Course Overview/Orbital Mechanics

- Course Overview
  - Problems with launch and entry
  - Course goals
  - Web-based Content
  - Syllabus
  - Policies
  - Project Content
- An overview of orbital mechanics at "point five past lightspeed"

Space Launch - The Physics

- Minimum orbital altitude is ~200 km

$$\frac{\text{Potential Energy}}{\text{kg in orbit}} = gh = 1.96x10^6 \frac{J}{kg}$$

- Circular orbital velocity there is 7784 m/sec

$$\frac{\text{Kinetic Energy}}{\text{kg in orbit}} = \frac{1}{2}v^2 = 30x10^6 \frac{J}{kg}$$

- Total energy per kg in orbit

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = PE + KE = 32x10^6 \frac{J}{kg}$$

Theoretical Cost to Orbit

- Convert to usual energy units

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = 32x10^6 \frac{J}{kg} = 8.888 \frac{kWhrs}{kg}$$

- Domestic energy costs are ~$0.05/kWhr

⏩ Theoretical cost to orbit <u>$0.44/kg</u>

Actual Cost to Orbit

- Delta IV Heavy
  - 23,000 kg to LEO
  - $150 M per flight
- $6500/kg of payload
- Factor of 15,000x higher than theoretical energy costs!

What About Airplanes?

- For an aircraft in level flight,

$$\frac{\text{Weight}}{\text{Thrust}} = \frac{\text{Lift}}{\text{Drag}}, \text{ or } \frac{mg}{T} = \frac{L}{D}$$

- Energy = force x distance, so

$$\frac{\text{Total Energy}}{\text{kg}} = \frac{\text{Thrust x Distance}}{\text{mass}} = \frac{Td}{m} = \frac{gd}{L/D}$$

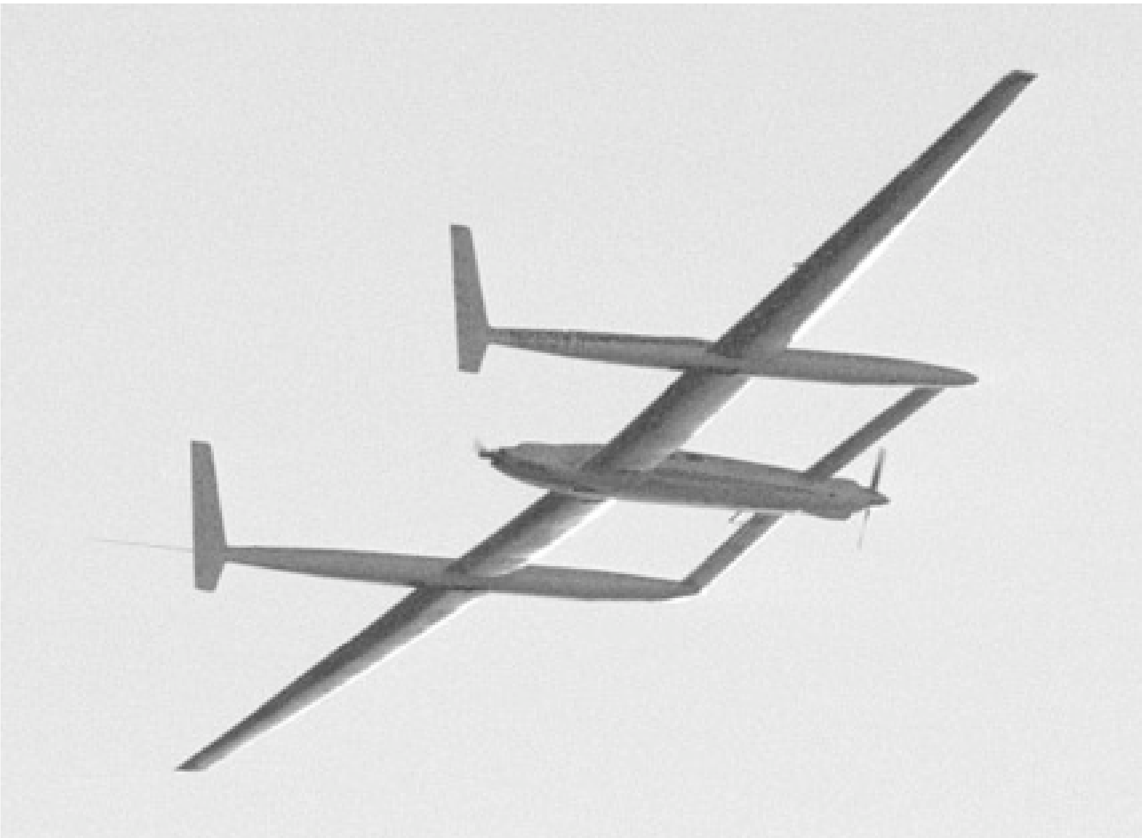
- For an airliner (L/D=25) to equal orbital energy, d=81,000 km (2 roundtrips NY-Sydney)

Equivalent Airline Costs?

- Average economy ticket NY-Sydney round-round-trip (Travelocity 1/28/04) ~$1300
- Average passenger (+ luggage) ~100 kg
- Two round trips = $26/kg
  - Factor of 60x electrical energy costs
  - Factor of 250x less than current launch costs
- But…

  you get to refuel at each stop!

Equivalence to Air Transport



- 81,000 kg ~ twice around the world
- Voyager - only aircraft to ever circle the world non-stop, non-refueled - *once!*

Orbital Entry - The Physics

- 32 MJ/kg dissipated by friction with atmosphere over ~8 min = 66kW/kg
- Pure graphite (carbon) high-temperature material: c_p=709 J/kg°K
- Orbital energy would cause temperature gain of 45,000°K!

The Vision

"Once you make it to low Earth orbit, you're halfway to anywhere!"

- Robert A. Heinlein

How To Approach Teaching This Course

- As a system user
- As a system provider
- As a venture capitalist
- As a system designer

Goals of ENAE 791

- Learn the underlying physics (orbital mechanics, flight mechanics, aerothermodynamics) which constrain and define launch and entry vehicles
- Develop the tools for preliminary design synthesis, including the fundamentals of systems analysis
- Provide an introduction to engineering economics, with a focus on the parameters affecting cost of launch and entry vehicles, such as reusability
- Examine specific challenges in the underlying design disciplines, such as thermal protection and structural dynamics

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Web-based Course Content

- Data web site at spacecraft.ssl.umd.edu
  - Course information
  - Syllabus
  - Lecture notes
  - Problems and solutions
- Interactive web site at www.ajconline.umd.edu
  - Communications for team projects
  - Surveys for course feedback

Syllabus Overview (1)

- Fundamentals of Launch and Entry Design
  - Orbital mechanics
  - Basic rocket performance
- Entry flight mechanics
  - Ballistic entry
  - Lifting entry
- Aerothermodynamics
- Thermal Protection System (TPS) analysis
- Entry, Descent, and Landing (EDL) systems

Syllabus Overview (2)

- Launch flight mechanics
  - Gravity turn
  - Targeted trajectories
  - Optimal trajectories
  - Airbreathing trajectories
- Launch vehicle systems
  - Propulsion systems
  - Structures and structural dynamics analysis
  - Payload accommodations

Syllabus Overview (3)

- Systems Analysis
  - Cost estimation
  - Engineering economics
  - Reliability issues
  - Safety design concerns
  - Fleet resiliency
- Case studies
- Design project

Policies

- Grade Distribution
  - 20% Problems
  - 25% Midterm Exam
  - 25% Term Project*
  - 30% Final Exam
- Late Policy
  - On time: Full credit
  - Before solutions: 70% credit
  - After solutions: 20% credit

* Team Grades

Term Project

- Teams of ~4 people (you pick)
- Given a mission model (mass per year), design a launch system which minimizes cost/kg of payload to LEO over the life of the project
- Detailed requirements forthcoming
- Design process should proceed throughout the term
- Formal design presentations at end of term

Orbital Mechanics: 500 years in 40 min.

