Introduction to Space Flight

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the planetary atmosphere, its velocity is at a maximum and the atmospheric density is at a minimum. As the vehicle penetrates the atmosphere, the atmospheric density increases rapidly, increasing the drag, reducing the velocity, and initiating deceleration. Since the deceleration is the product of two variables, one increasing and one decreasing, there will be an inflection point of maximum deceleration (see Fig. 7-3-5) where the velocity is decreasing more rapidly than the density is increasing. (With vehicles with a very large BC and entry angle, this inflection point might well be beneath the planetary surface.) Although the maximum deceleration, which can be very large and which increases as the entry velocity and entry angle increase (and as ρ_0 and g_0 increase), is independent of the characteristics of the E/V, the lighter vehicles decelerate at the higher altitudes and enter the curved terminal phase earlier and consequently have a shorter range than the heavier vehicles.

The entry trajectory can be further modified by the judicious use of lift, and we shall see in the next section that a little lift can be a very useful tool.

7-4 LIFTING ENTRY

Lift can be used to:

1. Increase the width of the entry corridor (see Fig. 7-2-4b)
2. Significantly reduce the decelerations experienced by an E/V
3. Enlarge the landing footprint, thus relaxing the deorbit and entry corridor requirements for the guidance and control system for a specified touchdown location
4. Provide additional entry trajectory options, such as skipping trajectories
5. Execute nonpropulsive plane changes with aerodynamic turns

Although the dynamic and kinetic equations of Section 7-2 are valid for lifting entry, closed-form solutions and analytic expressions that can give insight into the interaction between the trajectories and physical characteristics and parameters require approximations and assumptions, as was the case with ballistic entry.

An important lifting-entry trajectory is the *equilibrium glide*, which is a relatively flat glide in which the gravitational force is balanced by the combination of the lift and centrifugal forces. In addition to the "small" angle assumptions ($\sin\phi \cong 0$ and $\cos\phi \cong 1$) with respect to the elevation angle ϕ, it is further assumed that ϕ is changing slowly so that $d\phi/dt$ can be neglected (to a first approximation ϕ can be assumed to be constant) and that the lift-to-drag ratio is ≥ 0.5 or so.

Equations (7-2-2a) and (7-2-2b) can now be written as

$$\frac{dV}{dt} = -\frac{D}{m} = -\frac{\rho_0 C_D A}{2m}\sigma V^2 \tag{7-4-1}$$

$$\frac{V^2}{r} = -\frac{L}{m} + g = -\frac{\rho_0 C_L A}{2m}\sigma V^2 + g \tag{7-4-2}$$

where C_L and C_D are the lift and drag coefficients of the E/V; they are not necessarily constant, are interrelated, and are a function of the angle of attack of the vehicle.

Since Eq. (7-4-2) is algebraic, it can be solved directly for V as a function of σ, the atmospheric density ratio, and thus of h, the altitude. At this time it is convenient to introduce the lift-to-drag ratio (L/D) of the vehicle, which can be used to relate C_L and C_D as follows:

$$\frac{L}{D} = \frac{C_L}{C_D} \quad \text{and, therefore,} \quad C_L = \frac{L}{D} C_D \tag{7-4-3}$$

With $W = mg_0$ and Eq. (7-4-3), Eq. (7-4-2) can be rewritten as

$$V^2 = \frac{gr}{1 + [(L/D)(C_D A/W)(\rho_0 g_0/2)\sigma r]} \tag{7-4-4}$$

Since $(gr)^{1/2}$ is the circular-orbit velocity V_{cs} at r, $gr = V_{cs}^2$. Since a planetary atmosphere is thin with respect to the planetary radius, g and r (and V_{cs}) within the atmosphere are essentially constant and the surface values can be used, namely, g_0, r_0, and V_{cso}. (In the Earth's atmosphere, there is a 1% difference between r_{re} and r_0, a 2% difference between g_0 and g_{re}, and only a 0.5% difference in the orbital velocities.) With these assumptions, Eq. (7-4-4) can be solved to obtain an expression for the velocity in terms of the density ratio σ,

$$\boxed{\frac{V}{V_{cs}} = \frac{1}{\sqrt{1 + [(L/D)(C_D A/W)(\rho_0 g_0 r_0/2)\sigma]}}} \tag{7-4-6}$$

