Course Overview/Orbital Mechanics

- Course Overview
  - Challenges of launch and entry
  - Course goals
  - Web-based Content
  - Syllabus
  - Policies
  - Project Content
- An overview of orbital mechanics at "point five past lightspeed"

Space Launch - The Physics

- Minimum orbital altitude is ~200 km

$$\frac{\text{Potential Energy}}{\text{kg in orbit}} = gh = 1.96x10^6 \frac{J}{kg}$$

- Circular orbital velocity there is 7784 m/sec

$$\frac{\text{Kinetic Energy}}{\text{kg in orbit}} = \frac{1}{2}v^2 = 30x10^6 \frac{J}{kg}$$

- Total energy per kg in orbit

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = PE + KE = 32x10^6 \frac{J}{kg}$$

Theoretical Cost to Orbit

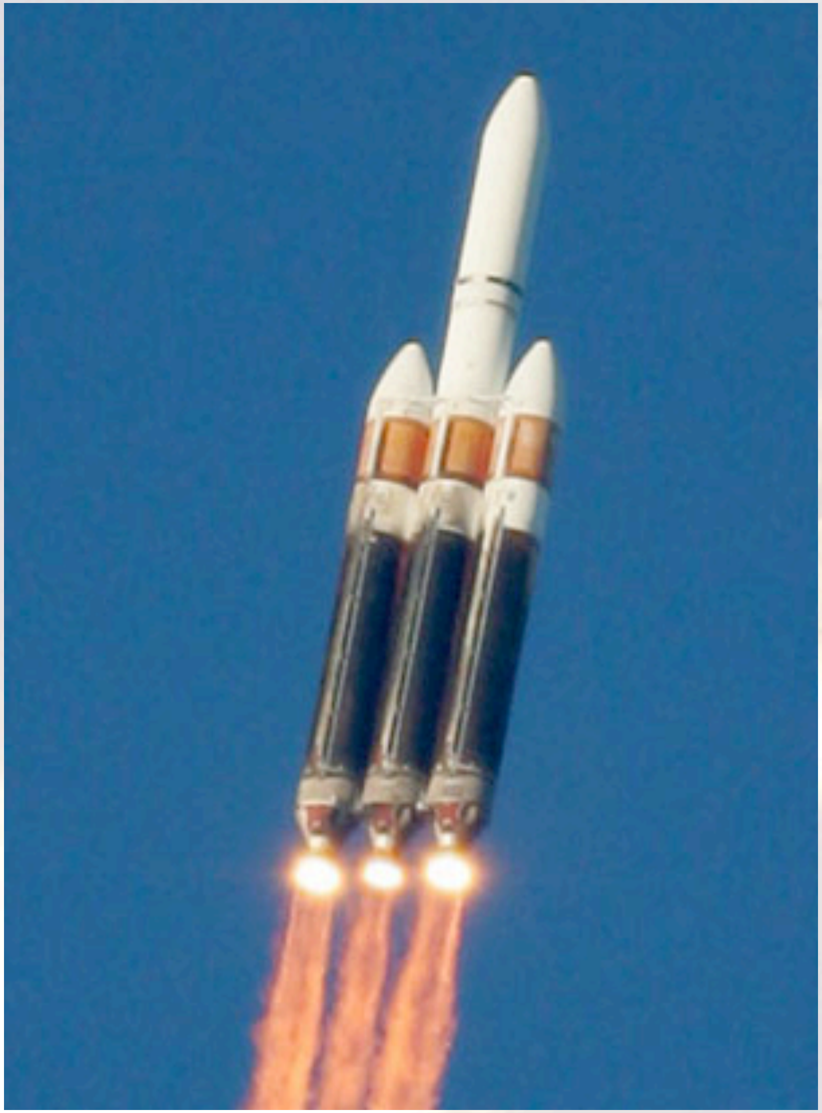
- Convert to usual energy units

$$\frac{\text{Total Energy}}{\text{kg in orbit}} = 32 \times 10^6 \ \frac{J}{kg} = 8.888 \frac{kWhrs}{kg}$$

- Domestic energy costs are ~$0.05/kWhr

    - ▸▸ Theoretical cost to orbit <u>$0.44/kg</u>

Actual Cost to Orbit



- Delta IV Heavy
  - 23,000 kg to LEO
  - $250 M per flight
- $10,870/kg of payload
- Factor of 25,000x higher than theoretical energy costs!