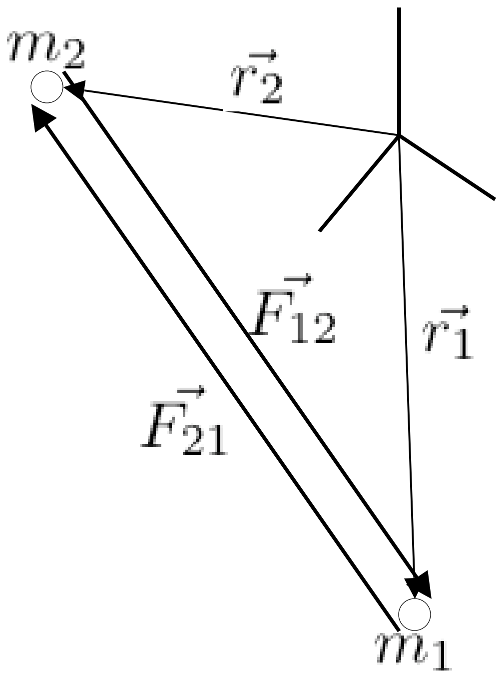
- Newton's Law of Universal Gravitation

$$F = \frac{Gm_1m_2}{r^2}$$

- Newton's First Law meets vector algebra

$$\vec{F} = m\vec{a}$$

Relative Motion Between Two Bodies



$$m_1 \frac{d^2 \vec{r}_1}{dt^2} = G \frac{m_1 m_2}{\left|\vec{r}_{21}\right|^2} \frac{\vec{r}_{21}}{\left|\vec{r}_{21}\right|}$$

$$= G \frac{m_1 m_2}{\left|\vec{r}_{21}\right|^3} \vec{r}_{21} = G \frac{m_1 m_2}{\left|\vec{r}_{21}\right|^3} \left(\vec{r}_2 - \vec{r}_1\right)$$

$$m_2 \frac{d^2 \vec{r}_2}{dt^2} = G \frac{m_1 m_2}{\left|\vec{r}_{12}\right|^3} \left(\vec{r}_1 - \vec{r}_2\right)$$

$\underline{F}_{12}$=force due to body 1 on body 2

Gravitational Motion

$$\frac{d^2\vec{r}}{dt^2} = \frac{G}{r^3}\left[m_2\left(-\vec{r}\right) - m_1\left(\vec{r}\right)\right] = \frac{-G}{r^3}\left(m_1 + m_2\right)\vec{r}$$

Let $r = |\vec{r}_{12}| = |\vec{r}_{21}|$ Let $\vec{r} = \vec{r}_1 - \vec{r}_2$

$$\boxed{\frac{d^2\vec{r}}{dt^2} + \mu\frac{\vec{r}}{r^3} = \vec{0}}$$

"Equation of Orbit" -
Orbital motion is simple harmonic motion

Orbital Angular Momentum

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \frac{d\vec{v}}{dt} + \mu\frac{\vec{r}}{r^3} = \vec{0}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} + \frac{\mu}{r^3}\left(\vec{r} \times \vec{r}\right) = \vec{0} \qquad \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}$$

$$= \vec{v} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \vec{0} \qquad \vec{r} \times \vec{v} = co\vec{nst}ant \qquad \vec{r} \times \vec{v} = \vec{h}$$

$\underline{h}$ is angular momentum vector - $\underline{r}$ and $\underline{v}$ planar

Fun and Games with Algebra

$$\frac{d\vec{v}}{dt} + \mu \frac{\vec{r}}{r^3} = \vec{0} \qquad \frac{d\vec{v}}{dt} \times \vec{h} + \frac{\mu}{r^3}\left(\vec{r} \times \vec{h}\right) = \vec{0}$$

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = \frac{d\vec{v}}{dt} \times \vec{h} + \cancelto{0}{\vec{v} \times \frac{d\vec{h}}{dt}}$$

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r} \times \vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r} \times \vec{r} \times \vec{v}\right)$$

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = -\frac{\mu}{r^3}\left[\left(\vec{r} \cdot \vec{v}\right)\vec{r} - \left(\vec{r} \cdot \vec{r}\right)\vec{v}\right]$$

$$\vec{r} \cdot \vec{v} = rv\cos\gamma = r\frac{dr}{dt}$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\frac{\mu}{r^3}\left[r\frac{dr}{dt}\vec{r} - r^2\frac{d\vec{r}}{dt}\right]$$

$$\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{\left(r\frac{d\vec{r}}{dt} - \vec{r}\frac{dr}{dt}\right)}{r^2} = \left(\frac{1}{r}\frac{d\vec{r}}{dt} - \frac{\vec{r}}{r^2}\frac{dr}{dt}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\mu\left(\frac{1}{r^2}\frac{dr}{dt}\vec{r} - \frac{1}{r}\frac{d\vec{r}}{dt}\right) = \mu\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h} - \mu\frac{\vec{r}}{r}\right) = \vec{0}$$

Orientation of the Orbit

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \vec{\text{constant}} \qquad \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \mu \vec{e}$$

$$\vec{e} \equiv \text{eccentricity vector, in orbital plane}$$

e points in the direction of periapsis

$$\vec{r} \cdot \vec{v} \times \vec{h} - \vec{r} \cdot \mu \frac{\vec{r}}{r} = \mu \left(\vec{r} \cdot \vec{e} \right)$$

$$\vec{r} \times \vec{v} \cdot \vec{h} - \mu \frac{\vec{r} \cdot \vec{r}}{r} = \mu r e \cos \theta$$

$$\vec{h} \cdot \vec{h} - \mu \frac{r^2}{r} = \mu r e \cos \theta$$

Position in Orbit

$$h^2 - \mu r = \mu r e \cos\theta$$

$$\boxed{r = \frac{h^2/\mu}{1 + e\cos\theta}}$$

θ = true anomaly: angular travel from perigee passage

$$\text{at } \theta = \pm\frac{\pi}{2};\ \cos\theta = 0;\ r = p \equiv h^2/\mu$$

$$\mu\vec{e} = \vec{v} \times \vec{h} - \mu\frac{\vec{r}}{r}$$

$$\mu\vec{e} \cdot \mu\vec{e} = \vec{v} \times \vec{h} \cdot \vec{v} \times \vec{h} - 2\mu\left(\vec{v} \times \vec{h}\right) \cdot \frac{\vec{r}}{r} + \mu^2\left(\frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r}\right)$$

$$\mu^2 e^2 = v^2 h^2 - 2\mu\frac{h^2}{r} + \mu^2$$

$$e^2 = \frac{v^2}{\mu}p - 2\frac{p}{r} + 1$$

$$p \equiv a(1 - e^2) = \frac{1 - e^2}{\frac{2}{r} - \frac{v^2}{\mu}}$$

Vis-Viva Equation

$$\boxed{v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)}$$

$$-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

Energy in Orbit

- Kinetic Energy

$$K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}$$

- Potential Energy

$$P.E. = -\frac{m\mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}$$

- Total Energy

$$Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

<--Vis-Viva Equation

Implications of Vis-Viva

- Circular orbit (r=a)

$$v_{circular} = \sqrt{\frac{\mu}{r}}$$

- Parabolic escape orbit (a tends to infinity)