where V_{cs} is used to denote V_{cso} and σ, the atmospheric density ratio, is approximated by the previously discussed exponential relationship

$$\sigma = e^{-\beta h} \tag{7-4-6}$$

Examining Eq. (7-4-5), we see the familiar ballistic coefficient of direct entry (BC) along with the L/D ratio. Defining a *lifting ballistic coefficient* (LBC) as

$$\text{LBC} = \frac{W}{C_D A(L/D)} = \frac{\text{BC}}{L/D} \tag{7-4-7}$$

Eq. (7-4-5) can now be written as an explicit function of h with the LBC as a major parameter.

$$\boxed{\frac{V}{V_{cs}} = \frac{1}{\sqrt{1 + [\rho_0 g_0 r_0/2(\text{LBC})]e^{-\beta h}}}} \tag{7-4-8}$$

Example 7-4-1

A lifting body entering the Earth's atmosphere in an equilibrium glide with a small elevation angle has a $W/C_D A = 5000$ Pa. Find the LBC and the nondimensional velocity at an altitude of 50 km for the following values of L/D:

(a) 1.0

(b) 1.5

(c) 2.0
(d) 2.5
(e) 3.0

Solution (a) With $L/D = 1.0$,

$$\text{LBC} = \frac{W/C_D A}{L/D} = \frac{5000}{L/D} = 5000 \text{ Pa}$$

At $h = 50$ km, with $\beta = 0.1378$ km^{-1},

$$\frac{\rho_0 g_0 r_0}{2(W/C_D A)} e^{-50\beta} = 7.785$$

and

$$\frac{V}{V_{cs}} = \frac{1}{\sqrt{1 + 7.785(L/D)}}$$

With $L/D = 1.0$, $V/V_{cs} = 0.337$.

(b) With $L/D = 1.5$, LBC = 3333 Pa and $V/V_{cs} = 0.281$.
(c) With $L/D = 2.0$, LBC = 2500 Pa and $V/V_{cs} = 0.246$.
(d) With $L/D = 2.5$, LBC = 2000 Pa and $V/V_{cs} = 0.220$.
(e) With $L/D = 3.0$, LBC = 1667 Pa and $V/V_{cs} = 0.206$.

This example shows that increasing the L/D decreases the LBC and shifts the deceleration to higher altitudes. This effect is confirmed in Fig. 7-4-1, which shows the variation of velocity with altitude for four values of the lifting ballistic coefficient (LBC). When compared with Fig. 7-3-3, a similar plot for direct entry, we see similar

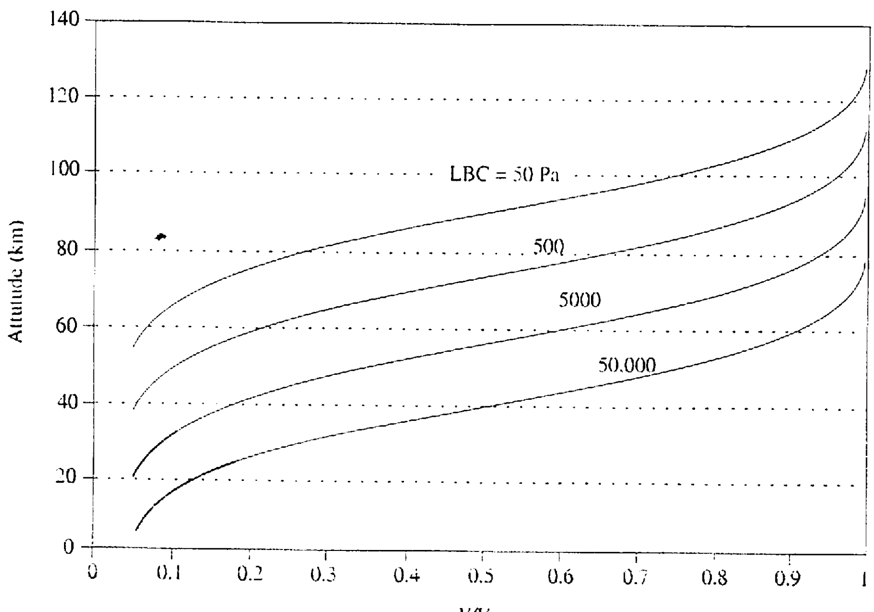

Figure 7-4-1. Velocity along a lifting reentry trajectory as a function of altitude for several values of the LBC.

shapes for the velocity curves but with deceleration occurring at lower altitudes when the L/D is zero (a ballistic entry).

