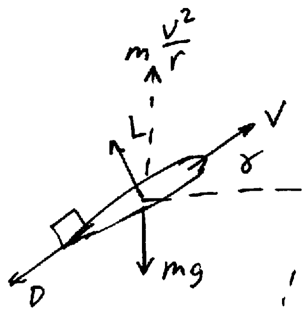
Lifting Entry - Equilibrium Glide

2/17/04

1

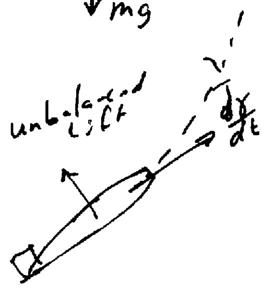


Along the velocity vector

$$m\frac{dV}{dt} = m\frac{V^2}{r}\sin\gamma - mg\sin\gamma - D$$

Perpendicular to velocity vector

$$mV\frac{d\gamma}{dt} = L + m\frac{V^2}{r}\cos\gamma - mg\cos\gamma$$



(unbalanced lift rotates flight path angle)

$$\frac{dV}{dt} = \left(\frac{V^2}{r} - g\right)\sin\gamma - \frac{D}{m}$$

$$V\frac{d\gamma}{dt} = \frac{L}{m} + \left(\frac{V^2}{r} - g\right)\cos\gamma$$

Equilibrium glide - Forces $\perp$ to velocity vector are balanced $\Rightarrow \frac{d\gamma}{dt} = 0$ $\quad \gamma = \text{constant}$

Typically very shallow glide $\rightarrow$ assume $\gamma \rightarrow 0$, $\sin\gamma \rightarrow 0$, $\cos\gamma \rightarrow 1$

Equations become

$$\frac{dV}{dt} = -\frac{D}{m} \qquad 0 = \frac{L}{m} + \left(\frac{V^2}{r} - g\right)$$

$$= -\frac{1}{2}\frac{\rho_0 V^2 C_D A}{m} e^{-h/h_s} \qquad \frac{V^2}{r} - g = \underbrace{\frac{1}{2}\frac{\rho_0 V^2 A C_L}{m}}_{} e^{-h/h_s}$$

$\frac{C_L}{C_D} = \frac{L}{D}$ $\leftarrow$ set by vehicle aerodynamics, flight velocity and angle of attack (assume constant!)

$$\frac{\rho_0 V^2}{2}\frac{A C_D}{m}\frac{C_L}{C_D} = \frac{\rho_0 V^2}{2}\frac{(L/D)}{\beta}$$

$\perp$ eqn becomes

$$\frac{V^2}{r} = \frac{\frac{L}{D}\rho_0 V^2}{-2\beta} e^{-h/h_s} + g$$

$$V^2 = \frac{\frac{L}{D}\rho_0 r V^2}{-2\beta} e^{-h/h_s} + gr$$

Let $e^{-h/h_s} = \sigma$ (density ratio) $\equiv \frac{\rho}{\rho_0}$

$$V^2\left[1+\frac{\rho_0 r \sigma (L/D)}{2\beta}\right] = g r$$

$$V^2 = \frac{g r}{1+\frac{\rho_0 r \sigma (L/D)}{2\beta}}$$

$V_{c_s} = \sqrt{\frac{\mu}{r}} = \sqrt{g_0 r_0}$ $(\mu = g_0 r_0^2)$

↑ surface $g \equiv g_0$

$$\frac{V}{V_{c_s}} = \frac{1}{\sqrt{1+\frac{\rho_0 r_0 \sigma (L/D)}{2\beta}}}$$

$$\boxed{\frac{V}{V_{c_s}} = \left[\sqrt{1+\frac{\rho_0 r_0}{2\beta}\frac{L}{D}e^{-h/h_s}}\right]^{-1}}$$

$\cong \frac{V}{V_e}$ (within 1-2% for Earth)

Entry trajectory function of altitude, ratio $\left(\frac{\beta}{L/D}\right)$

Show chart from Hale, page 227

How about deceleration?

$$\frac{L}{m} = g - \frac{V^2}{r} = g - \frac{gV^2}{gr} = \frac{gV^2}{V_{c_s}^2} = g\left[1-\left(\frac{V}{V_{c_s}}\right)^2\right]$$

$$\frac{D}{m} = \frac{L}{L/D}\frac{1}{m} \quad \text{and} \quad \frac{dV}{dt} = -\frac{D}{m}$$

$$\frac{dV}{dt} = \frac{-L}{m(L/D)} = -\frac{g}{L/D}\left[1-\left(\frac{V}{V_{c_s}}\right)^2\right]$$

let $n = \frac{1}{g}\frac{dV}{dt}$ ($\equiv$ deceleration in g's)

$$n = -\frac{1}{L/D}\left[1-\left(\frac{V}{V_{c_s}}\right)^2\right]$$

$$n = -\frac{1}{L/D}\left[1 - \frac{1}{1+\frac{\rho_0 r_0}{2\beta}\frac{L}{D}e^{-h/h_s}}\right]$$

