

Course Overview/Orbital Mechanics

- Course Overview
 - Problems with launch and entry
 - Course goals
 - Web-based Content
 - Syllabus
 - Policies
 - Project Content
- An overview of orbital mechanics at "point five past lightspeed"



Space Launch - The Physics

- Minimum orbital altitude is ~200 km

$$\frac{\text{Potential Energy}}{\text{kg in orbit}} = gh = 1.96 \times 10^6 \frac{J}{kg}$$

- Circular orbital velocity there is 7784 m/sec

$$\frac{\text{Kinetic Energy}}{\text{kg in orbit}} = \frac{1}{2}v^2 = 30 \times 10^6 \frac{J}{kg}$$

- Total energy per kg in orbit

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = PE + KE = 32 \times 10^6 \frac{J}{kg}$$



Theoretical Cost to Orbit

- Convert to usual energy units

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = 32 \times 10^6 \frac{J}{kg} = 8.888 \frac{kWhrs}{kg}$$

- Domestic energy costs are ~\$0.05/kWhr

▶▶ Theoretical cost to orbit \$0.44/kg



Actual Cost to Orbit



- Delta IV Heavy
 - 23,000 kg to LEO
 - \$150 M per flight
- \$6500/kg of payload
- Factor of 15,000x higher than theoretical energy costs!



What About Airplanes?

- For an aircraft in level flight,

$$\frac{\text{Weight}}{\text{Thrust}} = \frac{\text{Lift}}{\text{Drag}}, \text{ or } \frac{mg}{T} = \frac{L}{D}$$

- Energy = force x distance, so

$$\frac{\text{Total Energy}}{\text{kg}} = \frac{\text{Thrust x Distance}}{\text{mass}} = \frac{Td}{m} = \frac{gd}{L/D}$$

- For an airliner ($L/D=25$) to equal orbital energy,
 $d=81,000$ km (2 roundtrips NY-Sydney)



Equivalent Airline Costs?

- Average economy ticket NY-Sydney round-round-trip (Travelocity 1/28/04) ~\$1300
- Average passenger (+ luggage) ~100 kg
- Two round trips = \$26/kg
 - Factor of 60x electrical energy costs
 - Factor of 250x less than current launch costs
- But...
 - you get to refuel at each stop!



Equivalence to Air Transport



- 81,000 kg ~ twice around the world
- Voyager - only aircraft to ever circle the world non-stop, non-refueled - *once!*



Orbital Entry - The Physics

- 32 MJ/kg dissipated by friction with atmosphere over ~8 min = 66kW/kg
- Pure graphite (carbon) high-temperature material: $c_p=709 \text{ J/kg}^\circ\text{K}$
- Orbital energy would cause temperature gain of 45,000°K!



The Vision

"Once you make it to low Earth orbit, you're halfway to anywhere!"

- Robert A. Heinlein



How To Approach Teaching This Course

- As a system user
- As a system provider
- As a venture capitalist
- As a system designer



Goals of ENAE 791

- Learn the underlying physics (orbital mechanics, flight mechanics, aerothermodynamics) which constrain and define launch and entry vehicles
- Develop the tools for preliminary design synthesis, including the fundamentals of systems analysis
- Provide an introduction to engineering economics, with a focus on the parameters affecting cost of launch and entry vehicles, such as reusability
- Examine specific challenges in the underlying design disciplines, such as thermal protection and structural dynamics



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Course Overview/Orbital Mechanics
Launch and Entry Vehicle Design

Web-based Course Content

- Data web site at spacecraft.ssl.umd.edu
 - Course information
 - Syllabus
 - Lecture notes
 - Problems and solutions
- Interactive web site at www.ajconline.umd.edu
 - Communications for team projects
 - Surveys for course feedback



Syllabus Overview (1)

- Fundamentals of Launch and Entry Design
 - Orbital mechanics
 - Basic rocket performance
- Entry flight mechanics
 - Ballistic entry
 - Lifting entry
- Aerothermodynamics
- Thermal Protection System (TPS) analysis
- Entry, Descent, and Landing (EDL) systems



Syllabus Overview (2)

- Launch flight mechanics
 - Gravity turn
 - Targeted trajectories
 - Optimal trajectories
 - Airbreathing trajectories
- Launch vehicle systems
 - Propulsion systems
 - Structures and structural dynamics analysis
 - Payload accommodations



Syllabus Overview (3)

- Systems Analysis
 - Cost estimation
 - Engineering economics
 - Reliability issues
 - Safety design concerns
 - Fleet resiliency
- Case studies
- Design project



Policies

- Grade Distribution
 - 20% Problems
 - 25% Midterm Exam
 - 25% Term Project*
 - 30% Final Exam

- Late Policy

- On time: Full credit
- Before solutions: 70% credit
- After solutions: 20% credit

* Team Grades



Term Project

- Teams of ~4 people (you pick)
- Given a mission model (mass per year), design a launch system which minimizes cost/kg of payload to LEO over the life of the project
- Detailed requirements forthcoming
- Design process should proceed throughout the term
- Formal design presentations at end of term



Orbital Mechanics: 500 years in 40 min.

- Newton's Law of Universal Gravitation

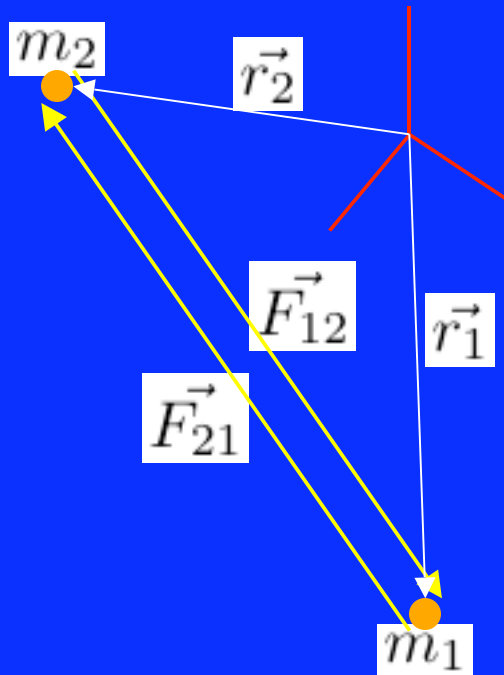
$$F = \frac{Gm_1m_2}{r^2}$$

- Newton's First Law meets vector algebra

$$\vec{F} = m\vec{a}$$



Relative Motion Between Two Bodies



$$m_1 \frac{d^2 \vec{r}_1}{dt^2} = G \frac{m_1 m_2}{|\vec{r}_{21}|^2} \frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

$$= G \frac{m_1 m_2}{|\vec{r}_{21}|^3} \vec{r}_{21} = G \frac{m_1 m_2}{|\vec{r}_{21}|^3} (\vec{r}_2 - \vec{r}_1)$$

$$m_2 \frac{d^2 \vec{r}_2}{dt^2} = G \frac{m_1 m_2}{|\vec{r}_{12}|^3} (\vec{r}_1 - \vec{r}_2)$$