An expression for the deceleration at any point along the trajectory in terms of V/V_{cs} can be found by first rearranging Eq. (7-4-2) to obtain

$$\frac{L}{m} = g - \frac{V^2}{r} = g - \frac{gV^2}{gr} = g - \frac{gV^2}{V_{cs}^2} = g\left[1 - \left(\frac{V}{V_{cs}}\right)^2\right] \qquad (7\text{-}4\text{-}9)$$

Since

$$\frac{D}{m} = \frac{L}{m(L/D)} \qquad (7\text{-}4\text{-}10)$$

and since, from Eq. (7-4-1), $dV/dt = -D/m$,

$$n = \frac{1}{g_0}\frac{dV}{dt} = -\frac{[1 - (V/V_{cs})^2]}{L/D} \qquad (7\text{-}4\text{-}11a)$$

where n is the deceleration expressed in g_0 units and $(V/V_{cs})^2$ is given by Eq. (7-4-8), which can be substituted into Eq. (7-4-11a) to obtain another expression for the deceleration, this time as a function of h directly. This expression can be written as

$$n = \frac{1}{g_0}\frac{dV}{dt} = \frac{-1}{(L/D) + [2(W/C_D A)e^{+\beta h}/\rho_0 g_0 r_0]} \qquad (7\text{-}4\text{-}11b)$$

Figure 7-4-2 is a plot of the deceleration as a function of the altitude for several values of the LBC and for two values of L/D. This figure and the deceleration expressions show that the deceleration increases continuously along the trajectory as the altitude decreases and that there is no maximum deceleration per se (no inflection point as with direct entry). The deceleration does, however, approach an asymptotic maximum that is simply the inverse of the lift-to-drag ratio:

$$n_{max} \cong \frac{-1}{L/D} \qquad (7\text{-}4\text{-}12)$$

Consequently, we see that the decelerations for E/Vs with only modest L/D ratios are significantly less than those for direct-entry (ballistic) E/Vs; a little lift does a lot.

Example 7-4-2

For the lifting E/V of Exercise 7-4-2 (BC = 5000 Pa), find the value of the deceleration at 50 km, using the values of V/V_{cs} and L/D from that example, as well as n_{max}.

Solution (a) With $L/D = 1$, $V/V_{cs} = 0.337$ and using Eq. (7-4-11a), $n = -0.886g$'s. From Eq. (7-4-12), $n_{max} = -1.0g$.

(b) With $L/D = 1.5$ and $V/V_{cs} = 0.281$, $n = -0.614g$ and $n_{max} = 0.666g$.

(c) With $L/D = 2.0$ and $V/V_{cs} = 0.246$, $n = -0.530g$ and $n_{max} = -0.50g$.

(d) With $L/D = 2.5$ and $V/V_{cs} = 0.220$, $n = -0.381g$ and $n_{max} = -0.40g$.

(e) With $L/D = 3.0$ and $V/V_{cs} = 0.206$, $n = -0.319g$ and $n_{max} = -0.333g$.

