Re-Entry Vehicle Dynamics

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6.7 Case V: Lifting Re-Entry

There are many special integrations of Eqs. (6.8a) and (6.8b) that can be made. We shall consider here one final closed-form solution—lifting re-entry. We shall continue to assume that gravity and centrifugal forces are approximately balanced. Consequently Eqs. (6.8a) and (6.8b) become

$$-V\frac{d\gamma}{dt} = \left[\frac{\rho_\infty g V^2}{2\beta}\right]\left[\frac{C_L}{C_D}\right] \tag{6.64a}$$

$$-\frac{dV}{dt} = \left[\frac{\rho_\infty g V^2}{2\beta}\right] \tag{6.64b}$$

Dividing Eq. (6.64a) by Eq. (6.64b) we get

$$\frac{dV/dt}{V} = \left[\frac{C_L}{C_D}\right]^{-1}\frac{d\gamma}{dt}$$

$$\frac{dV}{V} = \left[\frac{C_L}{C_D}\right]^{-1} d\gamma \tag{6.65}$$

Equation (6.65) integrates to

$$V = V_E \exp - \left[\frac{\gamma_E - \gamma}{C_L/C_D}\right] \tag{6.66}$$

By itself Eq. (6.66) is not of much use as the independent variable is Z, the geometric altitude, and V varies only implicitly with Z through $\gamma = \gamma(Z)$. Therefore we must find a function to represent the variation of γ with Z.

We have shown that

$$\frac{dZ}{dt} = -V\sin\gamma \tag{6.67}$$

We may rewrite Eq. (6.64a) to eliminate velocity magnitude V:

$$\sin\gamma d\gamma = \left[\frac{\rho_0 g H}{2\beta}\right]\left[\frac{C_L}{C_D}\right]e^{-Z/H} d\left[\frac{Z}{H}\right] \tag{6.68}$$

Integrating from γ_E to γ and Z_E to Z (entry conditions to local conditions)

$$\cos\gamma = \left[\frac{\rho_0 g H}{2\beta}\right]\left[\frac{C_L}{C_D}\right]\left[e^{-Z/H} - e^{-Z_E/H}\right] + \cos\gamma_E \tag{6.69}$$

Since Z_E is about 100 km and H is 6.7 km, we might assume that

$$e^{-Z_E/H} \simeq 0 \tag{6.70}$$

We may now solve for γ to get

$$\gamma = \cos^{-1}\left[\left(\frac{\rho_0 g H}{2\beta}\right)\left(\frac{C_L}{C_D}\right)e^{-Z/H} + \cos\gamma_E\right] \tag{6.71}$$

Equation (6.71) gives γ, the flight-path angle, as a function of geometric altitude Z. If Eq. (6.71) is inserted in Eq. (6.66) we get the velocity magnitude V as a function of altitude, ballistic factor β, and lift-to-drag ratio C_L/C_D:

$$V = V_E$$

$$\times \exp -\left\{\frac{1}{(C_L/C_D)}\left[\gamma_E - \cos^{-1}\left[\left(\frac{\rho_0 g H}{2\beta}\right)\left(\frac{C_L}{C_D}\right)e^{-Z/H} + \cos\gamma_E\right]\right]\right\} \tag{6.72}$$

If **a** is the acceleration vector we may write after Eqs. (6.8a and 6.8b) the deceleration components along the trajector $\mathbf{n}_v$ and perpendicular to the trajectory $\mathbf{n}_{\perp v}$ as

$$-\frac{\mathbf{a}}{g} = \mathbf{n} = -\left[\frac{\dot V}{g}\right]\mathbf{i}_m + \left[\frac{V\dot\gamma}{g}\right]\mathbf{k}_m = n_v\mathbf{i}_m + n_{\perp v}\mathbf{k}_m \tag{6.73}$$

From Eqs. (6.8a) and (6.8b) we have neglecting gravity,

$$n_v = -\frac{\dot V}{g} = \left[\frac{\rho_0 V^2}{2\beta}\right]e^{-Z/H} \tag{6.74a}$$

$$n_{\perp v} = -\frac{\dot\gamma V}{g} = \frac{\rho_0 V^2}{2\beta}\left[\frac{C_L}{C_D}\right]e^{-Z/H} \tag{6.74b}$$

