


Introduction to Space Flight

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7-3 BALLISTIC ENTRY

The basic assumption for a ballistic (direct) entry is the absence of lift ($L = L/D = 0$). Additional assumptions and approximations include the neglect of the gravitational (and centrifugal) force during the early initial high-velocity phase of the entry trajectory where the drag force is so much larger than the gravitational force and the E/V is decelerating. Since it is the gravitational force that curves the trajectory as the E/V slows down and approaches the planetary surface, with a flat-Earth model the trajectory during the deceleration phase can be approximated by a straight line with a constant elevation angle ($\phi = \phi_{re}$).

Implicit in the assumption that the lift is zero are the assumptions that the E/V is axisymmetric and that the angle of attack is zero (and remains equal to zero). Since an object in a trajectory maintains its initial attitude with respect to the inertial reference unless subjected to external forces, another implicit assumption is that the attitude of the E/V has somehow been appropriately adjusted prior to entry.

An expression for the drag is that

$$D = \frac{\rho_0 \sigma C_D A V^2}{2} \quad (7-3-1)$$

where ρ_0 is the surface (sea level) atmospheric density, σ the atmospheric density ratio (ρ/ρ_0), and C_D and A the drag coefficient and cross-sectional area of the E/V, respectively.

Applying the assumptions and Eq. (7-3-1), Eq. (7-2-2a) can be written in the form

$$\frac{dV}{dt} = -\frac{C_D A}{m} \frac{\rho_0 \sigma}{2} V^2 \quad (7-3-2)$$

Separating the variables and replacing m by W_0/g_0 (subscript 0 still denotes surface values) leads to

$$\frac{dV}{V^2} = -\frac{C_D A}{W_0} \frac{\rho_0 g_0}{2} \sigma dt \quad (7-3-3)$$

Rewriting Eq. (7-2-3) (with a constant $\phi = \phi_{re}$) as

$$dt = \frac{dh}{V \sin \phi_{re}} \quad (7-3-4)$$

and defining a *ballistic coefficient* (BC), with the dimensions of a pressure (pascal), as

$$BC = \frac{W_0}{C_D A} = \frac{mg_0}{C_D A} \quad (7-3-5)$$

Eq. (7-3-3) can be written as

$$\frac{dV}{V} = -\frac{\rho_0 g_0}{2(BC) \sin \phi_{re}} \sigma dh \quad (7-3-6)$$

To find an expression for the variation of σ , the atmospheric density ratio, with altitude requires some knowledge of the characteristics of the planetary atmosphere of interest. The Earth's atmosphere, for example, is modeled as a series of concentric layers; the exact number of layers defined is dependent on how the model is to be used and can range from three to seven. With respect to the atmospheric density, one useful model comprises four layers: the troposphere, the stratosphere, the ionosphere, and the exosphere. The troposphere and stratosphere are the two layers closest to the surface of the Earth and are of primary interest with respect to entry (and launch ascent); the remaining two layers are of interest with respect to orbital decay and lifetimes.

If the temperature profile in each layer is modeled by a constant temperature or by a constant temperature gradient (lapse rate) and hydrostatic equilibrium is assumed, the combination yields an exponential relationship that is a function of the density ρ , the acceleration due to gravity g , and the temperature T at a reference altitude. For the lower layers, from sea level to approximately 120 km (65 nmi), if g is assumed to be constant and an appropriate average value of T is used, the variation of σ with h can be approximated by the exponential expression

$$\sigma = \frac{\rho}{\rho_0} \cong e^{-\beta h} \quad (7-3-7)$$

where h is the altitude above the planetary surface and $1/\beta$ is the *scale height*. The validity of this exponential approximation decreases at the higher altitudes where the continuum tapers into free-molecule flow. Although it is possible to improve the accuracy of the approximation by using different values of β in certain regions, we shall use only one value of β for all altitudes. Furthermore, even though there is not complete agreement as to the best value of β , its value for the Earth will be taken to be $1/7524 \text{ m}^{-1}$ (0.1378 km^{-1}); typical values of β for Mars and Venus are given in Table 7-3-1.

Making use of Eq. (7-3-7), Eq. (7-3-6) can now be written in a form suitable for integration, namely,

$$\frac{dV}{V} = -\frac{\rho_0 g_0}{2(\text{BC}) \sin \phi_{re}} e^{-\beta h} dh \quad (7-3-8)$$

Before integrating, we need to look at the ballistic coefficient (BC) as defined in Eq. (7-3-5). In the absence of ablation or other mass and area changes, W/A can

TABLE 7-3-1 RELEVANT PLANETARY ATMOSPHERIC DATA

Planet	Radius (km)	Gravity (m/s^2)	Sea-level density (kg/m^3)	Beta (km^{-1})
Venus	6052	8.85	16.02	0.1606
Earth	6378	9.81	1.226	0.1378
Mars	3393	3.73	0.0993	0.0361

be considered to be remain constant over the range of interest, and C_D , the drag coefficient, is essentially constant in the hypersonic velocity regime. Consequently, BC will be assumed to be constant.

Integrating from the entry altitude (h_{re}) to any other altitude on the straight-line trajectory yields the expression

$$\ln \frac{V}{V_{re}} = + \left[\frac{\rho_0 g_0}{2(BC)\beta \sin \phi_{re}} \right] e^{-\beta h} \Big|_{h_{re}}^h \quad (7-3-9)$$

For the sake of simplification, define the bracketed term in Eq. (7-3-9) as B and evaluate the right-hand side to obtain

$$\ln \frac{V}{V_{re}} = B(e^{-\beta h} - e^{-\beta h_{re}}) \quad (7-3-10)$$

which can be written as

$$\frac{V}{V_{re}} = \exp[B(e^{-\beta h} - e^{-\beta h_{re}})] \quad (7-3-11)$$

Now for one last simplification. With $h < h_{re}$, as the E/V descends, the term $\exp(-\beta h)$, which is σ at the altitude of interest, rapidly becomes much larger than $\exp(-\beta h_{re})$, which is σ_{re} , so that the latter term can be neglected. Now the final approximation for the velocity along the entry trajectory can be written as

$$\boxed{V \cong V_{re} e^{B\sigma - \beta h}} \quad (7-3-12)$$

In addition to the assumption made above with respect to the density ratios (σ_{re} and σ), we have made the following assumptions and approximations in deriving Eq. (7-3-12):

1. Neglected the combination of the gravitational and centrifugal terms ($g - V^2/r$) (i.e., assumed a straight-line trajectory).
2. Used an exponential model for the atmospheric density.
3. Assumed $C_D/\sin \phi_{re}$ to be constant.