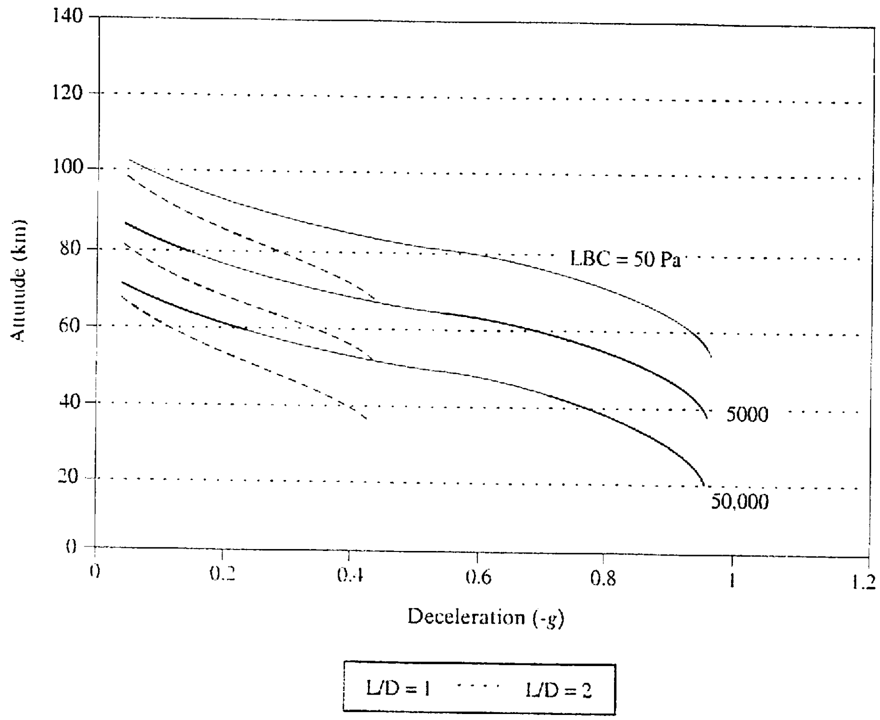

Figure 7-4-2. Deceleration along a lifting reentry trajectory for several values of the LBC and two L/D ratios.

The time along the entry trajectory as a function of the corresponding velocity can be found by rewriting Eq. (7-4-11a) as

$$dt = \frac{-(L/D)\,dV}{g[1 - (V/V_{cs})^2]} \tag{7-4-13}$$

Integrating from the velocity at the altitude of interest until the velocity is zero, the approximate time to touchdown from that altitude (and velocity) can be found from

$$\Delta t = \frac{1}{2}\sqrt{\frac{r_0}{g_0}}\frac{L}{D}\ln\frac{1 + (V/V_{cs})^2}{1 - (V/V_{cs})^2} \tag{7-4-14}$$

Equation (7-4-14) shows that the time in the entry trajectory, in the atmosphere, is directly proportional to the lift-to-drag ratio. Although this expression for time of flight is approximate, as are all of the values obtained from approximate solutions such as these, it provides comparative values. Figure 7-4-3 is a plot of Δt as a function of V/V_{cs} and L/D within the Earth's atmosphere.

Example 7-4-3

For the E/V of Exercise 7-4-2 (BC = 5000 Pa):

(a) Find the time to "touchdown" from a point where $V/V_{cs} = 0.80$ ($V = 6325$ m/s) for $L/D = 1.0$, 1.5, 2.0, 2.5, and 3.0.

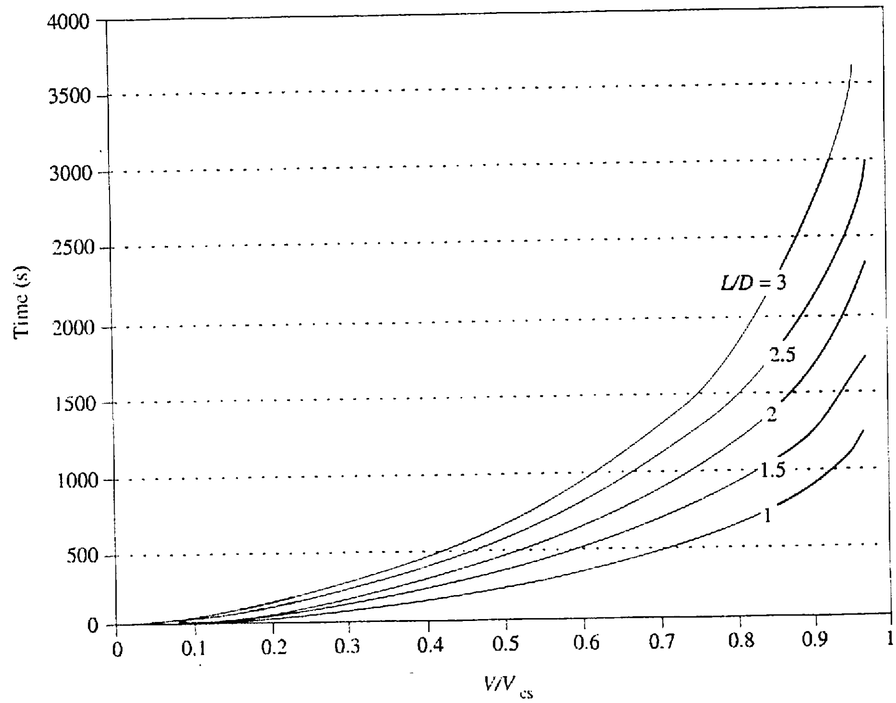

Figure 7-4-3. Reentry time as a function of the velocity along the trajectory for several values of L/D.

