine expected value of outcome

$$EV = P(A)U(A) + P(\overline{A})U(\overline{A})$$

Expected Value Example

- Maryland State Lottery - pick six numbers out of 49 (any order)

$$P(win) = 1 \Big/ \frac{49!}{6!43!} = 1/13,983,816$$

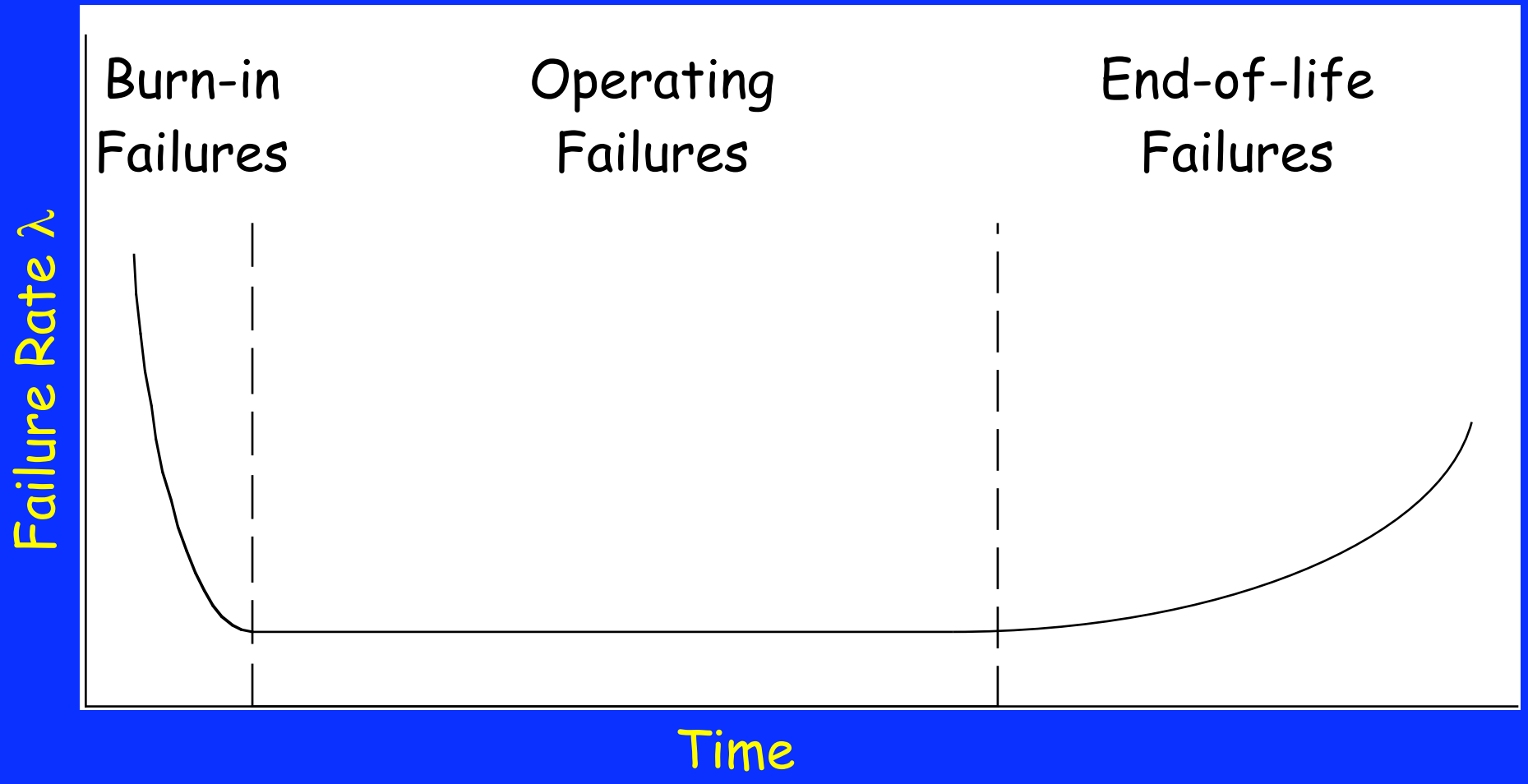
- Assume $10,000,000 jackpot

$$EV = (7.151 \times 10^{-8})(10^{7}) + (1)(-1) = -\$0.39$$

Utility Theory

- Numerical rating from expected value calculations does not fully quantify utility
- Lottery example previously: utility of (highly unlikely) win exceeds negative utility of small investment: *risk proverse*
- Imagine lottery where $1000 buys 1:500 chance at $1M - EV=(.998)(-$1000)+(.002)($.999M)=$1000 *risk adverse*

Component Reliability



Component Reliability Analysis

- Failure rate is defined as fraction of currently operating units failing per unit time

$$\lambda(t) = -\frac{1}{R(t)}\frac{d}{dt}R(t)$$

- The trend of operating units with time is then

$$\int_0^t \lambda(\tau)\,d\tau = -\int_1^{R(t)} \frac{dR(\tau)}{R(\tau)}$$

Component Reliability Analysis (continued)

- Evaluation of the definite integrals gives

$$\int_0^t \lambda(\tau)\, d\tau = -\ln[R(t)]$$

- Assuming that λ is constant over the operating lifetime,

$$R(t) = \exp\left[-\int_0^t \lambda(\tau) d\tau\right] = e^{-\lambda t}$$