Let us return and look at the variable B , which is defined as

$$B = \frac{\rho_0 g_0}{2(BC)\beta \sin \phi_{re}} \quad (7-3-13)$$

B is dimensionless and is negative, since the entry angle ϕ_{re} is negative and all other terms are positive. With a specified planet so that β , ρ_0 , and g_0 are specified, the magnitude of B is determined by the BC and by the entry angle (ϕ_{re}), being inversely proportional to both. Increasing the BC or the ϕ_{re} reduces the value of B and thus affects the entry velocity profile.

In comparing E/Vs with the same W/A ratio, the BC is a measure of the streamlining (the slenderness) of the body. For a blunt-body E/V, as sketched in Fig.

7-3-1a, $C_{D\infty}$ is on the order of unity ($C_{D\infty} \cong 1$), where the subscript ∞ denotes the hypersonic region. The qualitative variation of C_D with Mach number is shown in Fig. 7-3-1b. To a first approximation, $C_{D\infty}$ is a function of δ , the *half-cone angle*, as defined in Fig. 7-3-2, with

$$C_{D\infty} \cong 2 \sin^2 \delta \quad (7-3-14)$$

For example, if $\delta = 20^\circ$, $C_{D\infty} \cong 0.234$; if 30° , $C_{D\infty} \cong 0.50$; and if 45° (a blunt body), $C_{D\infty} \cong 1.0$. As the E/V becomes more slender, the half-cone angle decreases as does $C_{D\infty}$, which in turn increases the BC (for a given W/A) and B becomes smaller.

Let us return to Eq. (7-3-12) and look at some velocity profiles for E/V's entering the *Earth's atmosphere*, where $\rho_0 = 1.226 \text{ kg/m}^3$ and $\beta = 1/7254 = 1.378 \times 10^{-4} \text{ m}^{-1}$.

Example 7-3-1

An E/V entering the Earth's atmosphere has a mass of 50 kg (490.5 N or 110 lb) and has a diameter of 3.534 m and a half-cone angle of 45° (a blunt body). $V_e = 8000 \text{ m/s}$ and $\phi_{e,r}$, the parameter of interest, is unspecified at this time.

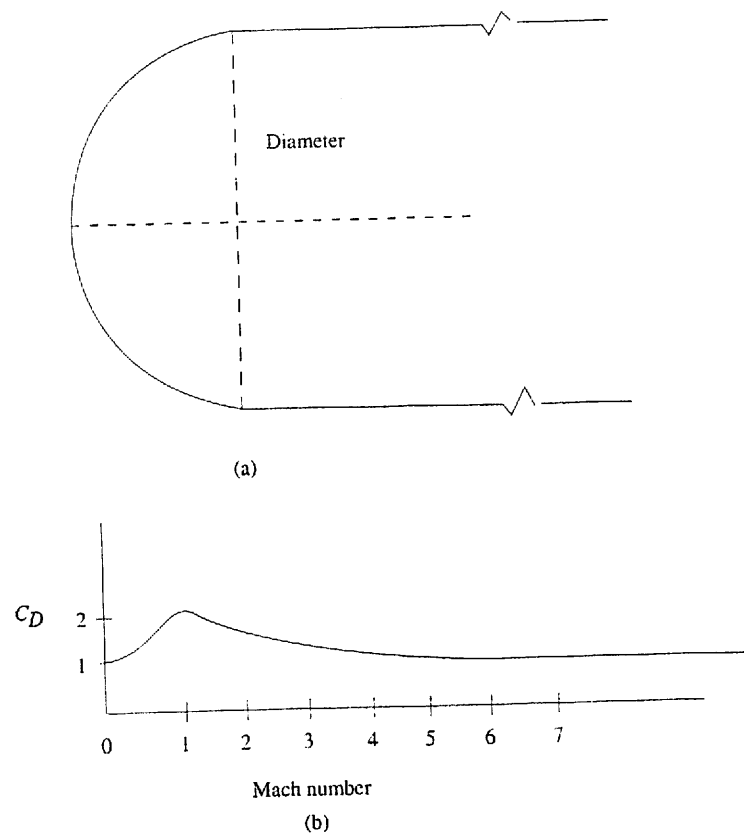


Figure 7-3-1. Blunt-body E/V; (a) configuration; (b) drag coefficient.

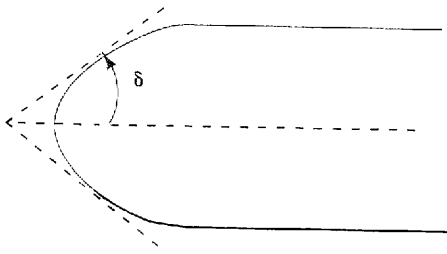


Figure 7-3-2. E/V half-cone angle.

- (a) Find the value of the ballistic coefficient (BC).
- (b) If $\phi_{re} = -22^\circ$, find the velocity (and V/V_{re}) at an altitude of 50 km.
- (c) Increase ϕ_{re} to -45° (steepen the entry) and determine the effect on the velocity at 50 km.

Solution (a) Note that with $V_{re0} = 7905$ m/s, $V_{re}^* = V_{re}/V_{re0} = 1.012$, indicating entry from a low Earth orbit.

$$C_{D\infty} = 2 \sin^2(45^\circ) = 1.0$$

The cross-sectional area is

$$A = \pi r^2 = 9.81 \text{ m}^2$$

and

$$\frac{W}{A} = 50 \text{ N/m}^2 = 50 \text{ Pa}$$

Therefore, the ballistic coefficient is

$$BC = \frac{W/A}{C_{D\infty}} = \frac{50}{1} = 50 \text{ N/m}^2 = 50 \text{ Pa}$$

- (b) With $\phi_{re} = -22^\circ$ and $BC = 50 \text{ Pa}$ (1.14 lb/ft^2),

$$B = \frac{1.226 \times 9.81 \times 7254}{2 \times 50 \times \sin(-22^\circ)} = -2328$$

and at 50,000 m,

$$V = 8000e^{-2328e^{-150,000/7254}} = 752.9 \text{ m/s}$$

so that

$$\frac{V}{V_{re}} = 0.094$$

We see that the most of the deceleration seems to have taken place before the E/V reached 50 km.