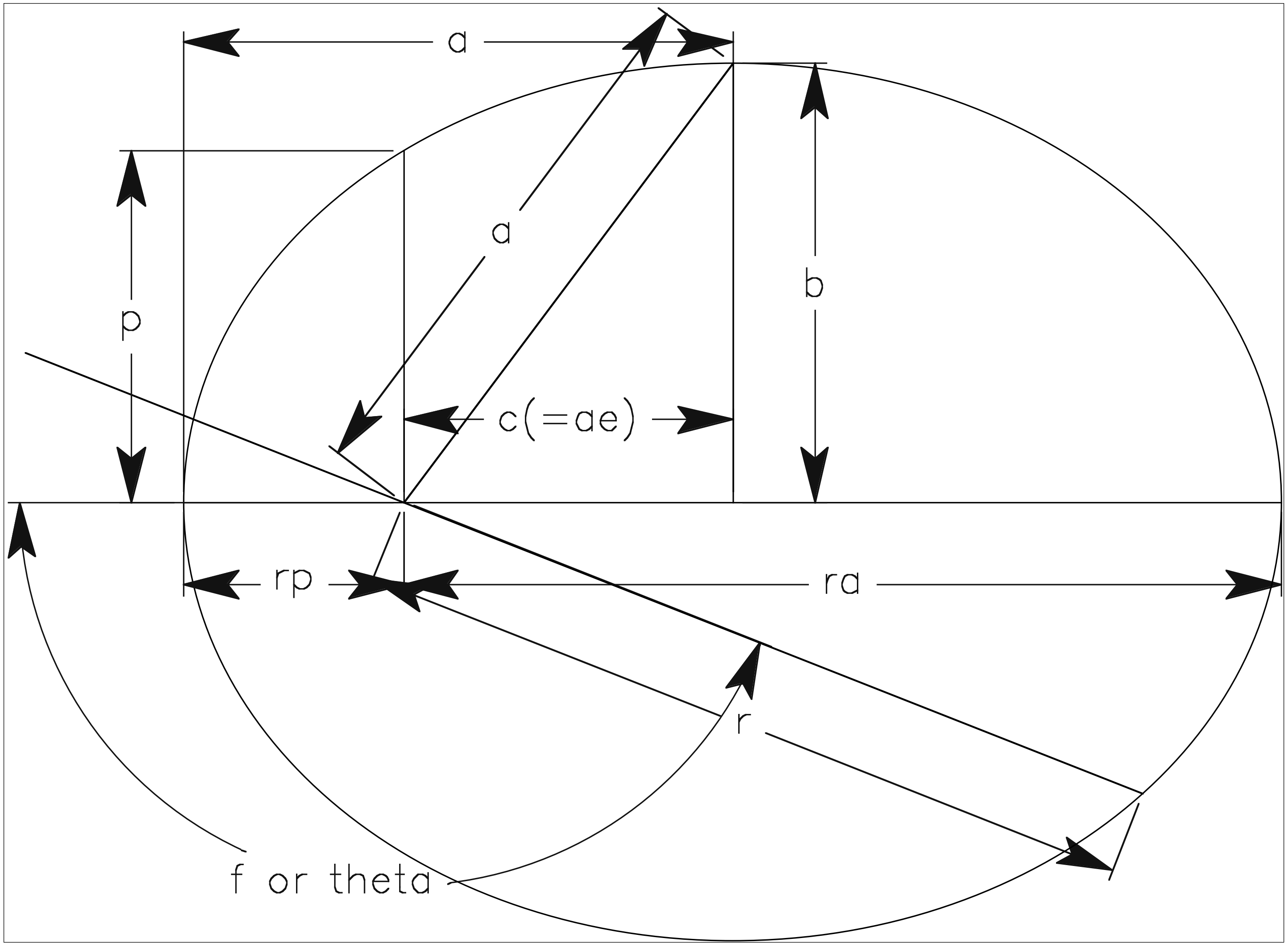
$$v_{escape} = \sqrt{\frac{2\mu}{r}}$$

- Relationship between circular and parabolic orbits $v_{escape} = \sqrt{2}v_{circular}$

Some Useful Constants

- Gravitation constant $\mu = GM$
  - Earth: 398,604 km^3/sec^2
  - Moon: 4667.9 km^3/sec^2
  - Mars: 42,970 km^3/sec^2
  - Sun: 1.327×10^{11} km^3/sec^2
- Planetary radii
  - $r_{Earth} = 6378$ km
  - $r_{Moon} = 1738$ km
  - $r_{Mars} = 3393$ km

Classical Parameters of Elliptical Orbits



Basic Orbital Parameters

- Semi-latus rectum (or parameter)