What About Airplanes?

- For an aircraft in level flight,

$$\frac{\text{Weight}}{\text{Thrust}} = \frac{\text{Lift}}{\text{Drag}}, \text{ or } \frac{mg}{T} = \frac{L}{D}$$

- Energy = force x distance, so

$$\frac{\text{Total Energy}}{\text{kg}} = \frac{\text{Thrust x Distance}}{\text{mass}} = \frac{Td}{m} = \frac{gd}{L/D}$$

- For an airliner (L/D=25) to equal orbital energy, d=81,000 km (2 roundtrips NY-Sydney)

Equivalent Airline Costs?

- Average economy ticket NY-Sydney round-round-trip (Travelocity 1/28/08) ~$1850
- Average passenger (+ luggage) ~100 kg
- Two round trips = $37/kg
  - Factor of 85x electrical energy costs
  - Factor of 300x less than current launch costs
- But...

  you get to refuel at each stop!

Equivalence to Air Transport

- 81,000 km ~ twice around the world
- Voyager - one of two aircraft to ever circle the world non-stop, non-refueled - once!

Orbital Entry - The Physics

- 32 MJ/kg dissipated by friction with atmosphere over ~8 min = 66kW/kg
- Pure graphite (carbon) high-temperature material: c_p=709 J/kg°K
- Orbital energy would cause temperature gain of 45,000°K!
- Thus proving the comment about space travel, "It's utter bilge!" (Sir Richard Wooley, Astronomer Royal of Great Britain, 1956)

The Vision

"Once you make it to low Earth orbit, you're halfway to anywhere!"

- Robert A. Heinlein

Goals of ENAE 791

- Learn the underlying physics (orbital mechanics, flight mechanics, aerothermodynamics) which constrain and define launch and entry vehicles
- Develop the tools for preliminary design synthesis, including the fundamentals of systems analysis
- Provide an introduction to engineering economics, with a focus on the parameters affecting cost of launch and entry vehicles, such as reusability
- Examine specific challenges in the underlying design disciplines, such as thermal protection and structural dynamics

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Web-based Course Content

- Data web site at spacecraft.ssl.umd.edu
  - Course information
  - Syllabus
  - Lecture notes
  - Problems and solutions
- Interactive web site at bb.eng.umd.edu
  - Communications for team projects (forums, wiki, blogs)
  - Surveys for course feedback
  - Videos of lectures

Syllabus Overview (1)

- Fundamentals of Launch and Entry Design
  - Orbital mechanics
  - Basic rocket performance
- Entry flight mechanics
  - Ballistic entry
  - Lifting entry
- Aerothermodynamics
- Thermal Protection System (TPS) analysis
- Entry, Descent, and Landing (EDL) systems

Syllabus Overview (2)

- Launch flight mechanics
  - Gravity turn
  - Targeted trajectories
  - Optimal trajectories
  - Airbreathing trajectories
- Launch vehicle systems
  - Propulsion systems
  - Structures and structural dynamics analysis
  - Avionics
  - Payload accommodations

Syllabus Overview (3)

- Systems Analysis
  - Cost estimation
  - Engineering economics
  - Reliability issues
  - Safety design concerns
  - Fleet resiliency
- Case studies
- Design project

Policies

- Grade Distribution
  - 25% Problems
  - 20% Midterm Exam
  - 25% Term Project
  - 30% Final Exam
- Late Policy
  - On time: Full credit
  - Before solutions: 70% credit
  - After solutions: 20% credit

Term Project - Space Tourism

- Design a system to carry humans into space and return them to Earth safely
- Must carry at least three people (one crew, two passengers) - larger at your discretion
- Annual market is based on price
  - $10M - 10 people/year
  - $1M - 200 people/year
  - $100K - 4000 people/year
  - $10K - 80,000 people/year

Term Project

- Design a launch and entry system which maximizes rate of return with reasonable investment
- More detailed requirements as needed
- Work as individuals or pairs
- Design process should proceed throughout the term
- Formal design presentations at end of term

Orbital Mechanics: 500 years in 40 min.