- (c) With ϕ_{re} increased to -45° but with the BC remaining at 50 Pa, B decreases to -1233 and V at 50 km is 2288 m/s ($V/V_{re} = 0.286$). We see that with the steeper entry angle the E/V is traveling faster when it reaches 50 km and is still decelerating.

Example 7-3-2

The BC of the E/V of Example 7-3-2 has been increased to 94.4 Pa (1.97 lb/ft^2), but V_{re} is still 8000 m/s.

- (a) Find the velocity at 50 km with $\phi_{re} = -22^\circ$ and compare the answer with that of Example 3-7-1(c).
 (b) Find the velocity at 50 km with $\phi_{re} = -45^\circ$ and compare with that of Example 7-3-1(c).

Solution

$$(a) \quad B = \frac{1.226 \times 9.81 \times 7254}{2 \times 94.4 \times \sin(-22^\circ)} = -1234$$

Substituting into Eq. (7-3-12) gives a $V/V_{re} = 0.2860$ and $V = 2288$ m/s. These values are identical with those found in the previous example for an E/V with BC = 50 Pa and $\phi_{re} = -45^\circ$. Increasing the BC has the same effect on the velocity as increasing the entry angle.

(b) With $\phi_{re} = -45^\circ$, B decreases, becoming -653.7 , and V/V_{re} and V at 50 km increase to 0.515 and 4120 m/s.

In these two examples, we see that increasing either the entry angle or the ballistic coefficient decreases B , thus increasing the value of V at a specified altitude. The implication is that the E/V with the higher BC or ϕ_{re} does not slow down as quickly and continues its deceleration to lower altitudes. Because B is in essence a similarity parameter, changes in BC and ϕ_{re} result in equivalent changes in B .

Since B , which is not a physical variable, is inversely proportional to the product of BC and $\sin \phi_{re}$, it is not unusual to see the negative of this product defined as the *ballistic factor*¹ and used in lieu of the BC. The negative of the variable B itself could be used as a *planetary ballistic factor* inasmuch as it includes ρ_0 and g_0 as well as the E/V characteristics and entry values. We shall, however, use only the ballistic coefficient BC along with the entry angle ϕ_{re} , keeping them separate, since the BC represents the E/V characteristics and ϕ_{re} is a characteristic of the entry trajectory.

It is interesting to plot the nondimensionalized velocity (V/V_{re}) as a function of the altitude for various values of BC for a specified value of the entry angle, as is done in Fig. 7-3-3 for $\phi_{re} = -22^\circ$. Notice that as the BC ($W/C_D A$) is increased, the E/V penetrates deeper into the atmosphere before it starts to slow down (decelerate) and that with BC = 50,000 Pa, the E/V is still decelerating as it hits the surface. *Increasing the entry angle has the same effect as increasing the BC.*

Let us now examine the deceleration ($-dV/dt$) along an entry trajectory. Returning to Eq. (7-3-12) and differentiating with respect to time yields

$$\frac{dV}{dt} = V_{re} B (e^{\beta e^{-\beta h}}) (-\beta e^{-\beta h}) \frac{dh}{dt} \quad (7-3-15)$$

But

$$\frac{dh}{dt} = V \sin \phi_{re} = V_{re} \sin \phi_{re} e^{\beta e^{-\beta h}} \quad (7-3-16)$$

¹To add to the confusion, at times the ballistic coefficient, as defined in this book, may be called the ballistic factor.

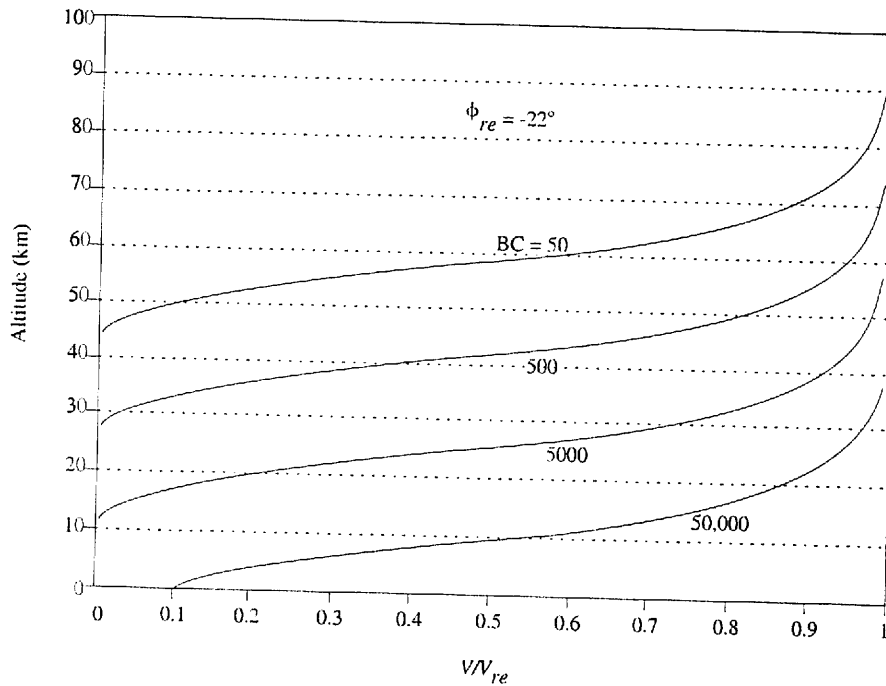


Figure 7-3-3. Altitude and velocity along a reentry trajectory for several values of the BC with $\phi_{re} = -22^\circ$.

so that Eq. (7-3-15) becomes

$$\frac{dV}{dt} = -\beta B V_{re}^2 \sin \phi_{re} e^{-\beta h} e^{2Be^{-\beta h}} \quad (7-3-17)$$

In Eq. (7-3-17), both B and $\sin \phi_{re}$ are negative and therefore dV/dt will be negative, showing that the E/V is decelerating, as is to be expected. The units of dV/dt are m/s^2 ; it is a common practice to describe accelerations in terms of Earth-surface g_0 using the symbol n , where $n = (dV/dt)/g_0$.