(Aside:) $1 - \frac{1}{1+K} = \frac{1+K-1}{1+K} = \frac{K}{1+K}$

$$= -\frac{1}{\cancel{L/D}}\,\frac{\frac{\rho_0 r_0}{2\beta}\cancel{\frac{L}{D}}e^{-h/h_s}}{1+\frac{\rho_0 r_0}{2\beta}\frac{L}{D}e^{-h/h_s}}$$

$$= \frac{-\rho_0 r_0 e^{-h/h_s}}{2\beta + \rho_0 r_0 \frac{L}{D} e^{-h/h_s}}$$

$$\boxed{n = \frac{-1}{\frac{2\beta}{\rho_0 r_0}e^{+h/h_s} + \frac{L}{D}}}$$

n monotonically increases with ~~alt~~ decreasing altitude

$$n_{limit} = \frac{-1}{\frac{2\beta}{\rho_0 r_0} + \frac{L}{D}}$$

$$\left.\begin{array}{l}\beta \sim O(10^3)\\ \rho_0 \sim O(1)\\ r_0 \sim O(10^6)\end{array}\right\}\ \frac{2\beta}{\rho_0 r_0} \sim O(10^{-3})$$

$$\boxed{n_{max} \cong \frac{-1}{L/D}}$$

Time for entry - go back to

$$\frac{dV}{dt} = -\frac{g}{L/D}\left[1-\left(\frac{V}{V_{cs}}\right)^2\right] \Rightarrow dt = \frac{-(L/D)\,dV}{g\left[1-\left(\frac{V}{V_{cs}}\right)^2\right]}$$

$$\int_0^t dt = \int_{V_e}^0 \frac{-L/D\,dV}{g\left[1-\left(\frac{V}{V_{cs}}\right)^2\right]}$$

let $u = \frac{V}{V_{cs}}$

$du = \frac{dV}{V_{cs}}$

$\int \frac{du}{1-u^2} = \frac{1}{2}\ln\left|\frac{u+1}{u-1}\right|$

$$\Delta t = -\frac{L/D}{2gV_{cs}}\ln\left|\frac{\frac{V}{V_{cs}}+1}{\frac{V}{V_{cs}}-1}\right|$$

$$= -\frac{L/D\,V_{cs}^2}{g}\int\frac{dV}{V_{cs}^2 - V^2} = -\frac{L/D\,V_{cs}^2}{g}\frac{1}{2V_{cs}}\ln\left|\frac{V+V_{cs}}{V-V_{cs}}\right|\Bigg|_{V_e}^{0}$$

Integrate to get

$$\Delta t = \frac{1}{2}\sqrt{\frac{r_0}{g_0}}\,\frac{L}{D}\ln\frac{1+(V/V_{cs})^2}{1-(V/V_{cs})^2}$$

Time for entry $\propto \frac{L}{D}$

Distance along flight path

$$\frac{ds}{dt} \equiv V \qquad ds = V\,dt$$

$$ds = \frac{-(L/D)\,V\,dV}{g\left[1-(V/V_{cs})^2\right]}$$

let $u = 1-\left(\frac{V}{V_{cs}}\right)^2$

$$du = -\frac{2V}{V_{cs}^2}\,dV$$

$$V\,dV = -\frac{V_{cs}^2\,du}{2}$$

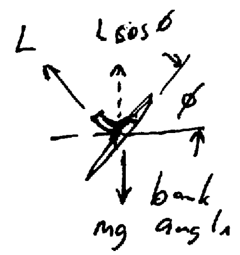
$$\int ds = +\frac{L/D}{g}\,\frac{1}{2}\int\frac{V_{cs}\,du}{u}$$

$$= \frac{L/D\;V_{cs}^2}{2g}\ln u = \frac{L/D\;\overset{g_0 r_0}{V_{cs}^2}}{2g}\ln\left[1-\left(\frac{V}{V_{cs}}\right)^2\right]$$

$$\boxed{\Delta s = L/D\,\frac{r_0}{2}\ln\left[1-\left(\frac{V}{V_{cs}}\right)^2\right]}$$

Take it as a miracle for now, but maximum cross range to a bank angle ϕ_{opt}

$$\phi_{opt} \cong \cot^{-1}\sqrt{1+.106\left(\frac{L}{D}\right)^2}$$



Max crossrange $\boxed{Y_{max} \cong \frac{r_0}{5.2}\left(\frac{L}{D}\right)^2\frac{1}{\sqrt{1+0.106(L/D)^2}}}$

Level turn:

$$L\cos\phi = mg$$

$$\frac{L}{mg} = \frac{1}{\cos\phi} = n_{bank}$$

(g's you pull in the bank)