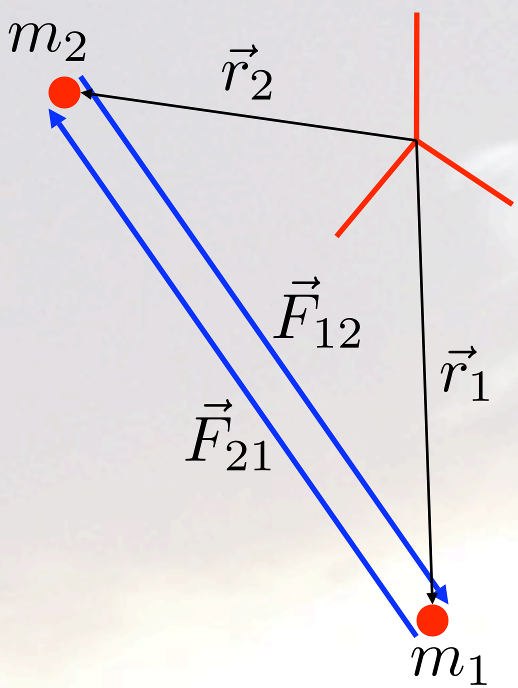
- Newton's Law of Universal Gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

- Newton's First Law meets vector algebra

$$\overrightarrow{F} = m \overrightarrow{a}$$

Relative Motion Between Two Bodies



$$m_1 \frac{d^2\vec{r}_1}{dt^2} = G\frac{m_1 m_2}{|\vec{r}_{21}|^2}\frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

$$= G\frac{m_1 m_2}{|\vec{r}_{21}|^3}\vec{r}_{21} = G\frac{m_1 m_2}{|\vec{r}_{21}|^3}(\vec{r}_2 - \vec{r}_1)$$

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = G\frac{m_1 m_2}{|\vec{r}_{12}|^3}(\vec{r}_1 - \vec{r}_2)$$

$\vec{F}_{12} =$ force due to body 1 on body 2

Gravitational Motion

$$\frac{d^2\vec{r}}{dt^2} = \frac{G}{r^3}\left[m_2\left(-\vec{r}\right) - m_1\left(\vec{r}\right)\right] = \frac{-G}{r^3}\left(m_1 + m_2\right)\vec{r}$$

$$\text{Let } r = |\vec{r}_{12}| = |\vec{r}_{21}| \qquad \text{Let } \vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\text{Let } \mu = G(m_1 + m_2)$$

$$\boxed{\frac{d^2\vec{r}}{dt^2} + \mu\frac{\vec{r}}{r^3} = \vec{0}}$$

"Equation of Orbit" -
Orbital motion is simple harmonic motion

Orbital Angular Momentum

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \frac{d\vec{v}}{dt} + \mu \frac{\vec{r}}{r^3} = \vec{0}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} + \frac{\mu}{r^3}\left(\vec{r} \times \vec{r}\right) = \vec{0} \qquad \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}$$

$$= \vec{v} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \vec{0} \qquad \vec{r} \times \vec{v} = \vec{constant} \qquad \vec{r} \times \vec{v} = \vec{h}$$

$\vec{h}$ is angular momentum vector (constant) $\Longrightarrow$
$\vec{r}$ and $\vec{v}$ are in a constant plane

Fun and Games with Algebra

$$\frac{d\vec{v}}{dt} + \mu\frac{\vec{r}}{r^3} = \vec{0} \qquad \frac{d\vec{v}}{dt} \times \vec{h} + \frac{\mu}{r^3}\left(\vec{r} \times \vec{h}\right) = \vec{0}$$

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = \frac{d\vec{v}}{dt} \times \vec{h} + \vec{v} \times \frac{d\vec{h}}{dt}$$