$$p = a(1 - e^2)$$

- Radial distance as function of orbital position

$$r = \frac{p}{1 + e\cos\theta}$$

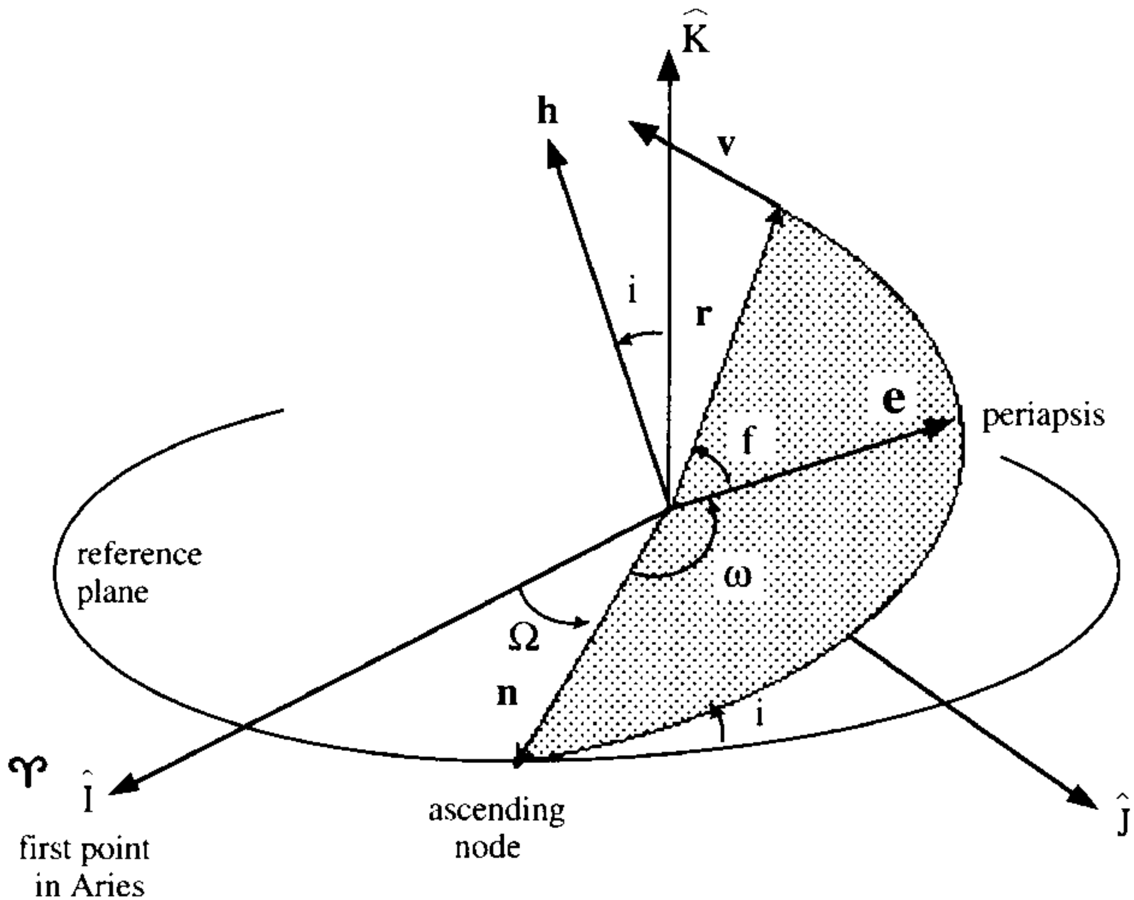
- Periapse and apoapse distances

$$r_p = a(1 - e) \qquad r_a = a(1 + e)$$

- Angular momentum

$$\vec{h} = \vec{r} \times \vec{v} \qquad h = \sqrt{\mu p}$$

The Classical Orbital Elements



Ω : longitude of the ascending node

ω : argument of periapsis

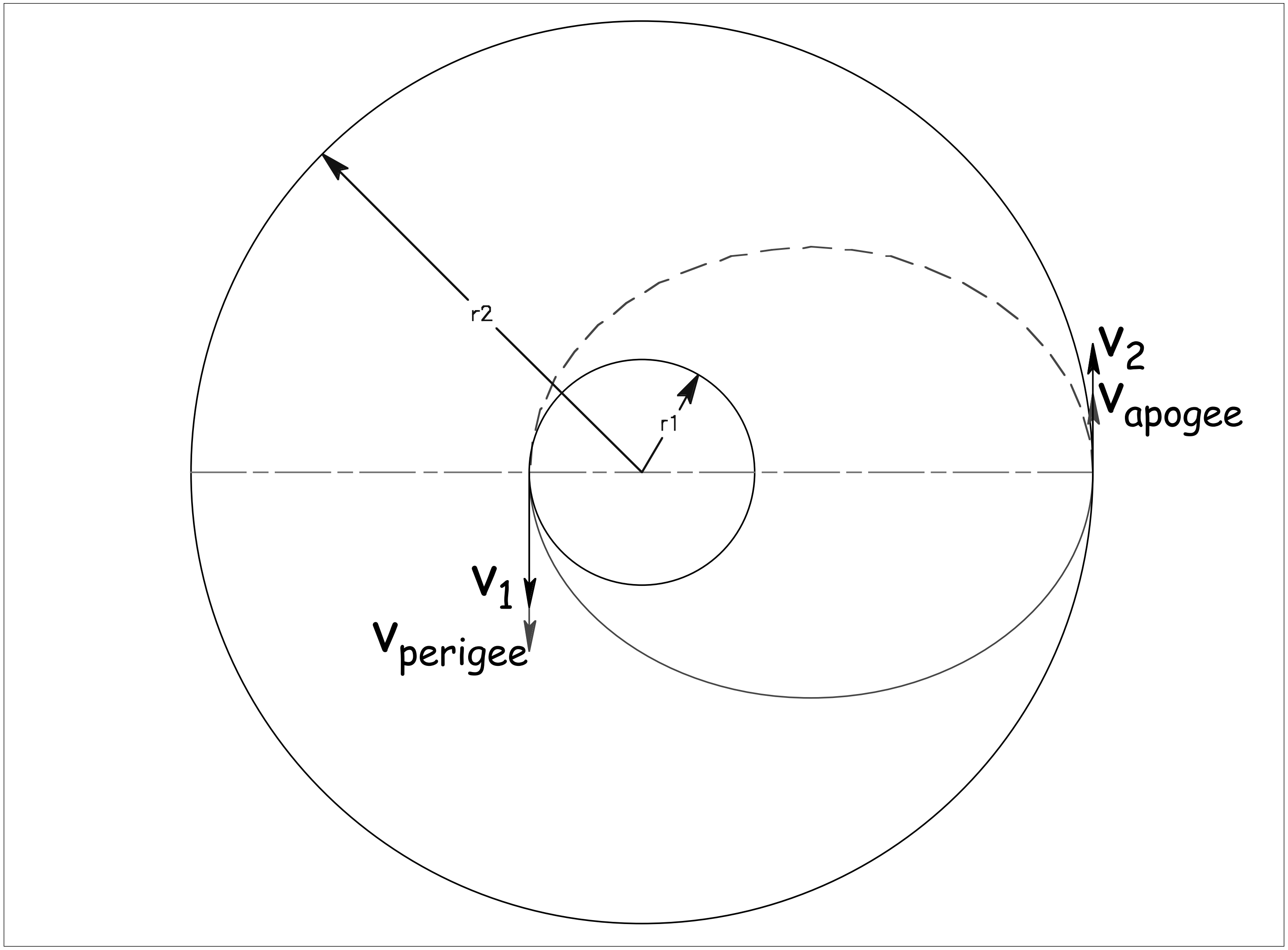
$\tilde{\omega} = \Omega + \omega$: longitude of periapsis

f : true anomaly

$L = \tilde{\omega} + f$: true longitude

Ref: J. E. Prussing and B. A. Conway, *Orbital Mechanics* Oxford University Press, 1993