\underline{F}_{12} = force due to body 1 on body 2



Gravitational Motion

$$\frac{d^2 \vec{r}}{dt^2} = \frac{G}{r^3} [m_2 (-\vec{r}) - m_1 (\vec{r})] = \frac{-G}{r^3} (m_1 + m_2) \vec{r}$$

$$\text{Let } r = |\vec{r}_{12}| = |\vec{r}_{21}| \quad \text{Let } \vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\frac{d^2 \vec{r}}{dt^2} + \mu \frac{\vec{r}}{r^3} = \vec{0}$$

“Equation of Orbit” -

Orbital motion is simple harmonic motion



Orbital Angular Momentum

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \frac{d\vec{v}}{dt} + \mu \frac{\vec{r}}{r^3} = \vec{0}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} + \frac{\mu}{r^3} (\vec{r} \times \vec{r}) = \vec{0}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt} (\vec{r} \times \vec{v}) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}$$

$$= \vec{v} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt} (\vec{r} \times \vec{v}) = \vec{0}$$

$$\vec{r} \times \vec{v} = \text{constant}$$

$$\vec{r} \times \vec{v} = \vec{h}$$

h is angular momentum vector - r and v planar



Fun and Games with Algebra

$$\frac{d\vec{v}}{dt} + \mu \frac{\vec{r}}{r^3} = \vec{0}$$

$$\frac{d\vec{v}}{dt} \times \vec{h} + \frac{\mu}{r^3} (\vec{r} \times \vec{h}) = \vec{0}$$

$$\frac{d}{dt} (\vec{v} \times \vec{h}) = \frac{d\vec{v}}{dt} \times \vec{h} + \vec{v} \times \frac{d\vec{h}}{dt}$$

$$\frac{d}{dt} (\vec{v} \times \vec{h}) = -\frac{\mu}{r^3} (\vec{r} \times \vec{h}) = -\frac{\mu}{r^3} (\vec{r} \times \vec{r} \times \vec{v})$$

$$\frac{d}{dt} (\vec{v} \times \vec{h}) = -\frac{\mu}{r^3} [(\vec{r} \cdot \vec{v}) \vec{r} - (\vec{r} \cdot \vec{r}) \vec{v}]$$

$$\vec{r} \cdot \vec{v} = rv \cos \gamma = r \frac{dr}{dt}$$



$$\frac{d}{dt} \left(\vec{v} \times \vec{h} \right) = -\frac{\mu}{r^3} \left[r \frac{dr}{dt} \vec{r} - r^2 \frac{d\vec{r}}{dt} \right]$$

$$\frac{d}{dt} \left(\frac{\vec{r}}{r} \right) = \frac{\left(r \frac{d\vec{r}}{dt} - \vec{r} \frac{dr}{dt} \right)}{r^2} = \left(\frac{1}{r} \frac{d\vec{r}}{dt} - \frac{\vec{r}}{r^2} \frac{dr}{dt} \right)$$

$$\frac{d}{dt} \left(\vec{v} \times \vec{h} \right) = -\mu \left(\frac{1}{r^2} \frac{dr}{dt} \vec{r} - \frac{1}{r} \frac{d\vec{r}}{dt} \right) = \mu \frac{d}{dt} \left(\frac{\vec{r}}{r} \right)$$

$$\frac{d}{dt} \left(\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} \right) = \vec{0}$$



Orientation of the Orbit

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \text{constant}$$

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \mu \vec{e}$$

$\vec{e} \equiv$ eccentricity vector, in orbital plane

e points in the direction of periapsis

$$\vec{r} \cdot \vec{v} \times \vec{h} - \vec{r} \cdot \mu \frac{\vec{r}}{r} = \mu (\vec{r} \cdot \vec{e})$$

$$\vec{r} \times \vec{v} \cdot \vec{h} - \mu \frac{\vec{r} \cdot \vec{r}}{r} = \mu r e \cos \theta$$

$$\vec{h} \cdot \vec{h} - \mu \frac{r^2}{r} = \mu r e \cos \theta$$



Position in Orbit

$$h^2 - \mu r = \mu r e \cos \theta$$

$$r = \frac{h^2 / \mu}{1 + e \cos \theta}$$

θ = true anomaly: angular travel from perigee passage

$$\text{at } \theta = \pm \frac{\pi}{2}; \cos \theta = 0; r = p \equiv h^2 / \mu$$



$$\mu \vec{e} = \vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r}$$

$$\mu \vec{e} \cdot \mu \vec{e} = \vec{v} \times \vec{h} \cdot \vec{v} \times \vec{h} - 2\mu \left(\vec{v} \times \vec{h} \right) \cdot \frac{\vec{r}}{r} + \mu^2 \left(\frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r} \right)$$

$$\mu^2 e^2 = v^2 h^2 - 2\mu \frac{h^2}{r} + \mu^2$$

$$e^2 = \frac{v^2}{\mu} p - 2 \frac{p}{r} + 1$$

$$p \equiv a(1 - e^2) = \frac{1 - e^2}{\frac{2}{r} - \frac{v^2}{\mu}}$$



Vis-Viva Equation

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

$$-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$



Energy in Orbit

- Kinetic Energy

$$K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}$$

- Potential Energy

$$P.E. = -\frac{m\mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}$$

- Total Energy

$$Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

<--Vis-Viva Equation



Implications of Vis-Viva

- Circular orbit ($r=a$)

$$v_{circular} = \sqrt{\frac{\mu}{r}}$$

- Parabolic escape orbit (a tends to infinity)