(b) Using the results of Example 7-4-2, find the time for each value of L/D from $V/V_{cs} = 0.8$ to an altitude of 50 km.

Solution (a) Substituting into Eq. (7-4-14) yields

L/D	TIME (s)	TIME (min)
1.0	611	10.2
1.5	917	15.3
2.0	1,223	20.4
2.5	1,528	25.5
3.0	1,834	30.6

(b) First, find the time from 50 km to touchdown and then subtract from the values of part (a).

L/D	V/V_{cs}	$t_{touchdown}$ (s)	Δt (s/min)
1.0	0.337	92.0	519/8.65
1.5	0.281	95.7	821/13.7
2.0	0.246	97.4	1126/18.8
2.5	0.220	97.7	1430/23.8
3.0	0.206	102.7	1731/28.8

With the small elevation angles associated with the equilibrium glide, the horizontal range of the E/V can be found from the approximation that $dS/dt \cong V$, so that $dS = V\,dt$. Solving Eq. (7-4-11a) for dt in terms of V and dV and substituting for dt produces

$$dS = \frac{-(L/D)V\,dV}{g[1 - (V/V_{cs})^2]} \tag{7-4-15}$$

Integrating from any point on the trajectory, where V is known, to touchdown yields an expression for the range from that point.

$$S = -\frac{r_G}{2}\frac{L}{D}\ln\left[1 - \left(\frac{V}{V_{cs}}\right)^2\right] \tag{7-4-16}$$

Therefore, the range, as was the case with the time of flight, is directly proportional to the lift-to-drag ratio. This distance is traveled in the direction of the entry velocity (in the vertical plane) and is known as the *down-range distance*.

Figure 7-4-4 shows the variation of the maximum down-range distance, expressed in planetary radii, with V/V_{cs} for several L/D ratios. The combination of the higher initial velocities and higher L/D values can result in quite large down-range distances, much larger than any attained with direct entry (no lift). Although our equilibrium glide analysis has implied a constant L/D, the L/D ratio can be varied to touch down at distances less than the maximum.

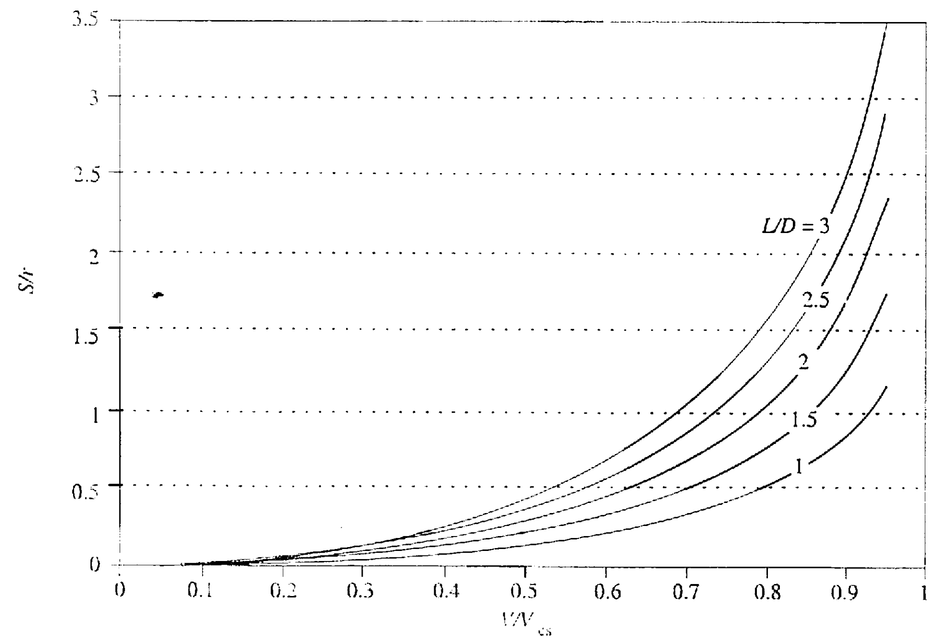

Figure 7-4-4. Downrange distance as a function of the velocity along the trajectory for several values of L/D.