- At $t=1/\lambda$, $1/e$ of the original units are still operating (defined as mean time between failures)

Component Reliability Analysis (continued)

- Frequently assess component reliability based on reciprocal of failure rate λ :

$$R(t) = e^{-\frac{t}{MTBF}}$$

where MTBF=mean time between failures

- For a mission duration of N hours, estimate of component reliability becomes

$$R(mission) = e^{-\frac{N}{MTBF}}$$

Verifying a Reliability Estimate

- Given a unit reliability of R, what is the probability P of testing it 20 times without a failure?
- What is the probability Q that you will see one or more failures?
  - R=.99 - P=.8179 - Q=.1821
  - R=.95 - P=.3584 - Q=.6416
  - R=.90 - P=.1216 - Q=.8784

Confidence

- The confidence C in a test result is equal to the probability that you should have seen worse results than you did

- P(observed and better outcomes) + C =1

Confidence Example - Saturn V

- NASA MSFC initially set requirement that Saturn V should demonstrate 95% reliability at 80% confidence prior to manned operations

$$0.95^n + 0.80 = 1 \Longrightarrow n = 31.4$$

- NASA actually flew 13 Saturn Vs without a failure - actual reliability at 80% confidence was

$$R^{13} + 0.80 = 1 \Longrightarrow R = 88.4\%$$

Example of Confidence - Space Shuttle

- 113 vehicle flights with 2 failures
- Assume a reliability value of R
- Confidence if no failures had occurred