$$v_{escape} = \sqrt{\frac{2\mu}{r}}$$

- Relationship between circular and parabolic orbits

$$v_{escape} = \sqrt{2}v_{circular}$$

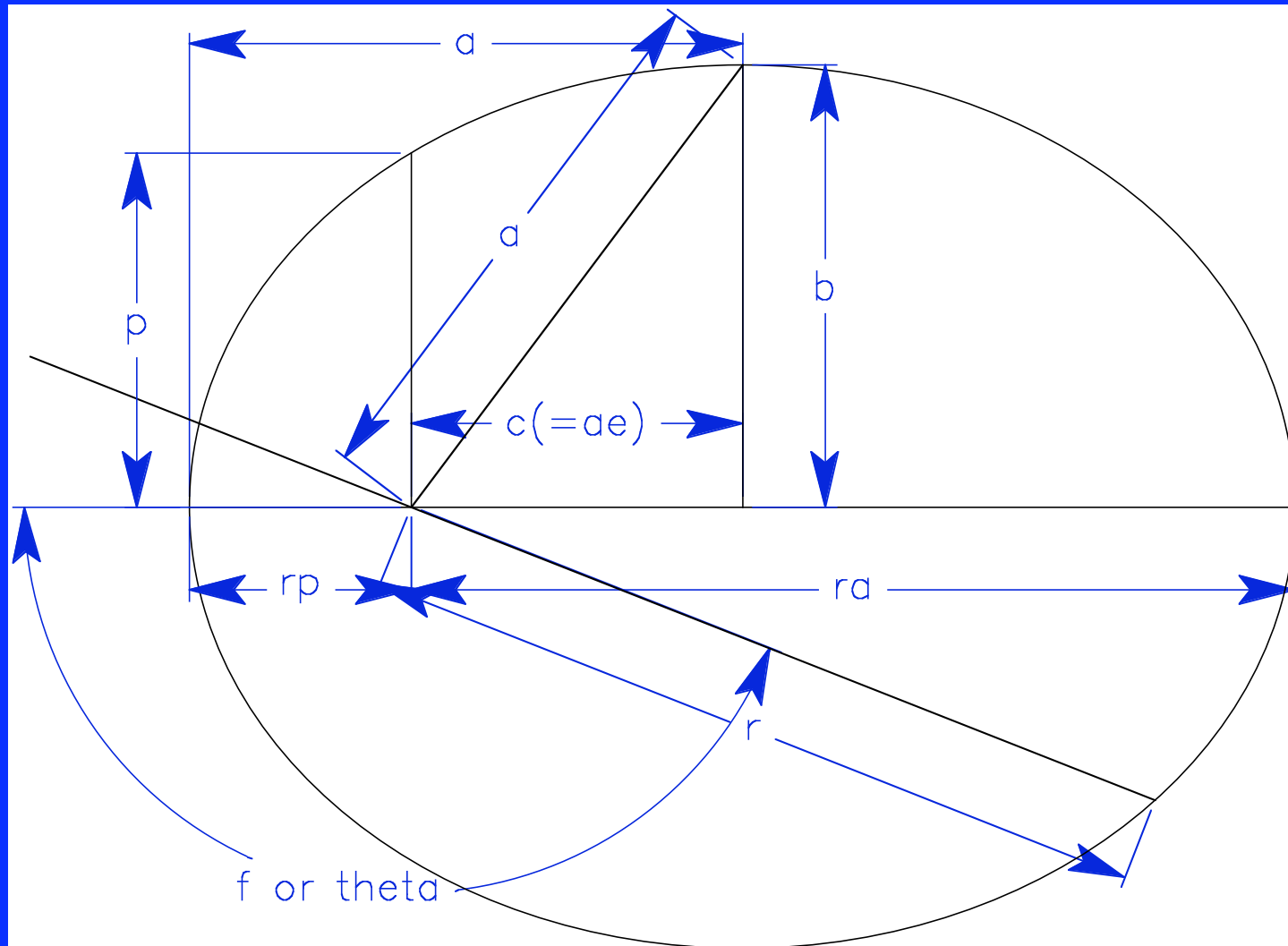


Some Useful Constants

- Gravitation constant $\mu = GM$
 - Earth: $398,604 \text{ km}^3/\text{sec}^2$
 - Moon: $4667.9 \text{ km}^3/\text{sec}^2$
 - Mars: $42,970 \text{ km}^3/\text{sec}^2$
 - Sun: $1.327 \times 10^{11} \text{ km}^3/\text{sec}^2$
- Planetary radii
 - $r_{\text{Earth}} = 6378 \text{ km}$
 - $r_{\text{Moon}} = 1738 \text{ km}$
 - $r_{\text{Mars}} = 3393 \text{ km}$



Classical Parameters of Elliptical Orbits



Basic Orbital Parameters

- Semi-latus rectum (or parameter)