An additional benefit of lift is the ability to make turns within the atmosphere, turns that can be used to reach a touchdown point not in the vertical plane (and to modify the duration of the entry). The out-of-plane distance is known as the *cross-range distance* and in conjunction with the down-range distance forms the *footprint* of the E/V. Without writing the equations for out-of-plane (lateral) flight, it can be shown that the maximum cross (lateral) range occurs when the *bank angle* Φ_{bank} (not to be confused with the elevation or entry angle) takes on an optimum value (Φ_{opt}), which is a function of L/D, namely,

$$\Phi_{opt} \cong \cot^{-1}\sqrt{1 + 0.106\left(\frac{L}{D}\right)^2} \tag{7-4-17}$$

The corresponding maximum cross range is

$$\boxed{Y_{max} \cong \frac{r_0}{5.2}\left(\frac{L}{D}\right)^2 \frac{1}{\sqrt{1 + 0.106(L/D)^2}}} \tag{7-4-18}$$

Equation (7-4-18) shows the variation of the maximum cross range (in planetary radii) and of the optimum bank angle as a function of the L/D ratio. To provide a footprint that will cover the globe (allow the E/V to land at a location of choice), Y/r_0 needs to be equal to $\pi/2$ rad (90°) and Eq. (7-4-18) shows that the required hypersonic L/D is on the order of 3.5. Designing an E/V that can handle the heating problem(s) associated with such an L/D is not a simple task, however.

A lifting vehicle must consider the *aerodynamic load factor*, which is defined as the lift-to-weight ratio (L/W) and has the dimensions of acceleration expressed in g_0 units. Although the standard symbol for the load factor is n, LF will be used to denote the load factor to avoid confusion with the n used for deceleration. Since the load factor is inversely proportional to the cosine of the bank angle, the load factor (g_0) in an optimum turn is

$$\text{LF} = \frac{1}{\cos \Phi_{opt}} \tag{7-4-19}$$

Example 7-4-4

Find and tabulate the values of the optimum bank angle, the maximum cross range, and load factor for L/D ratios of 1.0, 2.0, 3.0, and 3.5.

Solution

L/D	Φ_{opt} (deg)	Y_{max}/r_0	LF (g_0)
1.0	43	0.183	1.37
2.0	40	0.698	1.30
3.0	36	1.238	1.23
3.5	33.4	1.554	1.20

In addition to acquiring a cross-range capability, a steady-state turn can be used to reduce the descent time (and down-range distance) by effectively reducing the lift-to-drag ratio by a factor equal to cos Φ_{bank}. This may be a desirable maneuver to reduce the heating of the E/V during a critical portion of the entry.

It should be pointed out that the introduction of lift to reduce the deceleration and to provide a maneuvering capability also introduces the need to consider the dynamic stability and control of the E/V. The stability analysis is similar to that for conventional aircraft, with the added problem that at hypersonic velocities with low atmospheric densities, there is virtually no aerodynamic damping. If the oscillations in response to a command or a disturbance are to be controlled by the pilot (rather than by a flight control system), the vehicle must be designed so that frequency of any oscillation is low (i.e., the period of the oscillation is high).

As the lifting E/V penetrates the sensible atmosphere, it must function as a conventional aircraft, but to date without power. The flight regime of a lifting E/V covers a broad spectrum, from low touchdown speeds at sea level to hypersonic velocities at the outer fringes of the atmosphere.

Before leaving the equilibrium-glide entry, it should be pointed out that the approximate solutions above imply that the E/V is at a circular orbital velocity at the initiation of the entry. This means that an E/V returning from a high Earth orbit or from outside the SOI will be slowed down prior to entering the glide perhaps by a propulsive ΔV, an aerocapture maneuver using atmospheric braking, a series of grazing trajectories, or a combination thereof.