Equation (7-3-17) is somewhat complicated but at this point we are primarily interested in the magnitude of the maximum deceleration (n_{max}) and the altitude at which it occurs. Setting the derivative of dV/dt with respect to h equal to zero results in

$$\frac{d}{dh} \left(\frac{dV}{dt} \right) = B\beta^2 V_{re}^2 \sin \phi_{re} e^{-\beta h} e^{2Be^{-\beta h}} (2eBe^{-\beta h} + 1) = 0 \quad (7-3-18)$$

This is also a complicated equation but fortunately, we only need to set the terms within the parentheses equal to zero to satisfy this equation and the maxima-minima condition. Doing so and solving for h , the altitude where the maximum deceleration occurs, yields

$$h_{n_{\max}} = \frac{\ln(-2B)}{\beta} = \frac{1}{\beta} \ln \left[-\frac{\rho_0 g_0}{(BC)\beta \sin \phi_{re}} \right] \quad (7-3-19)$$

It is interesting to see in Eq. (7-3-19) that the altitude for n_{\max} is independent of the magnitude of V_{re} and is a function only of the ballistic coefficient and the entry angle.

Example 7-3-3

The E/V of Example 7-3-2 has $BC = 94.4 \text{ Pa}$ (1.97 lb/ft^2) and $V_{re} = 8000 \text{ ft/s}$.

- Find the altitude at which the deceleration reaches its maximum when $\phi_{re} = -22^\circ$.
- Do part (a) for a steeper entry angle, $\phi_{re} = -45^\circ$ and compare with part (a).
- Increase the BC to 5000 Pa (104.5 lb/ft^2) and compare the altitude for n_{\max} (with $\phi_{re} = -22^\circ$) with that of (a).
- If V_{re} is increased to $11,000 \text{ m/s}$, will the altitudes change?

Solution

$$(a) \quad B = \frac{1.226 \times 9.81 \times 7254}{2 \times 94.4 \times \sin(-22^\circ)} = -1234$$

and

$$h_{n_{\max}} = 7254 \ln[-2(-1234)] = 56,660 \text{ m} = 56.66 \text{ km}$$

(b) With $\phi_{re} = -45^\circ$, the magnitude of B decreases to -653.7 , and h for n_{\max} also decreases from $56,660 \text{ m}$ ($185,800 \text{ ft}$) to $52,050 \text{ m}$ ($170,700 \text{ ft}$), a decrease of 4610 m ($15,100 \text{ ft}$).

(c) With $BC = 5000 \text{ Pa}$ (104.4 lb/ft^2) and $\phi_{re} = -22^\circ$, B is equal to -23.30 and h for n_{\max} becomes $27,800 \text{ m}$ ($91,360 \text{ ft}$), which is on the order of half of the altitude found with the much lower value of BC (94.4 Pa).

(d) Changing V_{re} will not affect any of the altitudes found above.

Figure 7-3-4 shows the maximum deceleration altitude as a function of the ballistic coefficient (in kPa) for several entry angles. Notice the knee in the curves as BC increases as well as the decreased effect of increasing the entry angle for the larger values. If the BC were plotted on a logarithmic scale, the altitude curves would be straight lines.

Returning to Eq. (7-3-17), the expression for dV/dt , substituting the condition for the altitude for n_{\max} [$\exp(-\beta h) = -1/2B$] yields an expression for n_{\max} , namely,

$$n_{\max} = \frac{1}{g_0} \frac{dV}{dt} = \frac{\beta V_{re}^2 \sin \phi_{re}}{2eg_0} \quad (7-3-20)$$

where $e = 2.718$, the natural logarithm base. It is very interesting to note here that the maximum deceleration is independent of the ballistic coefficient and is a function only of the entry angle and the square of the entry velocity.

To find the velocity at n_{\max} , the condition for the altitude at that point is substituted into Eq. (7-3-12), the velocity equation, to obtain

$$V_{n_{\max}} = V_{re} e^{Be^{-\ln(-2B)}} \quad (7-3-21)$$

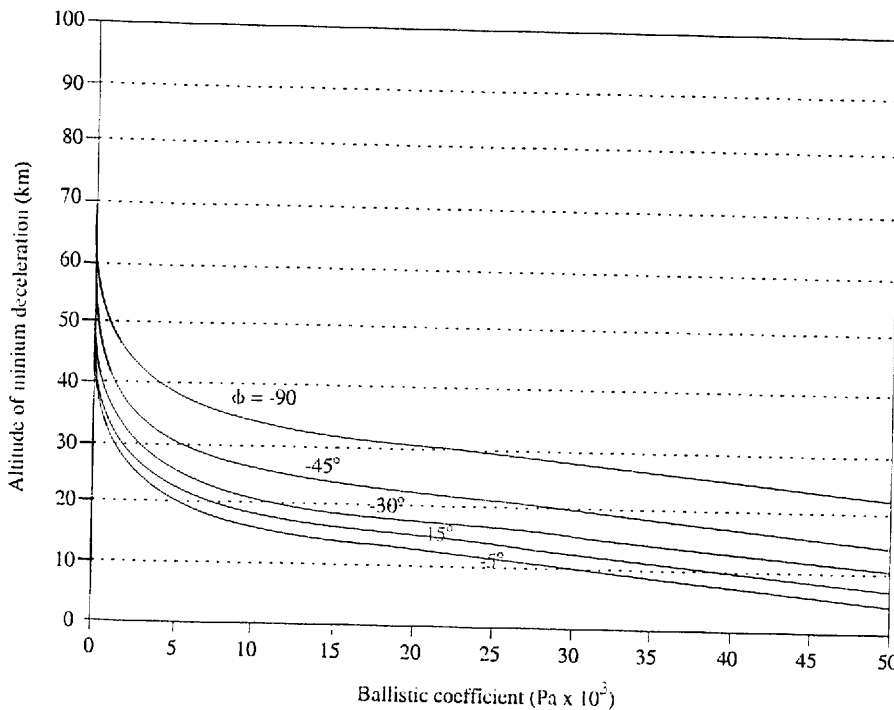


Figure 7-3-4. Altitude of maximum deceleration as a function of the BC for several values of ϕ_{re} .

which can be reduced to

$$V_{n_{max}} = V_{re} e^{-0.5} = 0.606 V_{re} \quad (7-3-22)$$

which shows that at n_{max} the velocity is a function only of the entry velocity, a somewhat surprising conclusion.

Example 7-3-4

An E/V has an entry angle of -22° and an entry velocity of 8000 m/s.