$$p = a(1 - e^2)$$

- Radial distance as function of orbital position

$$r = \frac{p}{1 + e \cos \theta}$$

- Periapse and apoapse distances

$$r_p = a(1 - e)$$

$$r_a = a(1 + e)$$

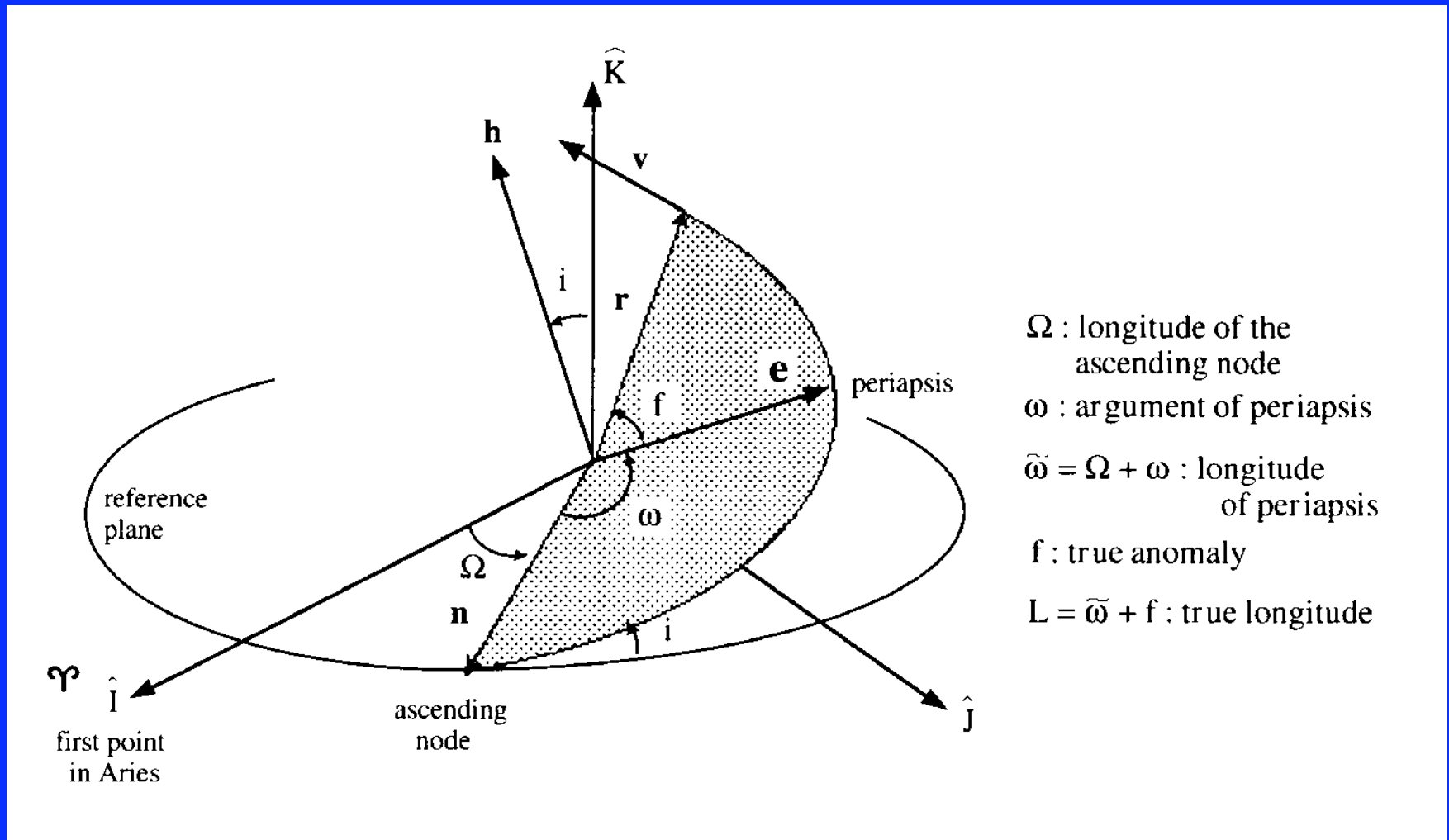
- Angular momentum

$$\dot{\mathbf{h}} = \mathbf{r} \times \dot{\mathbf{v}}$$

$$h = \sqrt{\mu p}$$



The Classical Orbital Elements



- Ω : longitude of the ascending node
- ω : argument of periapsis
- $\tilde{\omega} = \Omega + \omega$: longitude of periapsis
- f : true anomaly
- $L = \tilde{\omega} + f$: true longitude

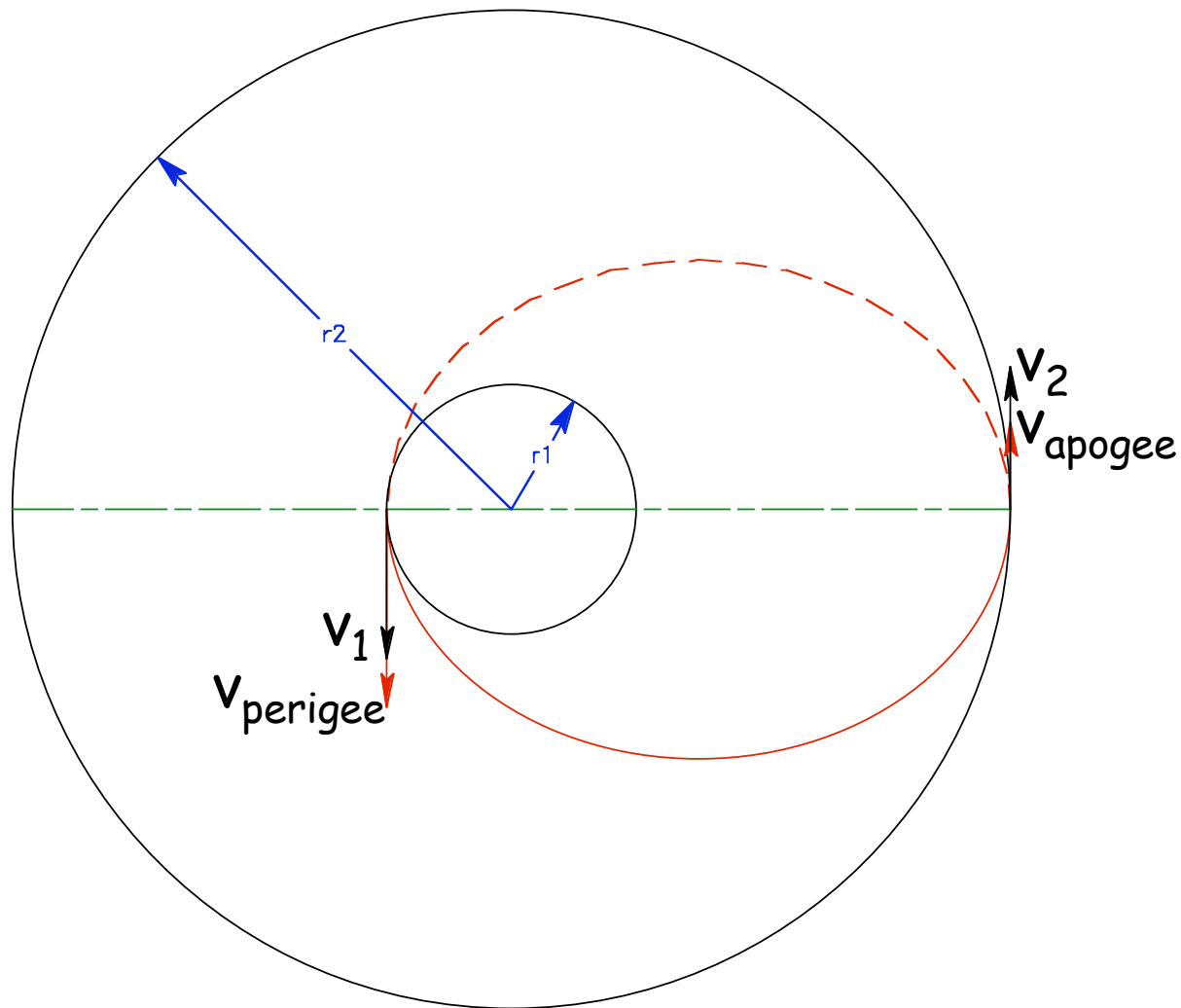
Ref: J. E. Prussing and B. A. Conway, *Orbital Mechanics* Oxford University Press, 1993



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Launch and Entry Vehicle Design

