Atmospheric Decay 2/10/04

Orbital energy $\equiv E = -\frac{\mu}{2a}$ $\qquad \frac{\partial E}{\partial t} = \frac{\mu}{2a^2}\frac{da}{dt}$

$$\frac{\partial E}{\partial t}\text{ due to drag} = \frac{F \cdot \text{distance}}{\text{time}} = (\text{Force})(\text{velocity}) = -V a_d$$

$$\frac{\partial E}{\partial t} = -V\frac{\rho V^2}{2\beta} = -\frac{\rho V^3}{2\beta} = -\frac{\rho}{2\beta}\left(\frac{\mu}{a}\right)^{3/2}$$

$$\frac{\mu}{2a^2}\frac{da}{dt} = -\frac{\rho}{2\beta}\left(\frac{\mu}{a}\right)^{3/2} \qquad \frac{da}{dt} = -\frac{\rho}{\cancel{2}\beta}\frac{\mu^{3/2}}{\mu}\frac{\cancel{2}a^2}{a^{3/2}}$$

$$\frac{da}{dt} = -\frac{\rho}{\beta}\sqrt{\mu a} \qquad \rho = \rho_0 e^{-h/h_s} = \rho_0 e^{-\frac{a-r_E}{h_s}}$$

$$\frac{da}{dt} = -\frac{\sqrt{\mu a}}{\beta}\rho_0 e^{-\frac{a-r_E}{h_s}}$$

Separable →

$$\frac{1}{\sqrt{a}} e^{\frac{a-r_E}{h_s}} da = -\frac{\sqrt{\mu}}{\beta}\rho_0 dt$$

$$\int_{a_0}^{a} \frac{1}{\sqrt{a}} e^{\frac{a-r_E}{h_s}} da = -\frac{\sqrt{\mu}}{\beta}\rho_0 \int_{t_0}^{t} dt$$

$$\int_{h_0}^{h} \frac{e^{h/h_s}}{\sqrt{r_E+h}} dh = -\frac{\sqrt{\mu}}{\beta}\rho_0 (t-t_0) \qquad \sqrt{r_E+h} \sim \sqrt{r_E} \quad (r_E \gg h)$$

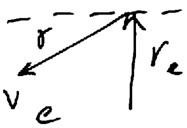
$$\frac{h_s}{\sqrt{r_E}}\left(e^{h/h_s} - e^{h_0/h_s}\right) = -\frac{\sqrt{\mu}}{\beta}\rho_0 (t-t_0)$$

$$\boxed{h(t) = h_s \ln\left[e^{h_0/h_s} - \frac{\sqrt{\mu r_E}}{\beta h_s}\rho_0 (t-t_0)\right]}$$

$$\frac{\sqrt{\frac{km^3}{sec^2} km}\ \frac{kg}{m^3}\ sec}{\frac{kg}{m^2}\ m} \qquad \frac{\frac{km^2}{\cancel{sec}}\ \frac{\cancel{kg}\ \cancel{sec}}{m^3}}{\frac{\cancel{kg}\ m}{\cancel{m^2}}} \qquad \frac{km^2}{m^2}$$

$$e^{\frac{h(t)-h_s}{}} = e^{h_0/h_s} - \frac{\sqrt{\mu r_E}}{\beta h_s}\rho_0 (t-t_0)$$

Deorbit ΔV 040209



Conservation of angular momentum $|\bar{r} \times \bar{v}| = r_e v_e \cos\gamma = r_a v_a$

Conservation of energy $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$ $\quad \frac{1}{a} = \frac{2}{r} - \frac{v^2}{\mu}$

want $v_a = f(r_a, r_e, \gamma)$ eliminate a, v_e

$$v_e^2 = \mu\left(\frac{2}{r_e} - \frac{1}{a}\right) = \mu\left[\frac{2}{r_e} - \left(\frac{2}{r_a} - \frac{v_a^2}{\mu}\right)\right]$$

$$r_e^2 v_e^2 \cos^2\gamma = r_a^2 v_a^2 \qquad v_a^2 = \left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma \, v_e^2$$

$$v_a^2 = \left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma \, \mu \left(\frac{2}{r_e} - \frac{2}{r_a} + \frac{v_a^2}{\mu}\right)$$

$$v_e^2\left[1 - \left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma\right] = \left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma \, \frac{2\mu}{r_a}\left(\frac{r_a}{r_e} - 1\right)$$

$$v_a^2 = \frac{\left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma \, \frac{2\mu}{r_a}\left(\frac{r_a}{r_e} - 1\right)}{\left\{1 - \left(\frac{r_e}{r_a}\right)^2 \cos^2\gamma\right\}}$$

$$v_a = \frac{\cos\gamma \sqrt{\frac{2\mu}{r_a}} \sqrt{\frac{r_e}{r_a} - \left(\frac{r_e}{r_a}\right)^2}}{\sqrt{1 - \left(\frac{r_e}{r_e}\right)^2 \cos^2\gamma}} \qquad \text{let } \frac{r_e}{r_a} = \rho_e \ (<1)$$

$$\Delta v = \sqrt{\frac{\mu}{r_a}} - v_a = \sqrt{\frac{\mu}{r_a}}\left[1 - \frac{\sqrt{2}\cos\gamma\sqrt{\rho - \rho^2}}{\sqrt{1 - \rho^2\cos^2\gamma}}\right]$$