- (a) Find the maximum deceleration (g_0) and the associated altitude and velocity for $BC = 5000 \text{ Pa}$ (104.4 lb/ft^2).
- (b) With $BC = 5000 \text{ Pa}$, find the velocity and deceleration at an altitude of 18 km (59,020 ft).
- (c) Do part (a) for a more slender and more pointed shape [i.e., for $BC = 50,000 \text{ Pa}$ (1044 lb/ft^2)].

Solution

(a)
$$n_{max} = \frac{(8000)^2 \sin(-22^\circ)}{2 \times 7254 \times 9.81 \times 2.718} = -62.0 g_0$$

Using Eq. (7-3-13), B is found to be -23.23 , so that, from Eq. (7-3-19),

$$h_{n_{max}} = 27,860 \text{ m} = 27.86 \text{ km} = 91,360 \text{ ft}$$

and

$$V_{n_{\max}} = 0.606 \times 8000 = 4848 \text{ m/s}$$

(b) With $V_{re} = 8000 \text{ m}$ and $B = -23.23$, V can be found from Eq. (7-3-12) to be

$$V = 8000e^{-23.23e^{-1.378 \times 10^{-4} \times 18,000}} = 1144 \text{ m/s}$$

From Eq. (7-3-17),

$$\frac{dV}{dt} = -131.8 \text{ m/s}^2 = -13.4g_0$$

(c) With the BC increased to 50,000 Pa (1044 lb/ft²), n_{\max} is unchanged at $-62.0g_0$, as is the velocity at 4848 m/s. The altitude, however, decreases inasmuch as B is now -2.323 , so that

$$h_{n_{\max}} = 7254 \ln(4.646) = 11,140 \text{ m} = 11.14 \text{ km} = 36,530 \text{ ft}$$

Figure 7-3-5 is a plot of the deceleration along a ballistic entry for a suborbital entry velocity of 7300 m/s and an entry angle of -22° with three values for the ballistic coefficient. Note that n_{\max} ($-51.6g_0$) is unaffected by the increase in the BC, but that its altitude decreases as the BC is increased, and the E/V penetrates deeper into the atmosphere. Increasing or decreasing ϕ_{re} would have the same effect as increasing or decreasing the BC. Increasing V_{re} would increase n_{\max} and its associated velocity but would not affect the corresponding altitude.

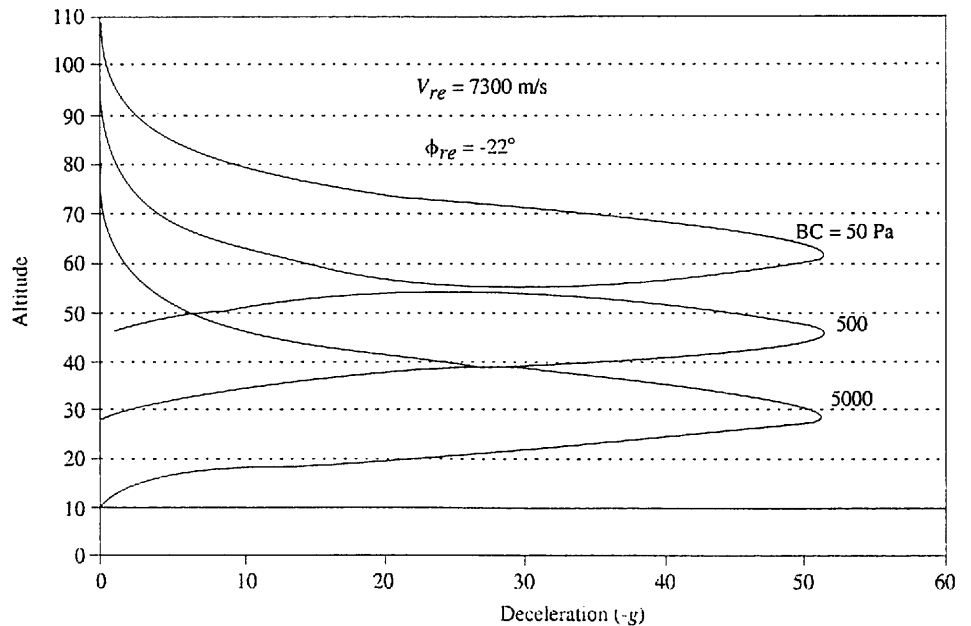


Figure 7-3-5. Deceleration along an entry trajectory as a function of altitude for several values of the BC with $V_{re} = 7300 \text{ m/s}$ and $\phi_{re} = -22^\circ$.

The n_{\max} of $-62g_0$ in Example 7-3-4 seems to be very large—and it is. However, n_{\max} can be much larger than that. For example, increasing V_{re} to 11,000 m/s, keeping $\phi_{re} = -22^\circ$, results in an n_{\max} of $-117g_0$. Leaving V_{re} at 11,000 m/s and increasing ϕ_{re} to 90° (a vertical entry) increases n_{\max} to $-313g_0$, a staggering figure. Figure 7-3-6 shows how n_{\max} varies with entry velocity and entry angle and emphasizes the importance of keeping the entry angle small if n_{\max} is to remain within a reasonable limit so that living occupants and equipment and the E/V itself might survive.

The maximum deceleration experienced by a human being (with survival but with some damage) strapped to a rocket-powered sled on a track was on the order of $12g_0$. The toleration of humans to decelerations is a function of magnitude and duration; decelerations in excess of $12g_0$ can be survived if the duration is sufficiently brief. Ballistic entry was used in the Mercury and Gemini programs, and the lift-to-drag ratio for the Apollo E/V was very small (<0.5). Consequently, the control of the entry angle was of the utmost importance.