The Hohmann Transfer



First Maneuver Velocities

- Initial vehicle velocity

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

- Needed final velocity

$$v_{perigee} = \sqrt{\frac{\mu}{r_1}} \sqrt{\frac{2r_2}{r_1 + r_2}}$$

- Delta-V

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right)$$



Second Maneuver Velocities

- Initial vehicle velocity

$$v_{apogee} = \sqrt{\frac{\mu}{r_2}} \sqrt{\frac{2r_1}{r_1 + r_2}}$$

- Needed final velocity

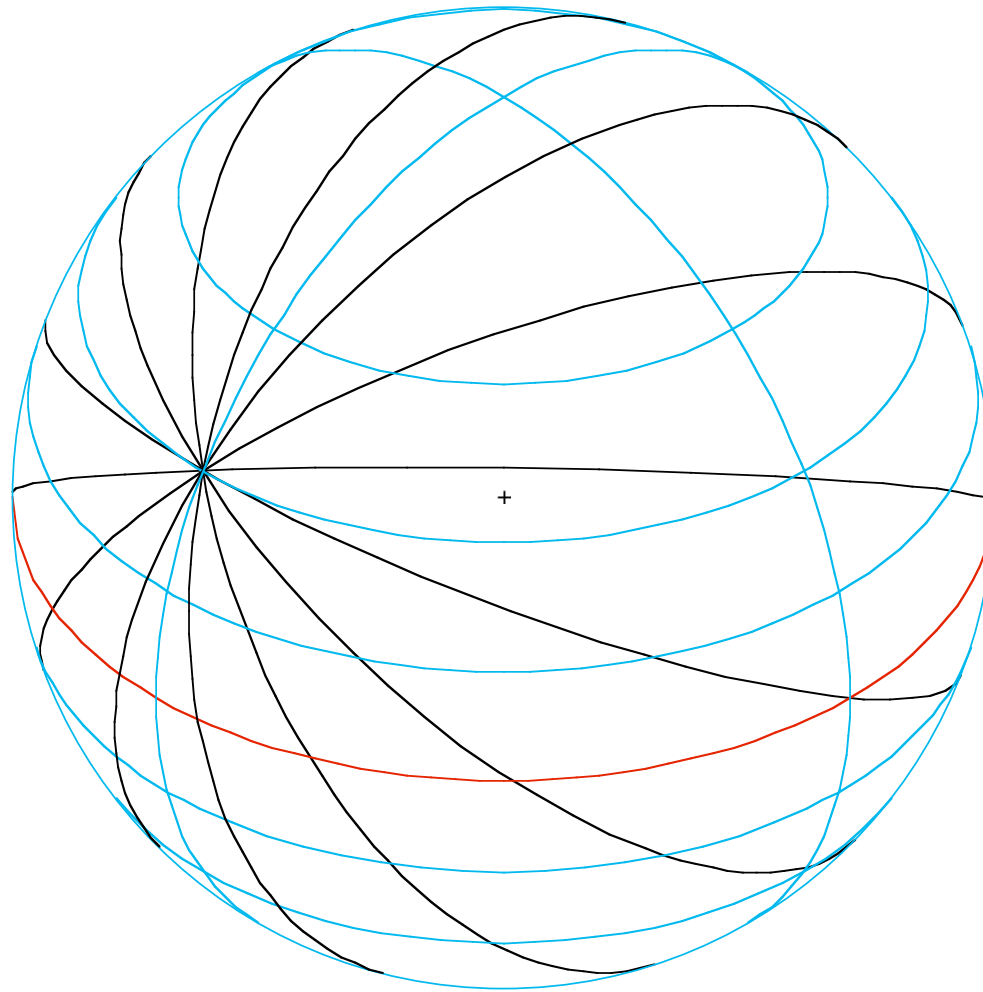
$$v_2 = \sqrt{\frac{\mu}{r_2}}$$

- Delta-V

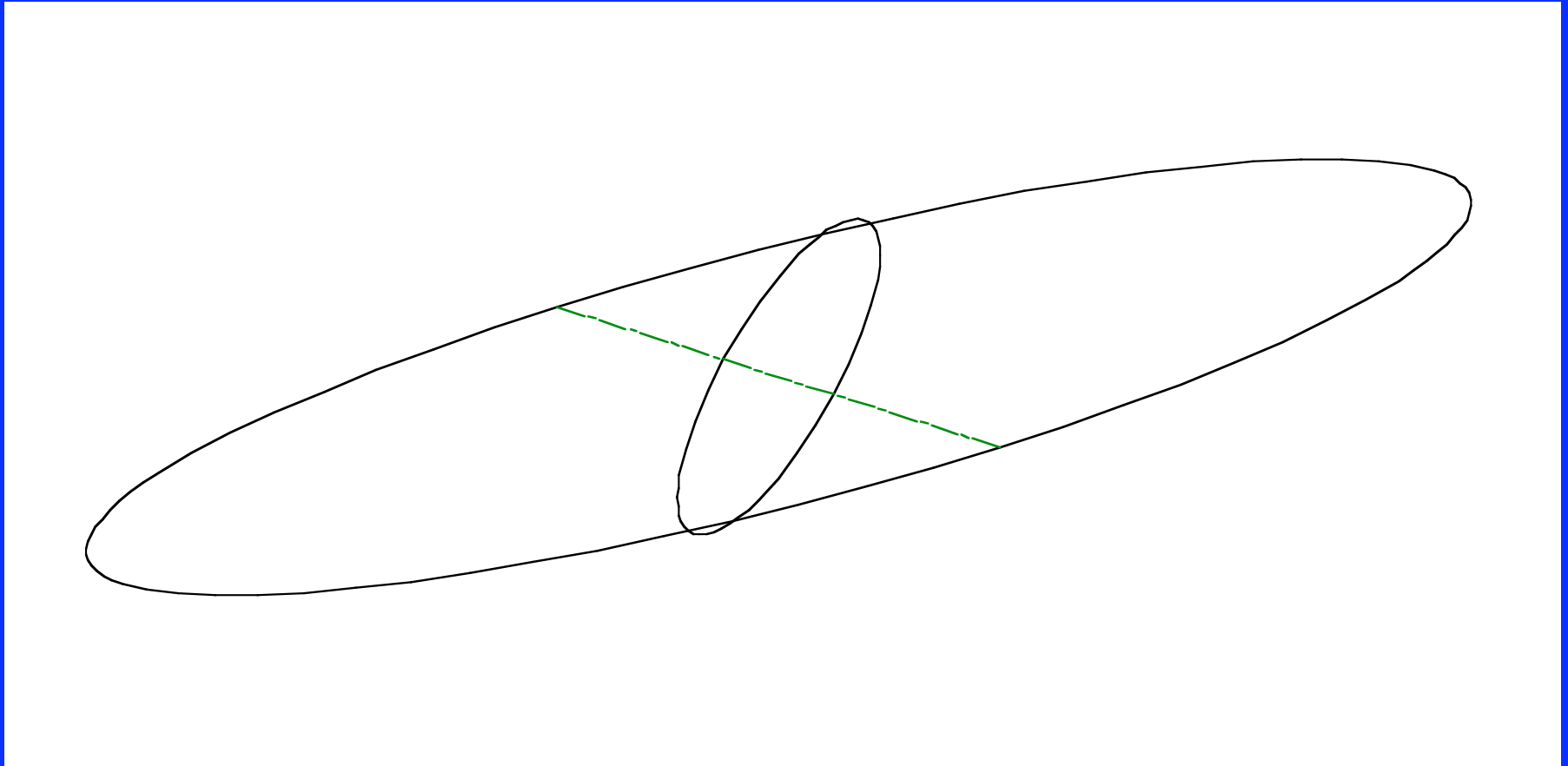
$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right)$$