The equilibrium glide entry gives insight into the characteristics and benefits of lifting entry and approximates the entry of the Space Shuttle (Space Transportation System). It is, however, not the only lifting entry trajectory. There are others of varying degrees of interest and usefulness that lend themselves to approximate closed-form solutions (and others that do not). Several of these are described briefly below with relevant comments where applicable.

The *skip entry* trajectory is a series of joined segments consisting of a penetration into the atmosphere (possibly with a turn), followed by an exit from the atmosphere at a lower velocity (the skipping phase) and a return to an elliptical Keplerian orbit that leads to another penetration, followed by another skip, and so on, until the desired conditions are reached for a descent to the planetary surface. During the skipping phase within the atmosphere, the aerodynamic lift is the dominant force, being much greater than the difference between the gravitational force and the centrifugal force ($mg - mV^2/r_0 \cong 0$). It can be shown that the exit elevation angle is equal to the negative of the entry angle ($\phi_{ex} = -\phi_{re}$). This relationship holds for each atmospheric leg in the overall trajectory so that the elevation angles can be considered to be constant and equal to ϕ.

The relationship between the exit and entry velocities for each skipping segment is given by

$$\frac{V_{ex}}{V_{re}} = \exp\left(\frac{-2\phi}{L/D}\right) \tag{7-4-20}$$

where ϕ is a reasonably large angle, similar to those of ballistic entries; the larger the angle (and the lower the L/D), the greater the velocity reduction. If, for example, the entry (and exit) angle ϕ is $-22°$ (0.384 rad) and the L/D is 2.0, the exit velocity will be 68% of the entry velocity; if the $L/D = 1$, then $V_{ex}/V_{re} = 0.46$.

A single skip followed by a return to a modified Keplerian orbit rather than another atmospheric penetration could be used to execute an aeroassisted transfer or an aerocapture maneuver.

Another lifting trajectory of possible interest is the *negative L/D glide*, which could be used to keep an E/V within the atmosphere (to widen the entry corridor) for reasonable entry velocities and large elevation angles or for large entry velocities with a wide range of elevation angles. In contrast to the equilibrium glide with positive L/D, the elevation angle does not remain constant along a negative-lift trajectory and there is an altitude at which the deceleration reaches a maximum and then decreases. This maximum deceleration is a function of the ballistic coefficient ($W/C_D A$), the L/D, and the entry angle.

The last category of entry trajectories to be mentioned includes those trajectories with *small positive or negative L/D*'s (-0.5 to $+0.5$) and *small entry angles* ($\leq 5°$). This category is applicable primarily to orbital decay trajectories (to include the ballistic decay case) and is characterized by low ($\lesssim 8g_0$) maximum decelerations which decrease in magnitude and increase in altitude with an increase in the magnitude of L/D and are influenced by the magnitude of the ballistic coefficient. Although we have considered the edge of the sensible atmosphere to be at an altitude of 75 km (41 nmi) or so, there is sufficient density and drag at higher altitudes to cause an orbit to decay eventually.

This section can be concluded with the simple statement that lift reduces the decelerations experienced by an E/V and increases its footprint, maneuverability, and duration of entry. However, as we shall see in the next section, this increase in duration introduces problems associated with the heating encountered during entry.

7-5 THE HEATING PROBLEM

An atmospheric entry is successful when an E/V and its contents arrive undamaged at a planetary surface, having survived the decelerations and heating associated with dissipating the large amount of kinetic energy possessed by the E/V at the time of entry. With a discussion of the deceleration problem and its possible solutions behind us, we can turn our attention to the heating problem and its possible solutions. We must not, however, be unaware of or ignore the impact of a heating problem solution upon the deceleration magnitude and profile. *The two entry problems, deceleration and heating, are linked and must be jointly solved.*

The kinetic energy of an E/V is dissipated by transformation into thermal energy (heat) as the E/V decelerates. The magnitude of this thermal energy is such that if all of the heat were transferred to the E/V, the E/V could be severely damaged and conceivably even completely vaporized. Fortunately, not all of the thermal energy is transferred to the surface of the E/V. It is the fraction that is transferred that is of interest to the designer (and of special interest to the occupants).