$$R^{113} + C_0 = 1$$

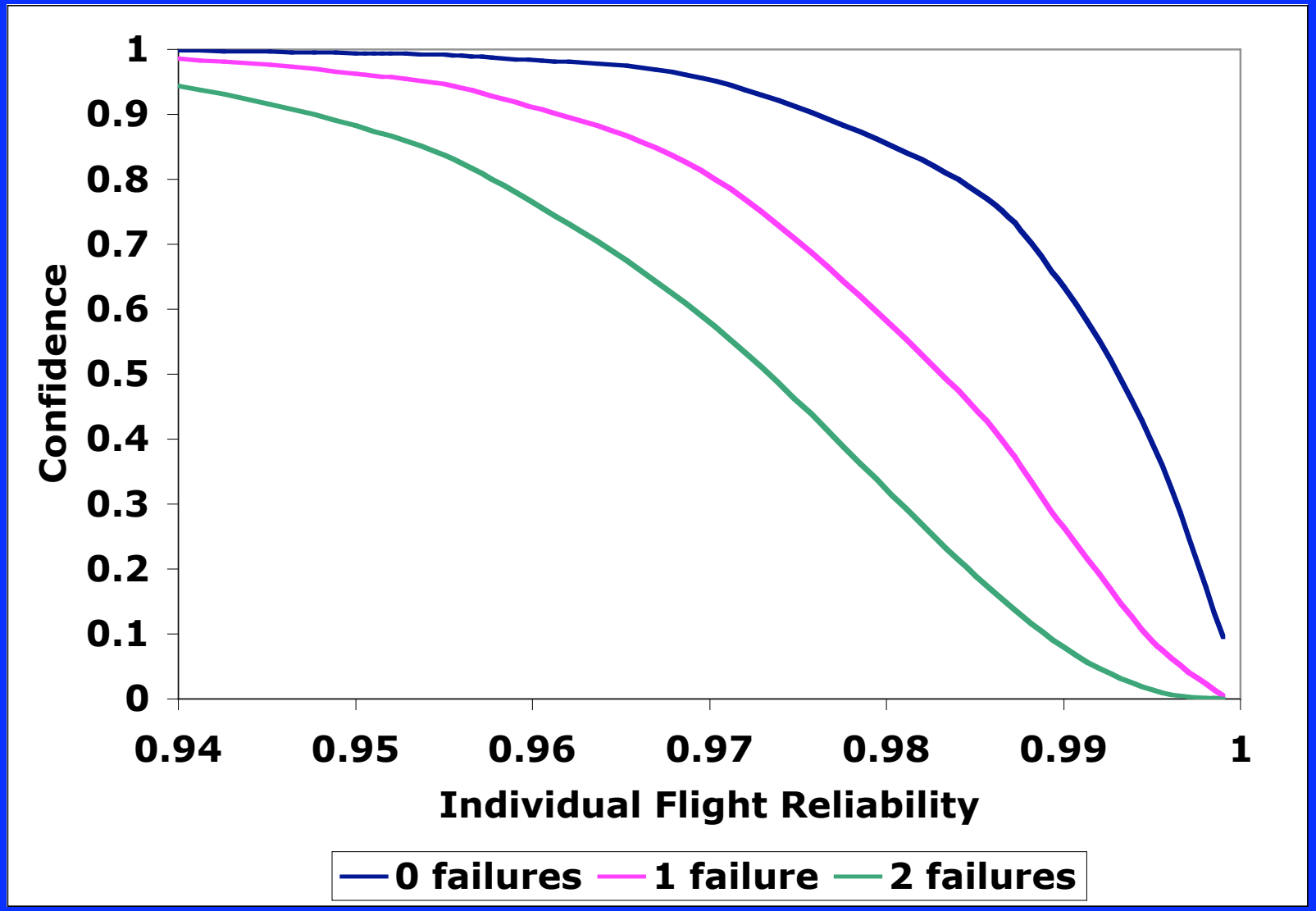
- Confidence if one failure had occurred

$$R^{113} + 113R^{112}(1-R) + C_1 = 1$$

- Confidence if two failures occurred

$$R^{113} + 113R^{112}(1-R) + \frac{(113)(112)}{2}R^{111}(1-R)^2 + C_2 = 1$$

Shuttle Reliability and Confidence



Definition of Redundancy

- Probability of k out of n units working = (number of permutations of k out of n) x P(k units work) x P(n-k units fail)

$$P\binom{k}{n} = \frac{n!}{k!(n-k)!} P^k (1-P)^{n-k}$$

Redundancy Example

3 parallel computers, each has reliability of 95%:

- Probability all three work

$$P(3) = P^3 = (.95)^3 = .8574$$

- Probability exactly two work

$$P(2) = 3P^2(1-P) = 3(.95)^2(.05) = .1354$$

- Probability exactly one works

$$P(1) = 3P(1-P)^2 = 3(.95)(.05)^2 = .0071$$

- Probability that none work

$$P(0) = (1-P)^3 = (.05)^3 = .0001$$

Redundancy Example

3 parallel computers, each has reliability of 95%:

- Probability all three work

$$P(3) = .8574$$

- Probability at least two work

$$P(3) + P(2) = .8574 + .1354 = .9928$$

- Probability at least one works

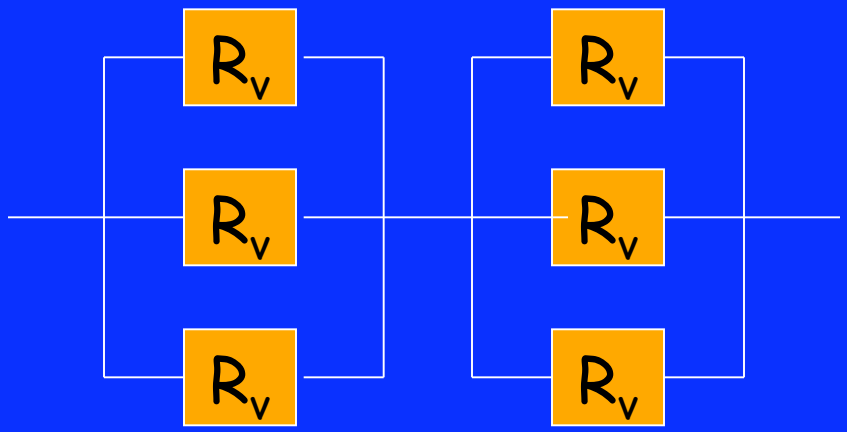
$$P(3) + P(2) + P(1) = .9928 + .0071 = .9999$$

- Probability that none work

$$P(0) = (1 - P)^3 = (.05)^3 = .0001$$

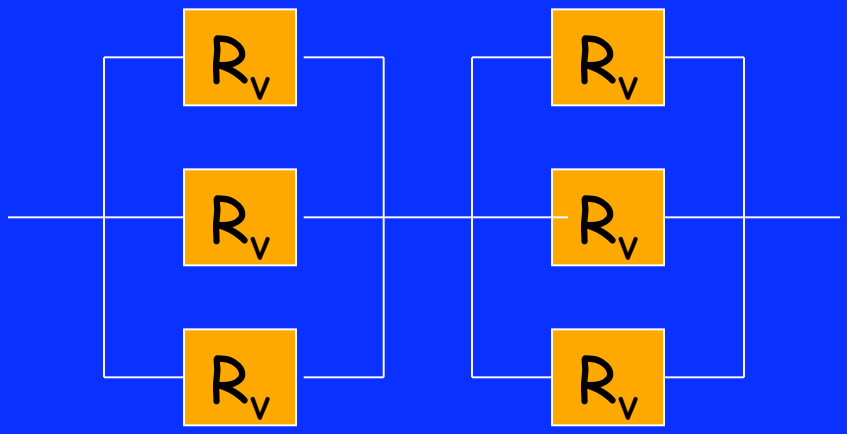
Reliability Diagrams

- Example of Apollo Lunar Module ascent engine
- Three valves in each of oxidizer and fuel lines
- One in each set of three must work
- R_v=0.9 --> R_{system}=.998



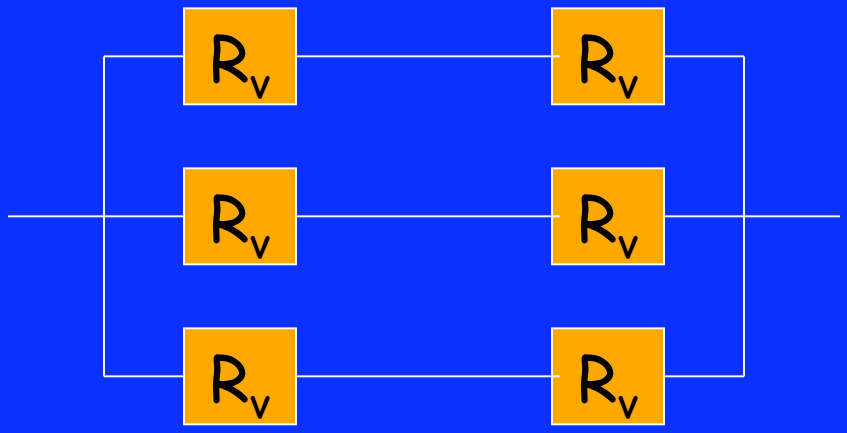
$$R_{system} = \left[1 - (1 - R_v)^3\right]^2$$