The Hohmann Transfer



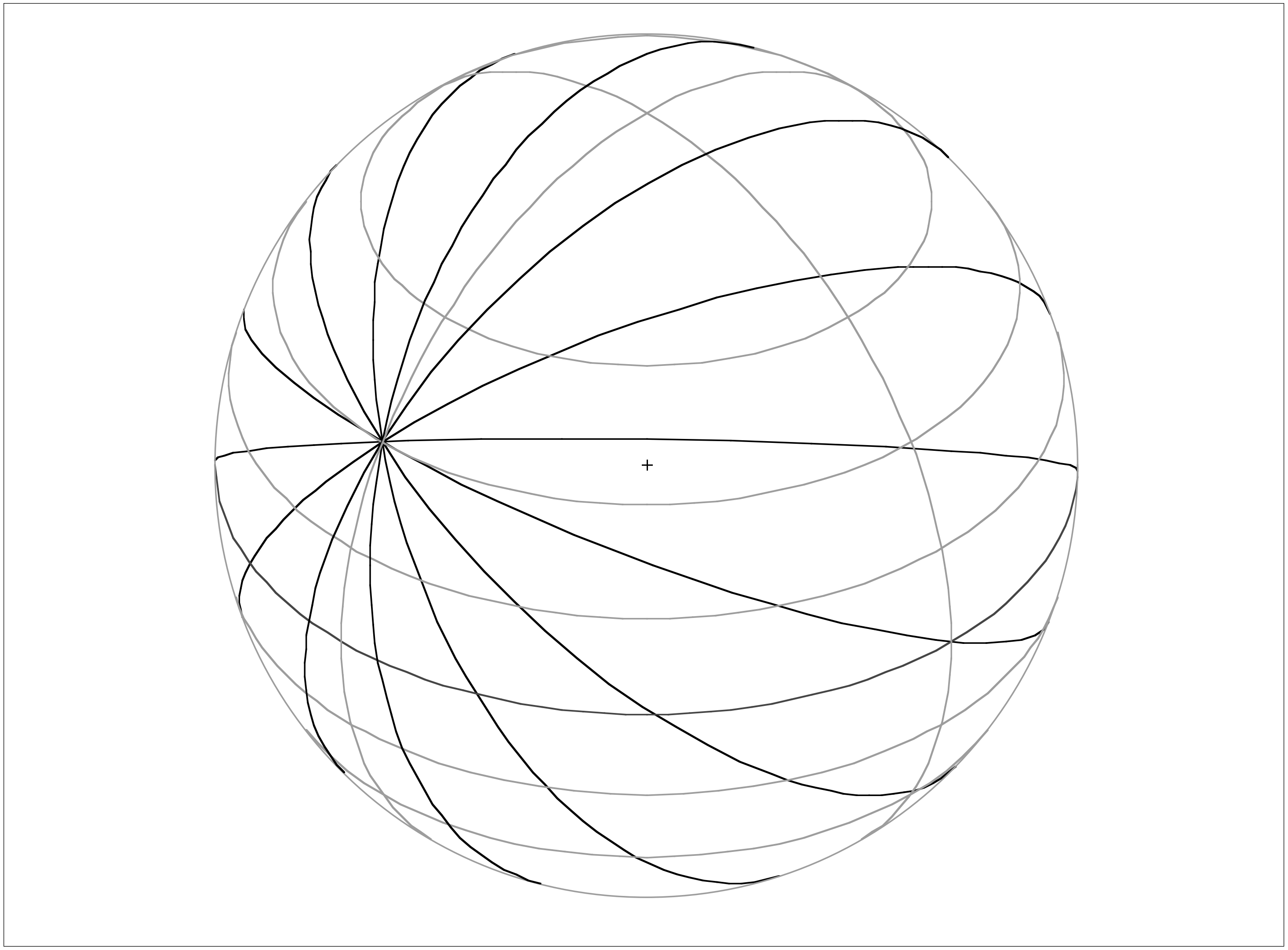
First Maneuver Velocities

- Initial vehicle velocity $$v_1 = \sqrt{\frac{\mu}{r_1}}$$
- Needed final velocity $$v_{perigee} = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$
- Delta-V $$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

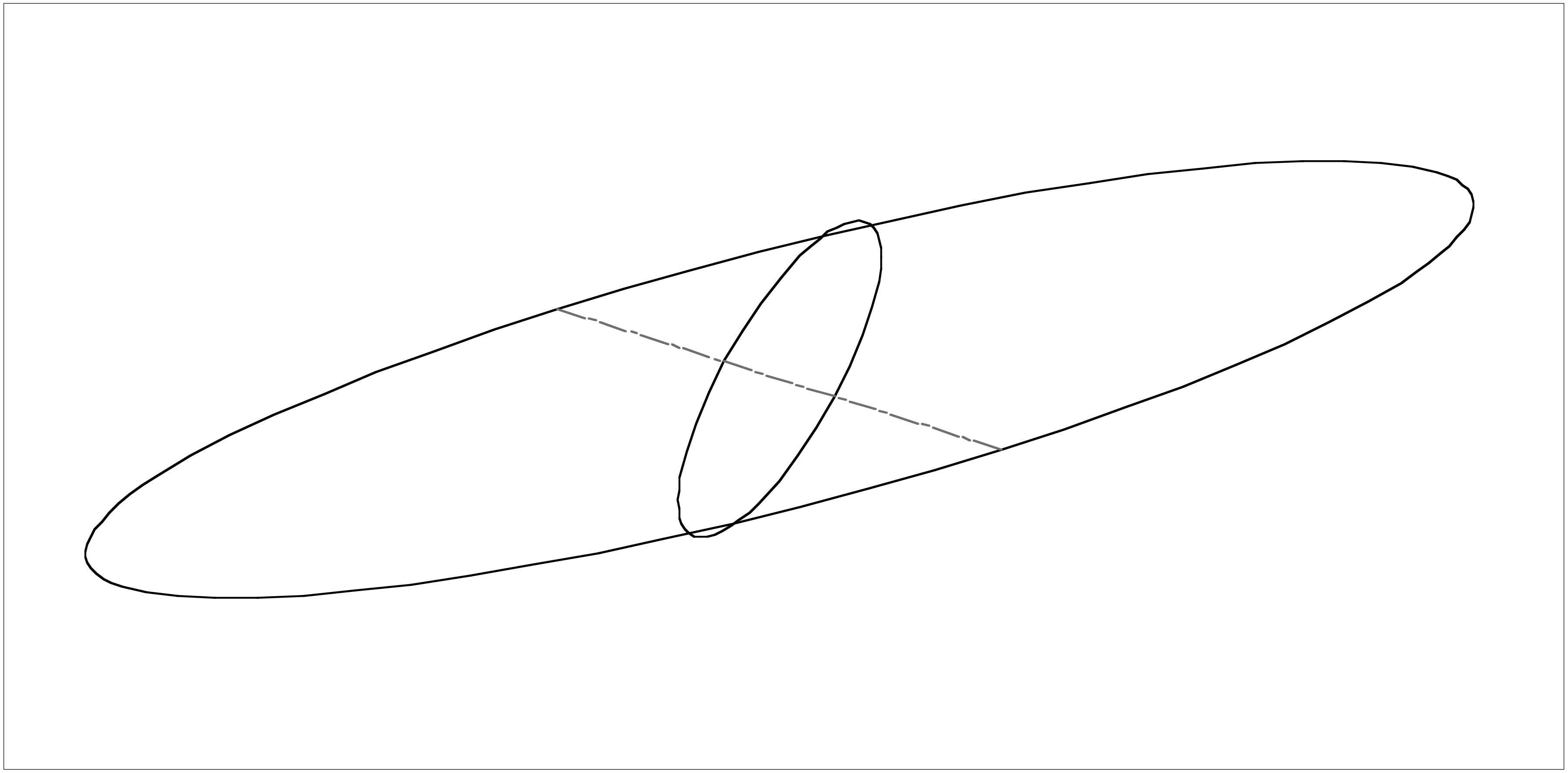
Second Maneuver Velocities

- Initial vehicle velocity $v_{apogee} = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$
- Needed final velocity $v_2 = \sqrt{\frac{\mu}{r_2}}$
- Delta-V $\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right)$