Example 7-3-5

With the assumption that the maximum allowable deceleration for a manned entry is $-6g_0$, find the maximum allowable entry angle for the following entry velocities:

- (a) 7300 m/s
- (b) 8000 m/s
- (c) 11,000 m/s

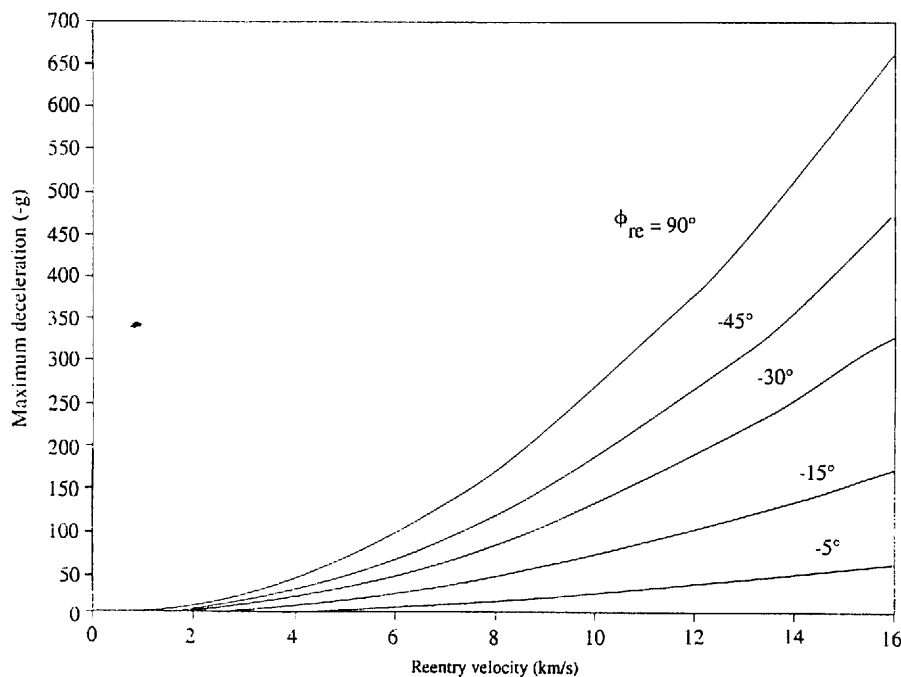


Figure 7-3-6. Maximum deceleration as a function of the reentry velocity for several values of ϕ_{re} .

Solution (a) Rearranging Eq. (7-3-20) to solve for ϕ_{re} produces

$$\phi_{re} = \sin^{-1} \frac{-2n_{\max} g_0 \ell}{\beta V_{re}^2} = \sin^{-1} \frac{-2.313 \times 10^6}{V_{re}^2}$$

With $V_{re} = 7300$ m/s,

$$\phi_{re, \max} = -2.50^\circ$$

(b) With $V_{re} = 8000$ m/s,

$$\phi_{re, \max} = -2.08^\circ$$

(c) With $V_{re} = 11,000$ m/s,

$$\phi_{re, \max} = -1.10^\circ$$

Example 7-3-6

If the entry angle is -5° , find the maximum deceleration for an Earth entry at the entry velocities of Example 7-3-5.

Solution Applying Eq. (7-3-20), as in the previous example:

$$\text{With } V_{re} = 7300 \text{ m/s, } n_{\max} = -12.0g_0.$$

$$\text{With } V_{re} = 8000 \text{ m/s, } n_{\max} = -14.4g_0.$$

$$\text{With } V_{re} = 11,000 \text{ m/s, } n_{\max} = -27.3g_0.$$

Although the relationship between time and altitude (or density ratio) during the linear deceleration phase can be found by using Eq. (7-2-3) ($dh/dt = V \sin \phi_{re}$) and Eq. (7-3-12) (the expression for V in terms of h), the integration is a bit complicated and will not be performed here inasmuch as the exact time along this straight-line portion of the entry trajectory is not of great importance. Suffice it to say that the time is a function of BC, ϕ_{re} , and V_{re} and is relatively short; for example, if the linear trajectory were extended to the surface of the Earth for an entry at -22° and 7300 m/s, the total entry time would be less than 1 minute.

The approximate distance (range) traveled during this time can easily be found by extending the straight-line approximation to the surface and combining Eqs. (7-2-3) and (7-2-4) to obtain

$$dS = V \cos \phi dt = \cot \phi dh \quad (7-3-23)$$

which, with a constant $\phi = \phi_{re}$, is a simple relationship between range and altitude as a function of ϕ . As was the case with the time, the range is relatively short. For an entry angle of -22° and an entry altitude of 76.25 km, the range is on the order of 185 km (100 nmi), 2.9% of the Earth's radius.

Only when the BC values and entry angles are very large will the straight-line trajectory extend to the planetary surface with the E/V still decelerating as it impacts. The usual case is for the straight-line trajectory to undergo a gravity-turn transition into a vertical trajectory at terminal velocity with the gravitational force balanced by the drag force. With the assumption that the deceleration is small with respect to these forces, it is possible to obtain closed-form expressions for this

vertical portion, which can then be joined with the straight-line solutions if one wished to obtain an approximate solution for the entire entry trajectory. It should be noted that the lower the BC and the lower the entry angle, the earlier the vertical transition will occur and the greater the reduction in the range.

Although the examples in this section pertain to entry into the atmosphere of Earth, the relationships and expressions developed can be used to investigate entry into any planetary (or lunar) atmosphere, provided that there is some knowledge of the planetary characteristics. Table 7-3-1 lists relevant characteristics for Venus and Mars; it must be emphasized that these characteristics are estimates only and do not necessarily represent the most current estimates. They do, however, give a feeling for the relationship to the values for the Earth, which are included in Table 7-3-1 for purposes of comparison. For example, this simple table indicates that the atmospheres of Earth and Venus have similar distributions, with the latter being more dense (high drag devices such as parachutes will be more effective on Venus). Mars, on the other hand, is characterized by a slower decrease in density with altitude than the other two planets and has a much lower density, decreasing the effectiveness (and increasing the size) of drag devices used for braking.

Figure 7-3-7 compares the variation of the velocity ratio (V/V_{re}) with altitude for entry into the three planetary atmospheres with a $W/C_D A$, the ballistic coefficient, equal to 500 Pa (10.44 lb/ft²) and an entry angle of -22° . Figure 7-3-8 shows the variation of the deceleration (in Earth g_0 units) with altitude for the same E/V