Reliability Diagrams (how not to…)



$$R_{system} = \left[1 - (1 - R_v)^3\right]^2$$

R_v=0.9 --> R_{system}=.998



$$R_{system} = \left[1 - (1 - R_v^{\,2})^3\right]$$

R_v=0.9 --> R_{system}=.993

Intercorrelated Failures

- Some failures in redundant systems are common to all units
  - Software failures
  - "Daisy-chain" failures
  - Design defects
- Following a failure, there is a probability f that the failure causes a total system failure

Intercorrelated Failure Example

3 parallel computers, each has reliability of 95%, and a 30% intercorrelated failure rate:

- Probability all three work

$$P(3) = P^3 = (.95)^3 = .8574$$

- Probability exactly two work (one failure)

$$P(2) = 3P^2(1-P) = 3(.95)^2(.05) = .1354$$

  - Probability the failure is benign (system works)

$$P\left(2_{safely}\right) = .7(.1354) = .0948$$

  - Probability of intercorrelated failure (system dies)

$$P\left(2_{system\ failure}\right) = .3(.1354) = .0406$$

Intercorrelated Failure Example

(continued from previous slide)

- Probability exactly one works (2 failures)

$$P(1) = 3P(1-P)^2 = 3(.95)(.05)^2 = .0071$$

  - Probability that both failures are benign

$$P\left(1_{safely}\right) = .7^2(.0071) = .0035$$

  - Probability that a failure is intercorrelated

$$P\left(1_{system\ failure}\right) = \left(1-.7^2\right)(.0071) = .0036$$

Redundancy Example with Intercorrelation

3 parallel computers, each has reliability of 95%, and a 30% intercorrelated failure rate:

- Probability all three work

$$P(3) = .8574$$

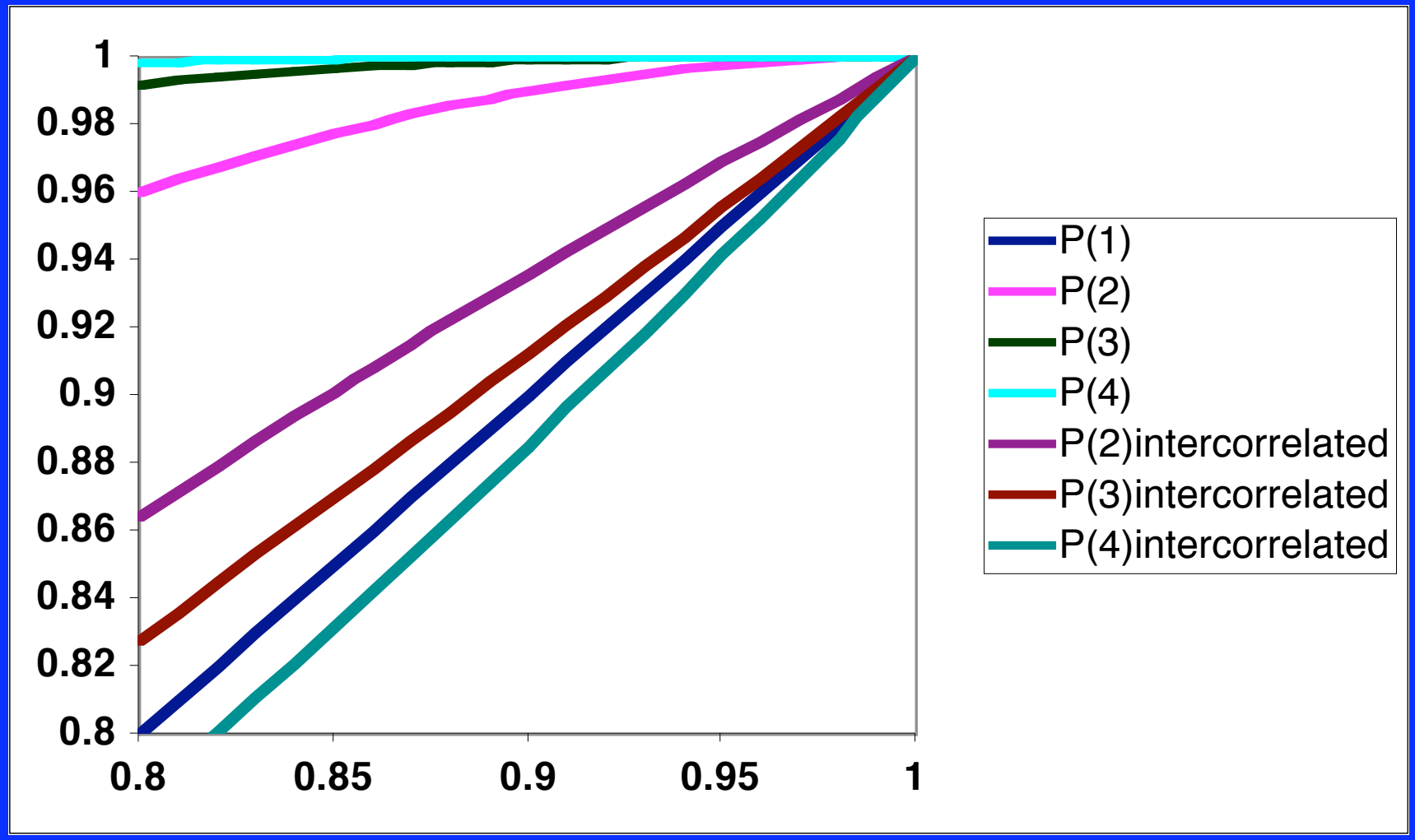
- Probability at least two work

$$= .8574 + .0948 = .9522 \quad (was\ .9928)$$

- Probability at least one works

$$= .9522 + .0035 = .9557 \quad (was\ .9999)$$

System Reliability with 30% Intercorrelation



Probabilistic Risk Assessment

- Identification and delineation of the combinations of events that, if they occur, could lead to an accident (or other undesired event)
- Estimation of the chance of occurrence for each combination
- Estimation of the consequences associated with each combination.

Concept of System Resiliency

- Initial flight schedule

- Hiatus period following a failure

- Backlog of payloads not flown in hiatus

- Surge to fly off backlog

- Resilient if backlog is cleared before next failure occurs (on average)

Resiliency Variables

r - nominal flight rate, flts/yr

d - down time following failure (yrs)

k - fraction of flights in backlog retained

S - surge flight rate/nominal flight rate

m - average/expected flights between failures

rd - number of missed flights

krd - number of flights in backlog

(S-1)r - backlog flight rate

Definition of Resiliency

$$\frac{Srkd}{S-1} \leq m$$

- ⇨ Example for Delta launch vehicle
- r = 12 flts/yr
- d = 0.5 yrs
- k = 0.8
- S = 1.5
- m = 30
- Srkd/(S-1) = 14.4 < 30 - system is resilient!

Shuttle Resiliency (post-*Challenger*)

r = 9 flts/yr

d = 2.5 yrs

k = 0.8

S = .67 (6 flts/yr)

m = 25

➥System has negative surge capacity due to reduction in fleet size - cannot *ever* recover from hiatus without more extreme measures

Modified Resiliency

k' - retention rate of all future payloads
(k'≤S for S<1)

- New governing equation for resiliency:

$$\frac{Srk'd}{S-k'} \leq m$$

- Implication for shuttle case:

➥k<.417 to achieve modified resiliency

Shuttle Resiliency (post-*Columbia*)

r = 5 flts/yr

d = 2 yrs

S = .8 (4 flts/yr)

m = 56 (average missions/failure)

➥Modified resiliency requires k'≤0.7 for all future payloads