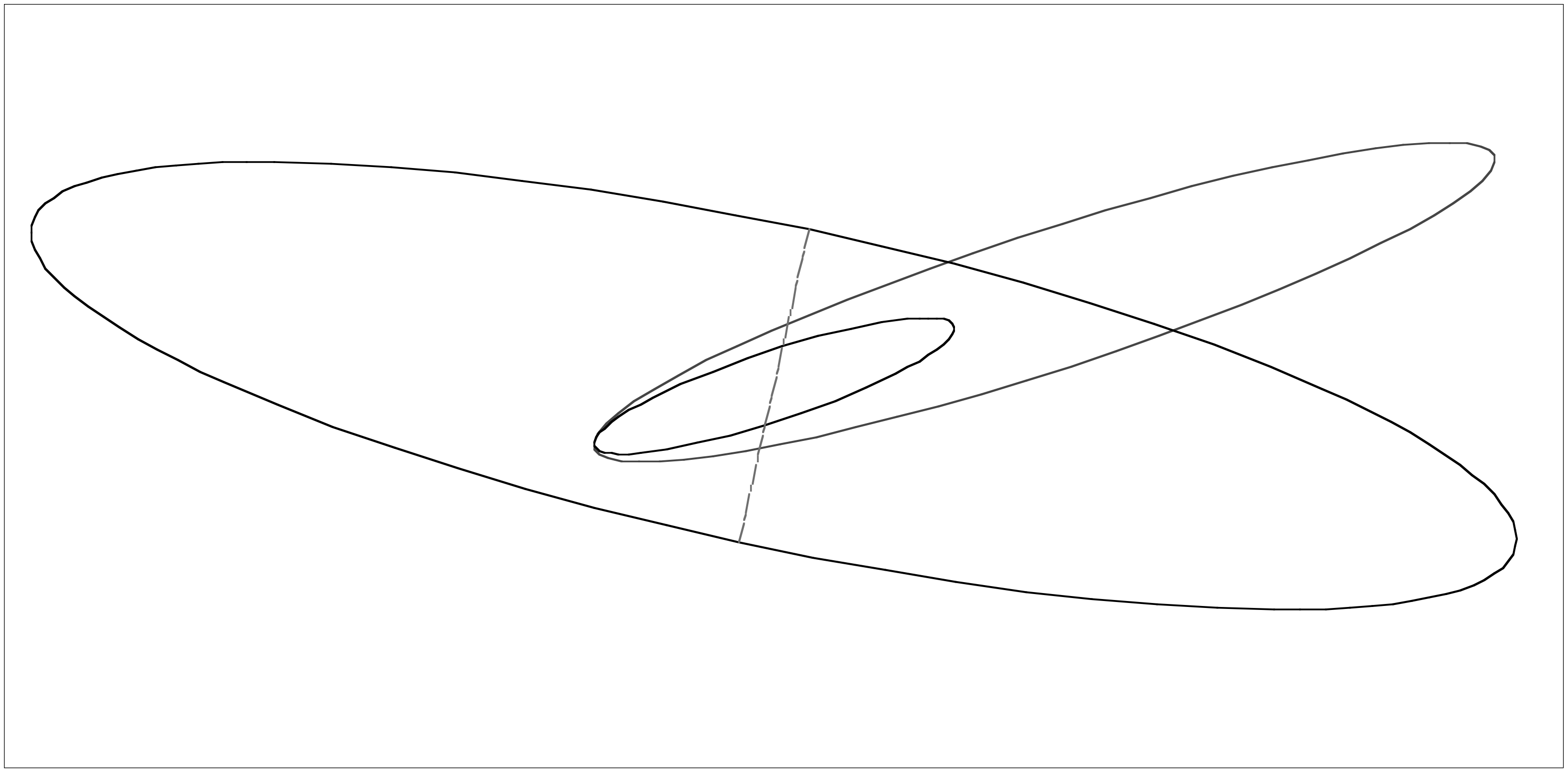
Limitations on Launch Inclinations



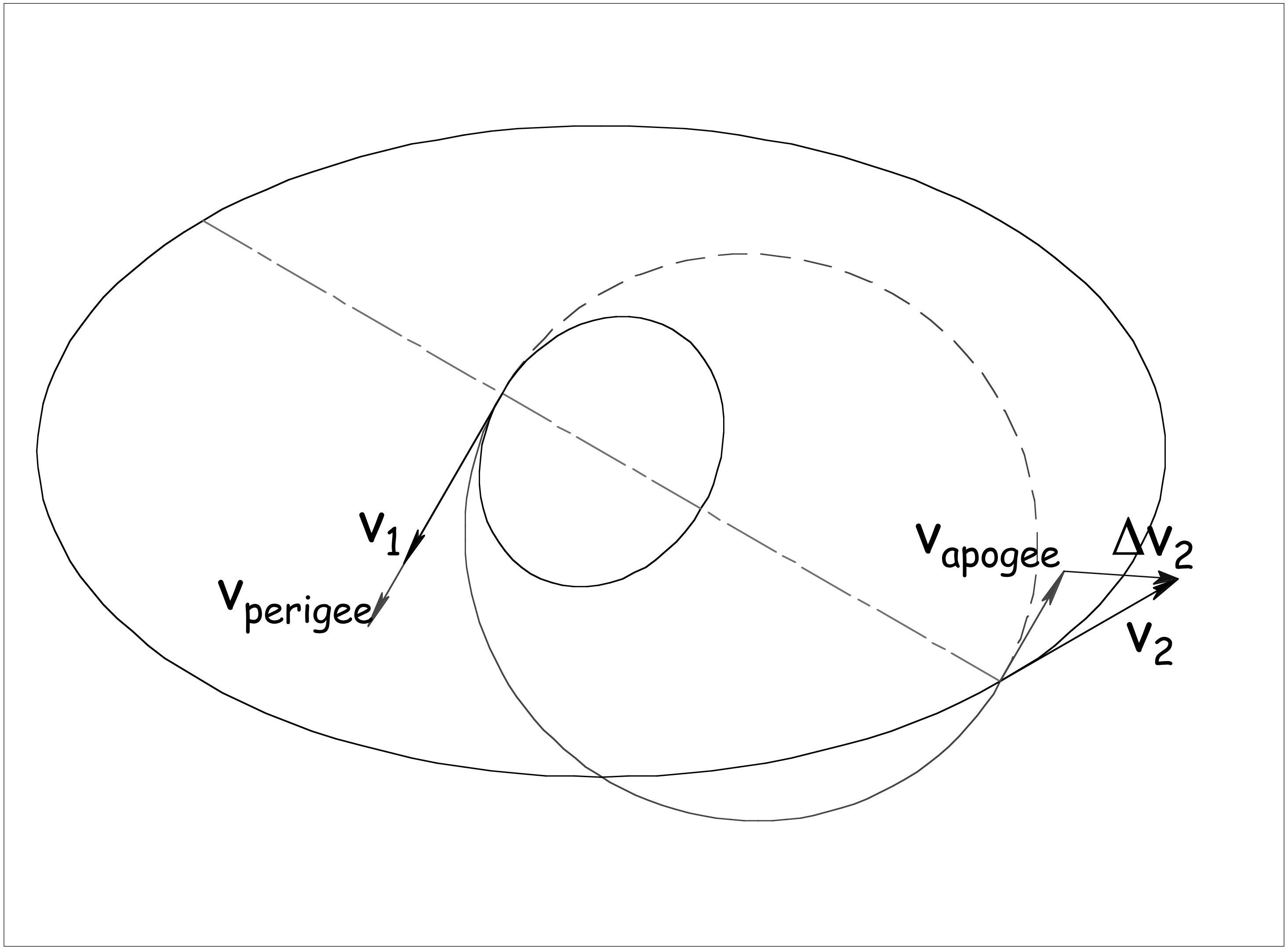
Differences in Inclination



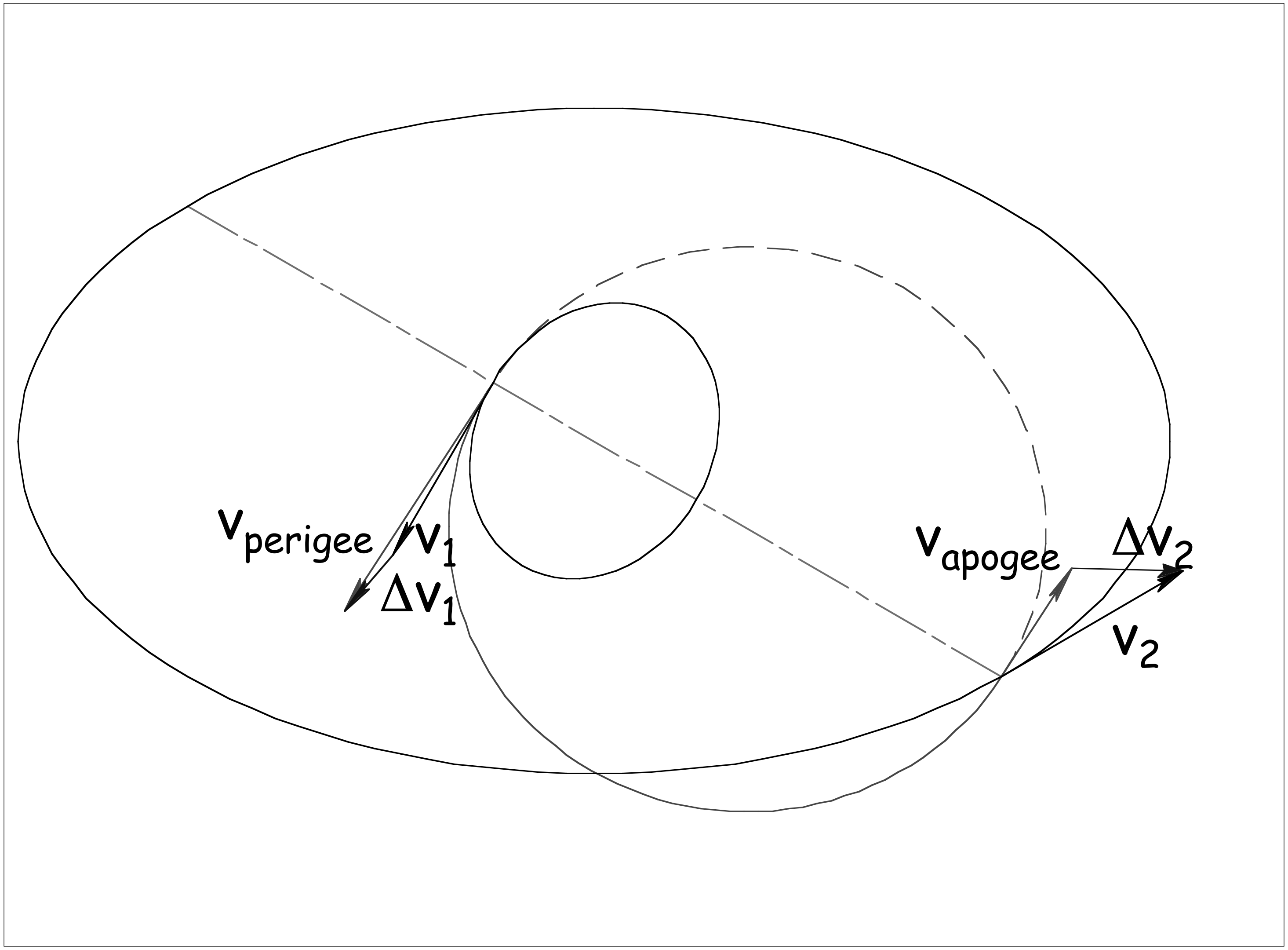
Choosing the Wrong Line of Apsides



Simple Plane Change



Optimal Plane Change



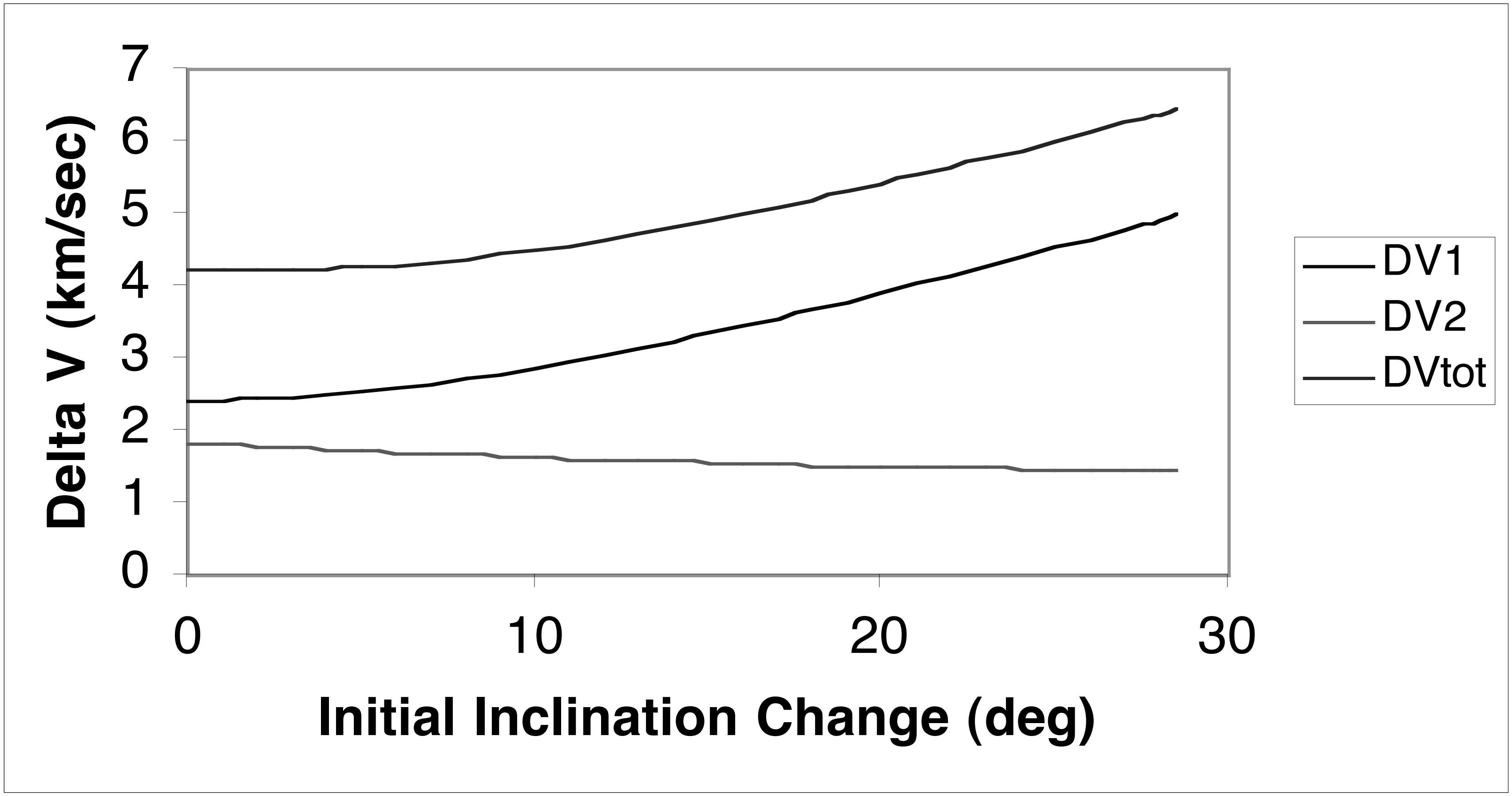
First Maneuver with Plane Change Δi_1

- Initial vehicle velocity $$v_1 = \sqrt{\frac{\mu}{r_1}}$$
- Needed final velocity $$v_p = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$