$\gamma_e = 0 \Rightarrow \Delta v = .0807$ km/sec $=$ Hohmann Δv ✓

Ballistic Entry

2/10/04

$$D = \frac{1}{2}\rho V^2 A C_D = \frac{1}{2}\rho_0 \frac{\rho}{\rho_0} V^2 A C_D$$

$$\frac{dV}{dt} = -\frac{D}{m} = \frac{\rho V^2}{2\beta}$$

Separable equation

$$\frac{dV}{V^2} = \frac{\rho}{2\beta} dt$$

Assume: $D \gg mg$ – straight line motion

Also assume exp. atm, flat earth

$$\delta h = V \sin\gamma\, dt \Rightarrow dt = \frac{dh}{V\sin\gamma}$$

$$\frac{dV}{V^2} = \frac{\rho}{2\beta}\frac{dh}{V\sin\gamma} \Rightarrow \frac{dV}{V} = \frac{\rho_0}{2\beta\sin\gamma} e^{-h/h_s} dh$$

$$\int_{V_e}^{V} \frac{dV}{V} = \frac{\rho_0}{2\beta\sin\gamma}\int_{h_e}^{h} e^{-h/h_s} dh$$

$$\ln\frac{V}{V_e} = \frac{\rho_0 h_s}{2\beta\sin\gamma} e^{-h/h_s}\Big|_{h_e}^{h}$$

$$= \frac{\rho_0 h_s}{2\beta\sin\gamma}\left[e^{-h/h_s} - e^{-h_e/h_s}\right]$$

e^{-h_e/h_s} ~~∝~~ air density at entry $\ll e^{-h/h_s}$

$$\frac{V}{V_e} \cong \exp\left[\frac{\rho_0 h_s}{2\beta\sin\gamma} e^{-h/h_s}\right]$$

check: $\frac{\frac{\cancel{kg}}{\cancel{m^3}}\cancel{m}}{\frac{\cancel{kg}}{\cancel{m^2}}}$ = dimensionless ✓

let $B = \frac{\rho_0 h_s}{2\beta \sin\gamma}$ for convenience

$$V = V_e e^{B e^{-h/h_s}}$$

$$\frac{dV}{dt} = V_e B e^{B e^{-h/h_s}} \left(-\frac{e^{-h/h_s}}{h_s}\right) \frac{dh}{dt}$$

$$\frac{dh}{dt} = V \sin\gamma = V_e \sin\gamma \, e^{(B e^{-h/h_s})}$$

$$n \equiv \frac{dV}{dt} = -\frac{B V_e^2}{h_s} \sin\gamma \, e^{-h/h_s} e^{2B e^{-h/h_s}}$$

deceleration

$\gamma < 0 \Rightarrow \sin\gamma < 0 \Rightarrow B < 0 \Rightarrow \frac{dV}{dt} < 0 \Rightarrow n < 0$ (deceleration)

want to know, what's n_{max} and $h(n_{max})$?

$$\frac{dn}{dh} = \frac{d}{dh}\left(\frac{dV}{dt}\right) = B \frac{V_e^2}{h_s^2} \sin\gamma \, e^{-h/h_s} e^{2B e^{-h/h_s}} \underbrace{\left(2 e B e^{-h/h_s} + 1\right)}_{=0 \text{ for min/max}} = 0$$

$$h_{n_{max}} = h_s \ln(-2B) = \boxed{h_s \ln\left(\frac{-\rho_0 h_s}{\beta \sin\gamma}\right)}$$

$f(\beta, \gamma)$ only!

plug this value of h into n equation to get

$$n_{max} = -\frac{B V_e^2}{h_s} \sin\gamma \, e^{-\frac{h_s}{h_s} \ln\left(\frac{-\rho_0 h_s}{\beta \sin\gamma}\right)} \exp\left[2 \frac{\rho_0 h_s}{\beta \sin\gamma} e^{-\ln \frac{-\rho_0 h_s}{\beta \sin\gamma}}\right]$$

and eventually get $n_{max} = \frac{V_e^2 \sin\gamma}{2 h_s e}$ $f(V_e, \gamma)$ only!

Velocity at $\eta_{max} \Rightarrow$

$$V_{\eta_{max}} = V_e\, e^{8e^{-\ln(-28)}} = V_e\, e^{-.5} = .606\, V_e$$

$f(V_e)$ only!