The magnitude of the deceleration vector **n** is

$$\mathbf{n} = n = \left|n_v^2 + n_{\perp v}^2\right|^{1/2} = \left[\frac{\rho_0 V^2}{2\beta}\right]e^{-Z/H}\left[1 + \left[\frac{C_L}{C_D}\right]^2\right]^{1/2} \tag{6.75a}$$

The vector **n** makes an angle with respect to the flight path, i.e., with respect to the velocity vector **V** of

$$\arg\mathbf{n} = \sin(C_L/C_D)\tan^{-1}(C_L/C_D) \tag{6.75b}$$

Let us now find the maximum deceleration load which we shall designate $n_{max} = n_m$. The corresponding altitude, flight-path angle, and velocity magnitudes at which this maximum load occurs are also designated as Z_m, γ_m, and V_m, respectively. From Eqs. (6.75a) and (6.72) the load magnitude n is

$$n = \frac{\rho_0 V_E^2}{2\beta} e^{-Z/H} \left[1 + \left(\frac{C_L}{C_D} \right)^2 \right]^{1/2}$$

$$\times \exp - \left\{ \frac{2}{(C_L/C_D)} \left[\gamma_E - \cos^{-1} \left[\left(\frac{\rho_0 g H}{2\beta} \right) \left(\frac{C_L}{C_D} \right) e^{-Z/H} + \cos\gamma_E \right] \right] \right\} \tag{6.76}$$

If the derivative of the above expression is taken with respect to Z and equated to zero we get

$$\left[\frac{\rho_0 g H}{2\beta \sin\gamma_m} \right] e^{-Z_m/H} = 1/2 \tag{6.77}$$

Solving for $\sin\gamma_m$ and making use of Eq. (6.56) we have

$$\sin\gamma_m = (2a_E \sin\gamma_E) e^{-Z_m/H} \tag{6.78}$$

A second relationship between Z_m and γ_m may be found in Eq. (6.71). If we eliminate γ_m between these equations we get

$$Z_m = H \ln \left\{ a_E \left[\left[4 + \left(\frac{C_L}{C_D} \right)^2 \csc^2\gamma_E \right]^{1/2} + \left(\frac{C_L}{C_D} \right) \cot\gamma_E \right] \right\} \tag{6.79}$$

Equation (6.79) gives the altitude at which the maximum deceleration takes place. Inserting this expression into Eq. (6.71) we get the flight-path angle γ_m at which maximum deceleration takes place:

$$\gamma_m = \cos^{-1} \left\{ \frac{C_L/C_D \sin\gamma_E}{[4 + C_L/C_D^2 \csc^2\gamma_E]^{1/2} + (C_L/C_D)\cot\gamma_E} + \cos\gamma_E \right\} \tag{6.80}$$

The velocity magnitude at which the maximum deceleration takes place is from Eq. (6.66):

$$V_m = V_E \exp - \left[\frac{\gamma_E - \gamma_m}{C_L/C_D} \right] \tag{6.81}$$

and finally the value of the maximum deceleration n_m is from Eq. (6.76):

$$n_m = \frac{\rho_0 V_E^2}{2\beta} \left[1 + \left(\frac{C_L}{C_D} \right)^2 \right]^{1/2} V_m^2 \tag{6.82}$$

From Eq. (6.79) it is clear that the altitude at which maximum deceleration takes place depends upon the mass and lift/drag characteristics as well as the entry angle γ_E through the presence of a_E [see Eq. (6.56)]. However, the flight-path angle γ_m and the velocity magnitude V_m, at which maximum deceleration n_m takes place, depend only on the entry angle and the lift-to-drag ratio C_L/C_D. Remembering that C_L/C_D can be either positive or negative, it is apparent that the deceleration profile (with altitude) as well as the maximum value of deceleration and the altitude Z_m at which this maximum occurs can be modified by controlling C_L/C_D.