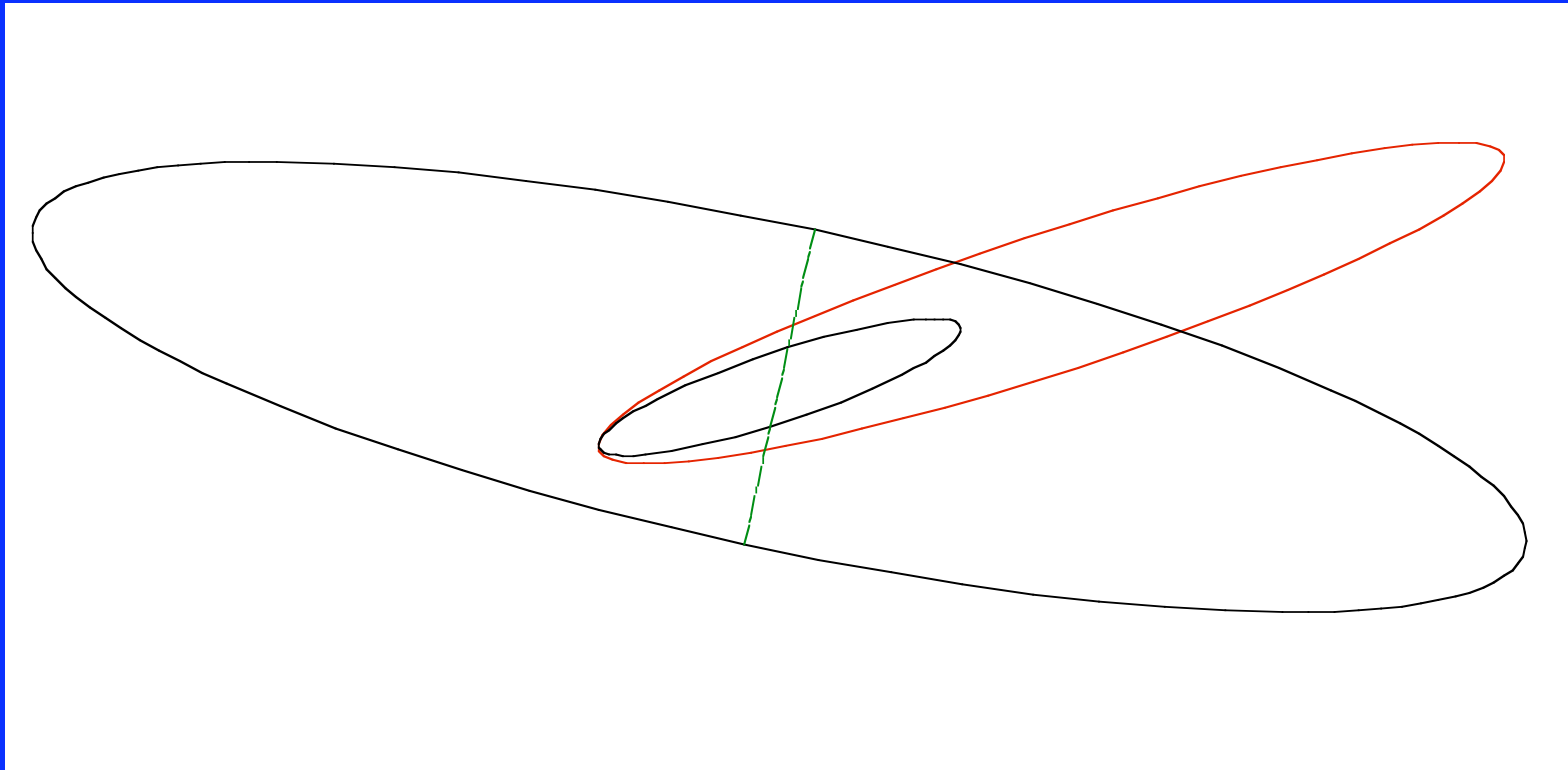
Limitations on Launch Inclinations



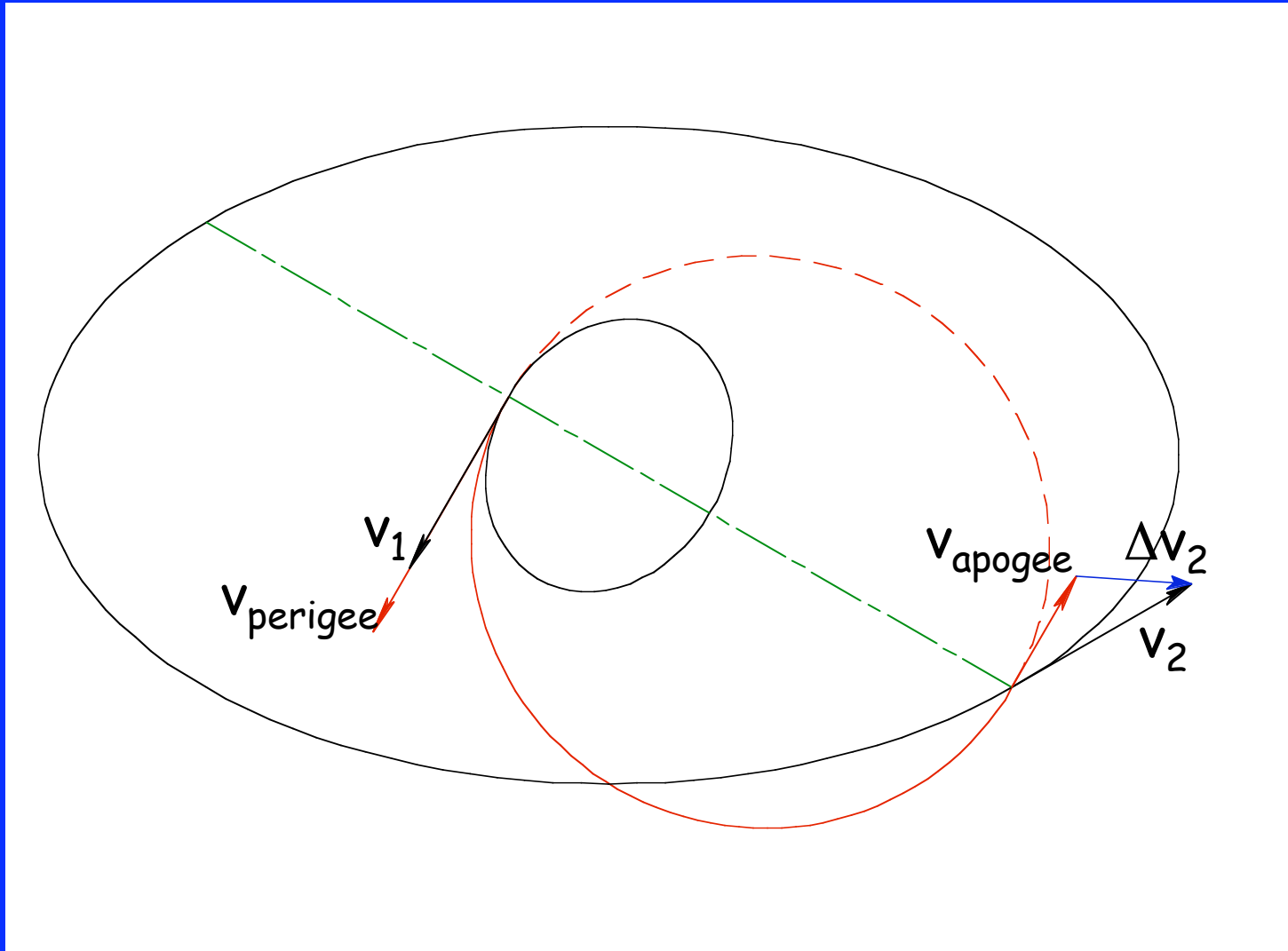
Differences in Inclination



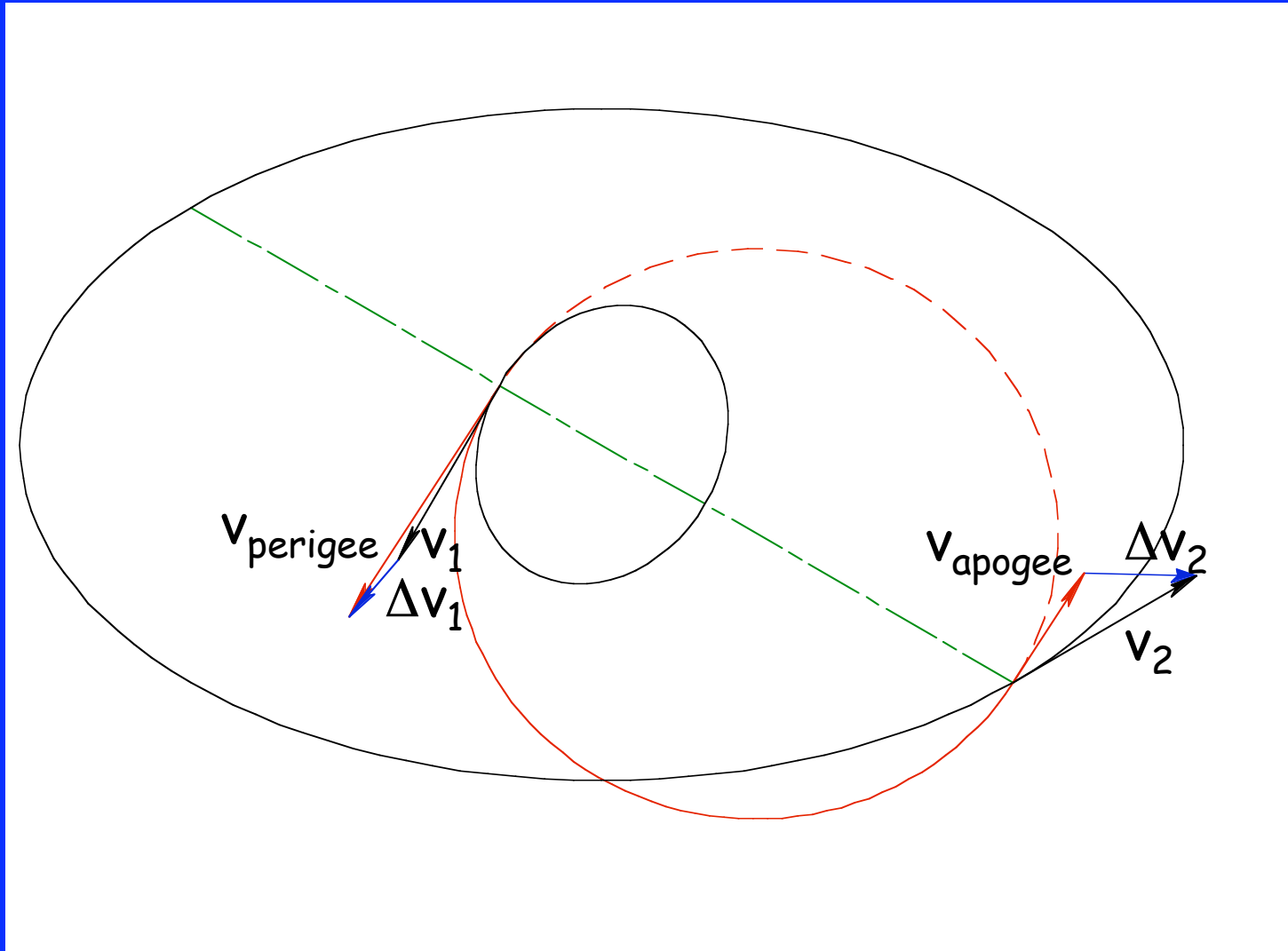
Choosing the Wrong Line of Apsides



Simple Plane Change



Optimal Plane Change



First Maneuver with Plane Change Δi_1

- Initial vehicle velocity

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

- Needed final velocity

$$v_p = \sqrt{\frac{\mu}{r_1}} \sqrt{\frac{2r_2}{r_1 + r_2}}$$

- Delta-V