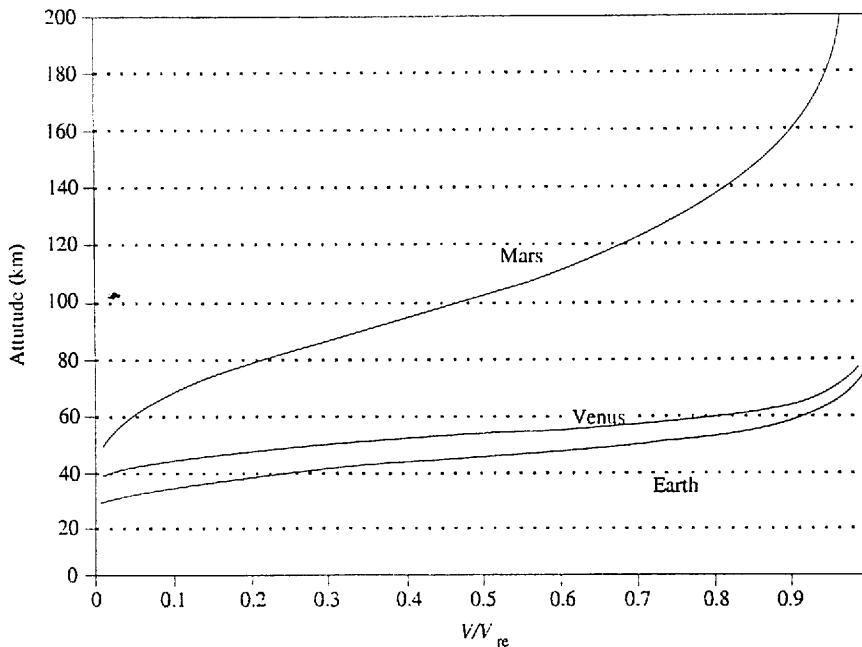


Figure 7-3-7. Velocity along a reentry trajectory in three planetary atmospheres.

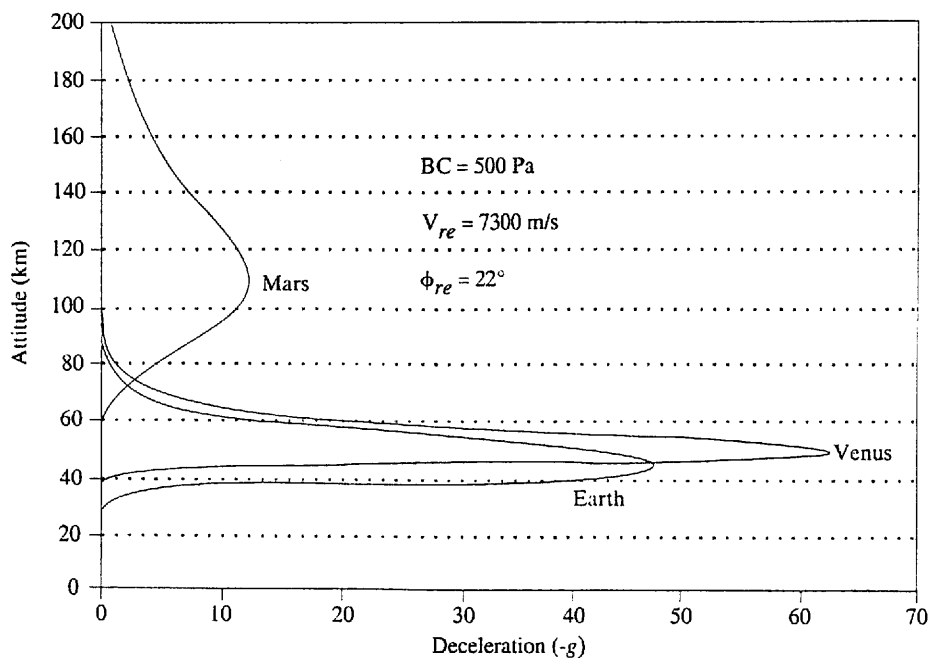


Figure 7-3-8. Maximum deceleration in three planetary atmospheres.

and entry angle with an entry velocity of 7300 m/s. We see that the curves for Venus and Earth are very similar because of the similar atmospheres, with the higher density of Venus manifesting itself in the slightly higher values of the maximum deceleration and its altitude. The lower density and slower variation of density with altitude of Mars, on the other hand, produce a slower variation of velocity with altitude and a much lower maximum deceleration at a higher altitude.

Since the gravitational attractions and the planetary radii of Venus and Earth are of the same order of magnitude, so are their escape velocities and the corresponding maximum decelerations, approximately $325g_0$ for both with a 90° entry angle. Mars, though, with its lower gravitational attraction, would have a lower escape velocity and a corresponding maximum deceleration on the order of only $20g_0$.

Let us return to the entry vehicle itself and use two simple examples to illustrate some of the configuration considerations for a ballistic E/V and to demonstrate the effect(s) of changing the ballistic coefficient while holding the weight constant.

Example 7-3-7

The total weight of an unmanned E/V is 9810 N (1000 kg or 2205 lb), including the structure and all equipment. The desired BC is 5000 Pa (104.4 lb/ft^2) and the selected half-cone angle δ is 25° . The preliminary shape is to be a conical nose section followed by a cylindrical afterbody, if one is needed, as sketched in Fig. 7-3-9, where the total length is $L = l' + l$. If possible, the specific gravity of the E/V is to be 0.5 or more (i.e., a desired density $\geq 499.8 \text{ kg/m}^3$).

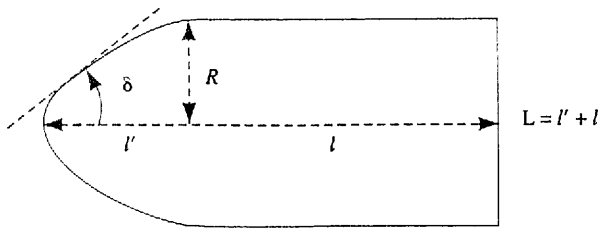


Figure 7-3-9. E/V schematic.

- (a) Find A , the cross-sectional area, and the diameter D and radius R of the E/V.
- (b) Find L , the total length, and V_T , the total volume, including the volume of the cone and of the afterbody. If the desired density cannot be achieved, determine the maximum value that can be obtained.