The preceding discussion covered five special cases of planetary entry. For each of these cases fairly simple analytic relationships were developed containing variables of interest as functions of time, arc length, or altitude. We can regard these analytic expressions as approximate solutions to the four differential equations given in Eqs. (6.8). If these equations were to be numerically integrated, a graphical or tabular presentation of the various dependent/independent variable pairs would be called the "exact" solutions to the same set of differential equations. The sometimes self-imposed burden on the engineering analyst is to compare his approximate solutions with the numerical or exact solutions. The graphical comparison seems to be even more impressive if sprinkled liberally with data symbols from appropriate experiments. Mutual corroboration among the three methodologies: analytic (approximate), numerical (exact), and experimental is the engineering equivalent of the navigators three-point-fix. An acceptable fix gives the analyst, like the navigator, great confidence in the essential correctness of his labors. If the analyst is an engineer, then he draws pride in his ability to discern the essential physics in the problem before committing his analytic skills; if the analyst feels that his forte is analytic virtuosity, then he will emphasize his repertoire of mathematical methods or manipulative skills. (It would be a bit embarrassing to struggle with a series solution only to find after going public that his great analytic insight was an established part of the tool kit of his nineteenth century counterpart.)

If there is an unacceptable deviation between the numerical and/or experimental solutions, then the analyst usually retreats and reconsiders his thought process. Of course the "exact" solution may also be dead wrong for a variety of reasons such as the use of inappropriate or incomplete differential equations in the first place or round-off errors during computation. Experimental studies could also be unreliable, perhaps resting on the shifting sands of instrument error or spurious or unaccounted for inputs. The analyst usually sees only graveyard humor in the shortfall of the computer modeler or the experimentalist. The analyst seeks a benchmark, if not a gold standard, in the work of his collaborators in the laboratory or at the computer terminal.

If we could be assured of the correctness of experiment and computer modeling, why, we may ask, do we place any emphasis on closed-form solutions? During the pre-Cambrian epoch when the electronic computer was only a gleam in the eye of Charles Babbage, approximate analytic solutions were probably the foundations of any design decisions. Today with the computer increasingly available and versatile, we need only call up the appropriate subroutine to tame some formidable nonlinear differential equations. Still there remains an essential place for the closed-form solution. Such solutions have the advantage of clearly indicating the relative importance of the various components of the physical model. We may ask, what are the design parameters that are of primary significance in meeting mission requirements? It may be seen, for example, in Eq. (6.61) the relative importance of vehicle drag, vehicle weight, and atmospheric properties in setting the altitude for maximum deceleration.

All of the analysis set forth in this chapter was drawn from a more extensive discussion by Loh.[1] Loh provides an almost encyclopedic coverage of closed-form analysis or re-entry body physics. The propriety of the various concepts may be found in his extensive bibliography. In addition, there is a discussion of re-entry body particle dynamics in Ashley's work[2] which is based upon Loh's earlier compilation; however, Ashley's treatment is more lucid, with an improvement in symbolism. In addition, Ashley also includes some interesting insights into re-entry body dynamics.

It might be of some interest to trace the development of Loh's effort in order to develop some insight of our own into re-entry body mechanics. Let us first restate Eqs. (6.8):

$$\frac{dV}{dt} = -\left[\frac{\rho g V^2}{2\beta}\right] + (g\sin\gamma) \tag{6.83a}$$

$$V\frac{d\gamma}{dt} = -V\frac{d\theta}{dt} + g\cos\gamma - \left[\frac{\rho g V^2}{2\beta}\right]\left[\frac{C_L}{C_D}\right] \tag{6.83b}$$

$$(R_E + Z)\frac{d\theta}{dt} \simeq R_E\frac{d\theta}{dt} = V\cos\gamma \tag{6.83c}$$

$$\frac{dZ}{dt} = -V\sin\gamma \tag{6.83d}$$

In atmospheric vehicle analysis the flight-path angle γ is usually taken positive when the velocity vector is *above* the local horizontal. However, for RV's, the velocity vector is usually *below* the local horizontal so we shall define γ, the flight-path angle, as positive under such circumstances. We can rewrite Eq. (6.83a) by changing the independent variable from time t to altitude Z as:

$$\frac{d(\,)}{dt} = \frac{dZ}{dt}\frac{d(\,)}{dZ} \tag{6.84}$$

Using Eq. (6.83d), the above chain rule becomes

$$\frac{d(\,)}{dt} = -V\sin\gamma\frac{d(\,)}{dZ} \tag{6.85}$$