$$\Delta v_1 = \sqrt{v_1^2 + v_p^2 - 2v_1v_p \cos(\Delta i_1)}$$



Second Maneuver with Plane Change Δi_2

- Initial vehicle velocity

$$v_a = \sqrt{\frac{\mu}{r_2}} \sqrt{\frac{2r_1}{r_1 + r_2}}$$

- Needed final velocity

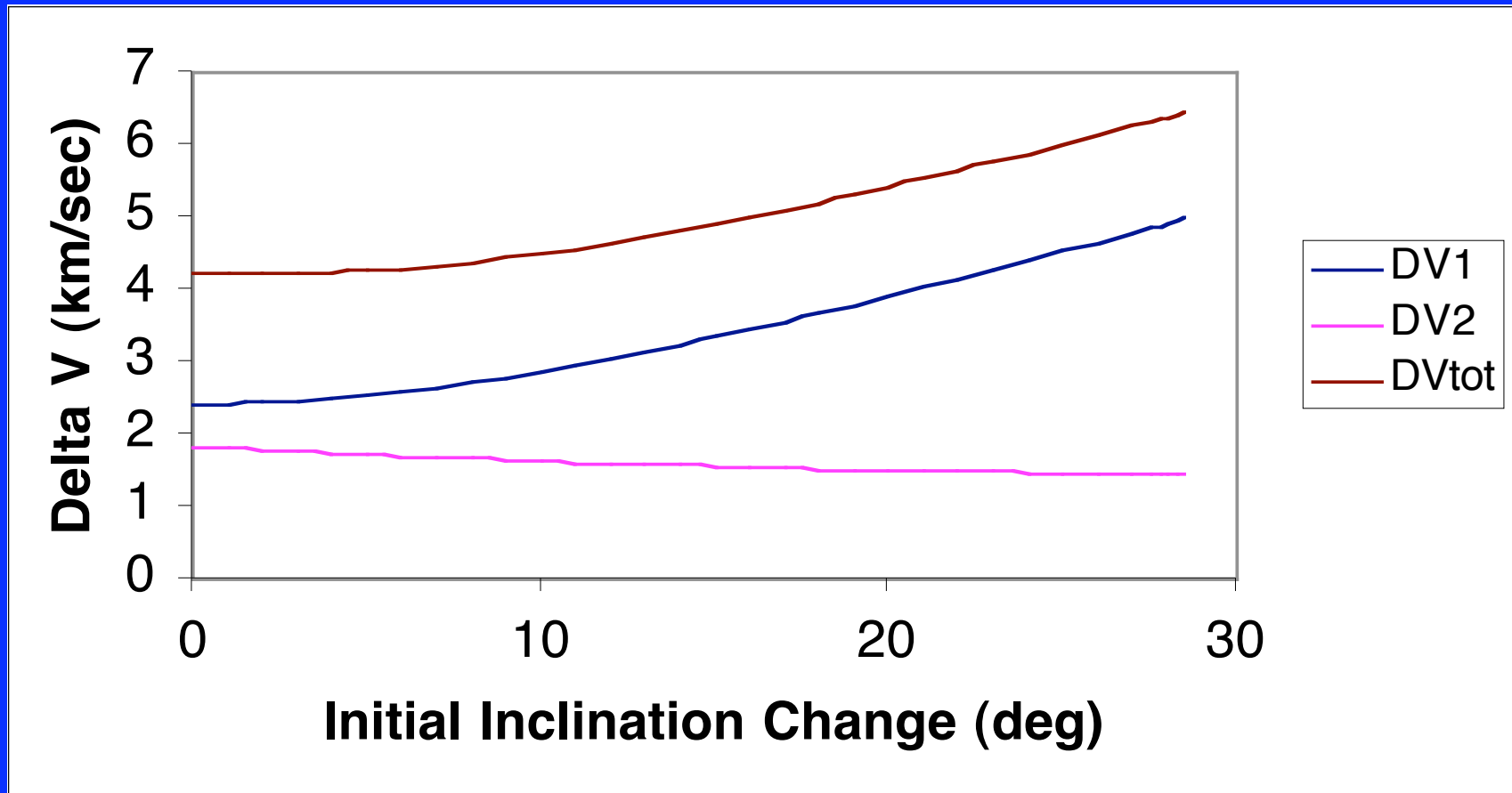
$$v_2 = \sqrt{\frac{\mu}{r_2}}$$

- Delta-V

$$\Delta v_2 = \sqrt{v_2^2 + v_a^2 - 2v_2v_a \cos(\Delta i_2)}$$



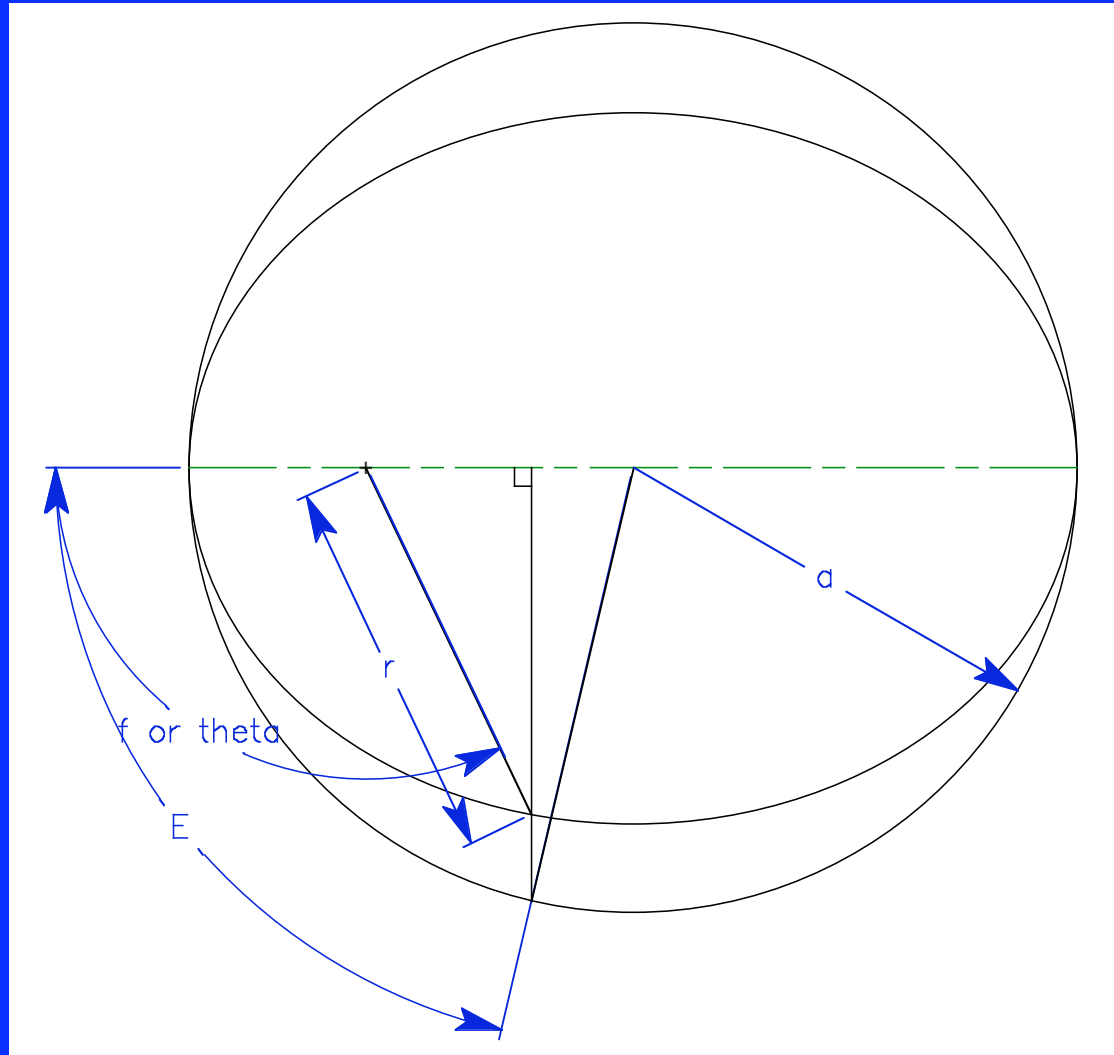
Sample Plane Change Maneuver



Optimum initial plane change = 2.20°



Calculating Time in Orbit



Time in Orbit

- Period of an orbit

$$P = 2\pi \sqrt{\frac{a^3}{\mu}}$$

- Mean motion (average angular velocity)

$$n = \sqrt{\frac{\mu}{a^3}}$$

- Time since pericenter passage

$$M = nt = E - e \sin E$$

↳ M = mean anomaly



Dealing with the Eccentric Anomaly

- Relationship to orbit

$$r = a(1 - e \cos E)$$

- Relationship to true anomaly

$$\tan \frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

- Calculating M from time interval: iterate

$$E_{i+1} = nt + e \sin E_i$$

until it converges