Solution

(a) $C_D = 2 \sin^2(25^\circ) = 0.357$

$$\frac{W}{A} = (BC) \times C_D = 5000 \times 0.357 = 1785 \text{ N/m}^2$$

$$A = \frac{9810}{1785} = 5.496 \text{ m}^2 \quad D = 2.64 \text{ m} \quad R = 1.32 \text{ m}$$

- (b) Looking at the conical nose section (the nose cone), we find that

$$l' = \frac{R}{\tan \delta} = 2.836 \text{ m}$$

and the volume of the cone, V_c , is

$$V_c = \frac{l' A}{3} = 5.196 \text{ m}^3$$

Since the total volume, V_T , is equal to the total mass divided by the vehicle density, with the desired density V_T should be

$$V_T = \frac{m}{\rho} = \frac{1000}{499.8} = 2.0 \text{ m}^3$$

Since this value is considerably less than the volume of the nose cone itself, it is not realistic. Consequently, $l = 0$, and L must be equal to $l' = 2.836 \text{ m}$; furthermore, the desired density cannot be attained. Therefore, this E/V is a cone with no afterbody, a cross-sectional area of 5.496 m^2 , a diameter of 2.64 m , and a length of 2.836 m . With the weight fixed at 9810 N , it has the density

$$\rho = \frac{m}{V_c} = \frac{1000}{5.195} = 192.5 \text{ kg/m}^3$$

which corresponds to a specific gravity of 0.192 .

Example 7-3-8

Keep the weight at 9810 N and the half-cone angle at 25° but increase the BC to 50.000 Pa .

- (a) Find the cross-sectional area and diameter of this E/V and compare with the results of Example 7-3-7(a).

(b) Use the density found in Example 7-3-7 (192.5 kg/m^3) to find the total volume and length.

(c) Do part (b) with a specific gravity of 0.5 ($\rho = 499.6 \text{ kg/m}^3$).

Solution (a) Using the relationships of Example 7-3-7 yields

$$\frac{W}{A} = 17,850 \text{ N/m}^2$$

$$A = 0.549 \text{ m}^2 \quad D = 0.8365 \text{ m} \quad R = 0.418 \text{ m}$$

The cross-sectional area and diameter of this E/V are considerably less than those of the E/V with the smaller BC.

(b) Since R is less than that of Example 7-3-6, so will l' and V_c be smaller.

$$l' = 0.897 \text{ m} \quad \text{and} \quad V_c = 0.164 \text{ m}^3$$

Using the density of the preceding example,

$$V_T = \frac{1000}{192.5} = 5.195 \text{ m}^3$$

Since $V_c = 0.164 \text{ m}^3$, the volume of the afterbody, V_a , will be 5.031 m^3 and is equal to lA . Therefore, $l = 9.164 \text{ m}$ and the total length of the E/V is 10.06 m , considerably longer than that of the preceding example.

(c) With the higher density of 499.8 kg/m^3 , $V_T = 2.0 \text{ m}^3$ and $V_c = 1.837$. Therefore, $l = 3.346$ and $L = 4.243 \text{ m}$, less than half of the length of the less dense E/V.

These two examples show that increasing the ballistic coefficient while keeping the weight constant lengthens and slenderizes the E/V and that increasing the density makes the E/V shorter and more compact, as would be expected. Although *caliber* is commonly used to denote the diameter of a bullet or gun barrel, it is also a measure of the slenderness of a body when defined as the ratio of the length to the diameter: The larger the caliber, the more slender the body. Applying this definition to the E/Vs of these two examples, the caliber of the pure cone with the lower density is 1.07, that of the lower-density cone plus cylindrical afterbody is 12.0, and that of the higher-density E/V is 5.07.

When entry angles become smaller than about -5° , as in Example 7-3-5, this straight-line analysis loses its validity and relevance and the entry trajectories become curved, the decelerations become smaller, and the times become larger. As the entry angle approaches zero, the trajectory becomes what is often referred to as an *orbital decay entry*, in which a spacecraft in a circular orbit gradually loses energy because of atmospheric drag (which is initially small at the orbital altitude in the outer fringes of the atmosphere) and spirals toward the planetary surface until it enters a terminal vertical phase. It can be shown that the maximum deceleration with orbital decay is on the order of $-8g_0$ for all BCs.

We conclude this section on ballistic (direct) entry with the observation that the gas dynamic drag on the E/V is the principal deceleration mechanism. Since the drag is directly proportional to the product of the atmospheric density and the square of the velocity (the dynamic pressure), so is the deceleration. As the E/V approaches

the planetary atmosphere, its velocity is at a maximum and the atmospheric density is at a minimum. As the vehicle penetrates the atmosphere, the atmospheric density increases rapidly, increasing the drag, reducing the velocity, and initiating deceleration. Since the deceleration is the product of two variables, one increasing and one decreasing, there will be an inflection point of maximum deceleration (see Fig. 7-3-5) where the velocity is decreasing more rapidly than the density is increasing. (With vehicles with a very large BC and entry angle, this inflection point might well be beneath the planetary surface.) Although the maximum deceleration, which can be very large and which increases as the entry velocity and entry angle increase (and as ρ_0 and g_0 increase), is independent of the characteristics of the E/V, the lighter vehicles decelerate at the higher altitudes and enter the curved terminal phase earlier and consequently have a shorter range than the heavier vehicles.

The entry trajectory can be further modified by the judicious use of lift, and we shall see in the next section that a little lift can be a very useful tool.

7-4 LIFTING ENTRY

Lift can be used to:

1. Increase the width of the entry corridor (see Fig. 7-2-4b)
2. Significantly reduce the decelerations experienced by an E/V
3. Enlarge the landing footprint, thus relaxing the deorbit and entry corridor requirements for the guidance and control system for a specified touchdown location
4. Provide additional entry trajectory options, such as skipping trajectories
5. Execute nonpropulsive plane changes with aerodynamic turns

Although the dynamic and kinetic equations of Section 7-2 are valid for lifting entry, closed-form solutions and analytic expressions that can give insight into the interaction between the trajectories and physical characteristics and parameters require approximations and assumptions, as was the case with ballistic entry.

An important lifting-entry trajectory is the *equilibrium glide*, which is a relatively flat glide in which the gravitational force is balanced by the combination of the lift and centrifugal forces. In addition to the "small" angle assumptions ($\sin \phi \cong 0$ and $\cos \phi \cong 1$) with respect to the elevation angle ϕ , it is further assumed that ϕ is changing slowly so that $d\phi/dt$ can be neglected (to a first approximation ϕ can be assumed to be constant) and that the lift-to-drag ratio is ≥ 0.5 or so.

Equations (7-2-2a) and (7-2-2b) can now be written as

$$\frac{dV}{dt} = -\frac{D}{m} = -\frac{\rho_0 C_D A}{2m} \sigma V^2 \quad (7-4-1)$$

$$\frac{V^2}{r} = -\frac{L}{m} + g = -\frac{\rho_0 C_L A}{2m} \sigma V^2 + g \quad (7-4-2)$$