- Delta-V $$\Delta v_1 = \sqrt{v_1^2 + v_p^2 - 2v_1 v_p \cos(\Delta i_1)}$$

Second Maneuver with Plane Change Δi_2

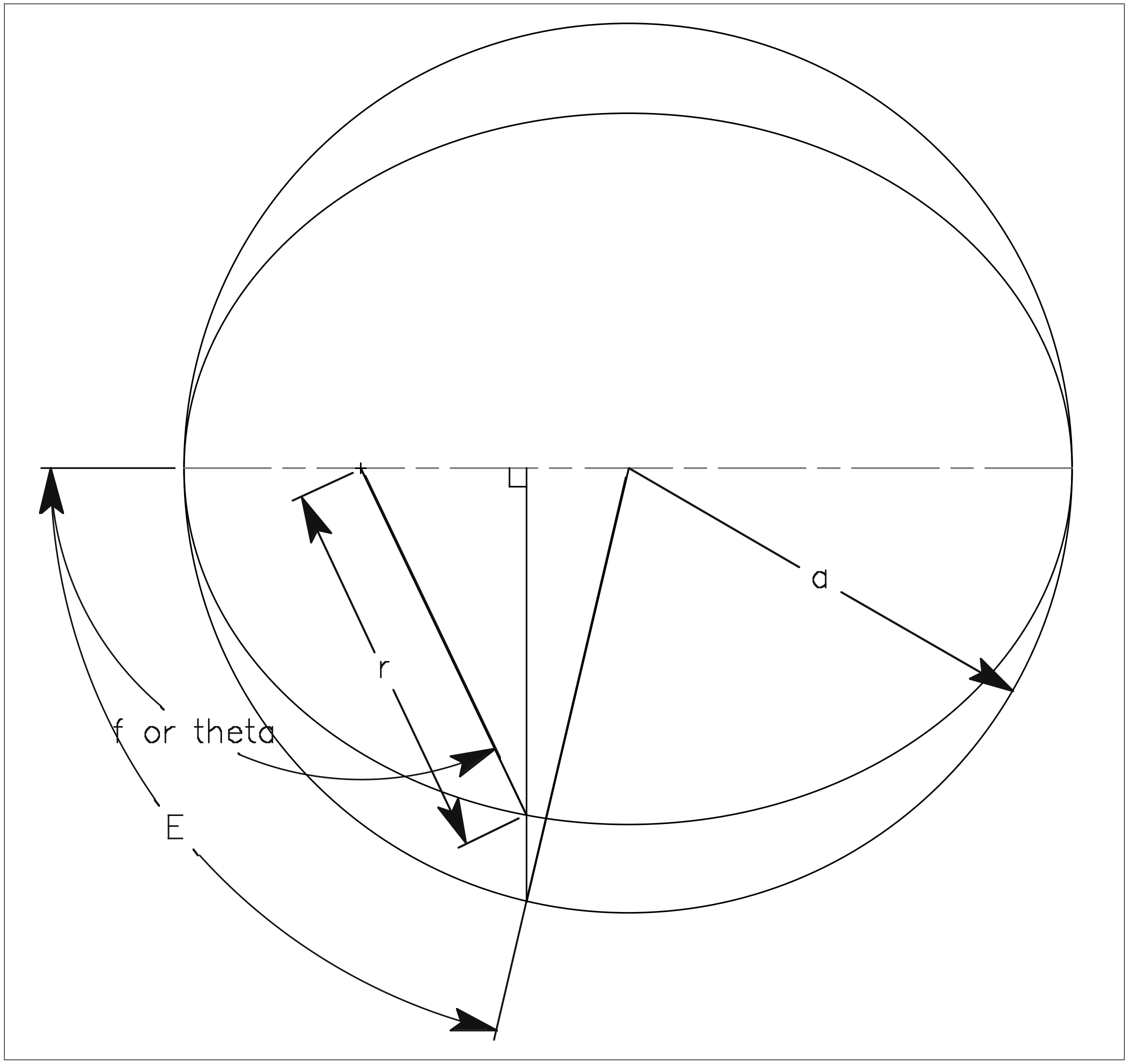
- Initial vehicle velocity $v_a = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$
- Needed final velocity $v_2 = \sqrt{\frac{\mu}{r_2}}$
- Delta-V $\Delta v_2 = \sqrt{v_2^2 + v_a^2 - 2v_2 v_a \cos(\Delta i_2)}$

Sample Plane Change Maneuver



Optimum initial plane change = 2.20°

Calculating Time in Orbit



Time in Orbit

- Period of an orbit

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

- Mean motion (average angular velocity)

$$n = \sqrt{\frac{\mu}{a^3}}$$

- Time since pericenter passage

$$M = nt = E - e\sin E$$

➥M=mean anomaly

Dealing with the Eccentric Anomaly

- Relationship to orbit

$$r = a(1 - e\cos E)$$

- Relationship to true anomaly

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

- Calculating M from time interval: iterate

$$E_{i+1} = nt + e\sin E_i$$

until it converges