0

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r} \times \vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r} \times \vec{r} \times \vec{v}\right)$$

$$\frac{d}{dt}\left(\vec{v} \times \vec{h}\right) = -\frac{\mu}{r^3}\left[\left(\vec{r} \cdot \vec{v}\right)\vec{r} - \left(\vec{r} \cdot \vec{r}\right)\vec{v}\right]$$

$$\vec{r} \cdot \vec{v} = rv\cos\gamma = r\frac{dr}{dt}$$

More Algebra, More Fun

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\frac{\mu}{r^3}\left[r\frac{dr}{dt}\vec{r} - r^2\frac{d\vec{r}}{dt}\right]$$

$$\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{\left(r\frac{d\vec{r}}{dt} - \vec{r}\frac{dr}{dt}\right)}{r^2} = \left(\frac{1}{r}\frac{d\vec{r}}{dt} - \frac{\vec{r}}{r^2}\frac{dr}{dt}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\mu\left(\frac{1}{r^2}\frac{dr}{dt}\vec{r} - \frac{1}{r}\frac{d\vec{r}}{dt}\right) = \mu\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h} - \mu\frac{\vec{r}}{r}\right) = \vec{0}$$

Orientation of the Orbit

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \vec{\text{constant}}$$

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \mu \vec{e}$$

$\vec{e} \equiv$ eccentricity vector, in orbital plane

$\vec{e}$ points in the direction of periapsis

$$\vec{r} \cdot \vec{v} \times \vec{h} - \vec{r} \cdot \mu \frac{\vec{r}}{r} = \mu \left(\vec{r} \cdot \vec{e} \right)$$

$$\vec{r} \times \vec{v} \cdot \vec{h} - \mu \frac{\vec{r} \cdot \vec{r}}{r} = \mu r e \cos \theta$$

$$\vec{h} \cdot \vec{h} - \mu \frac{r^2}{r} = \mu r e \cos \theta$$

Position in Orbit

$$h^2 - \mu r = \mu r e \cos\theta$$

$$r = \frac{h^2/\mu}{1 + e\cos\theta}$$

θ = true anomaly: angular travel from perigee passage

$$\text{at } \theta = \pm\frac{\pi}{2};\ \cos\theta = 0;\ r = p \equiv h^2/\mu$$

Relating Velocity and Orbital Elements

$$\mu\vec{e} = \vec{v} \times \vec{h} - \mu\frac{\vec{r}}{r}$$

$$\mu\vec{e} \cdot \mu\vec{e} = \vec{v} \times \vec{h} \cdot \vec{v} \times \vec{h} - 2\mu\left(\vec{v} \times \vec{h}\right) \cdot \frac{\vec{r}}{r} + \mu^2\left(\frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r}\right)$$

$$\mu^2 e^2 = v^2 h^2 - 2\mu\frac{h^2}{r} + \mu^2$$

$$e^2 = \frac{v^2}{\mu}p - 2\frac{p}{r} + 1$$

$$p \equiv a(1 - e^2) = \frac{1 - e^2}{\frac{2}{r} - \frac{v^2}{\mu}}$$

Vis-Viva Equation

$$v^2 = \mu \left(\frac{2}{r} - \frac{1}{a} \right)$$

$$-\frac{\mu}{2a} = \frac{v^2}{2} - \frac{\mu}{r}$$

Energy in Orbit

- Kinetic Energy

$$K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}$$

- Potential Energy

$$P.E. = -\frac{m\mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}$$

- Total Energy

$$Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

<--Vis-Viva Equation

How Close are we to Space Tourism?

- Energy for 100 km vertical climb

$$-\frac{\mu}{r_E + 100\ km} + \frac{\mu}{r_E} = 0.965\ \frac{km^2}{sec^2} = 0.965\ \frac{MJ}{kg}$$

- Energy for 200 km circular orbit

$$-\frac{\mu}{2(r_E + 200\ km)} + \frac{\mu}{r_E} = 32.2\ \frac{km^2}{sec^2} = 32.2\ \frac{MJ}{kg}$$

- Energy difference is a factor of 33.4!

Implications of Vis-Viva

- Circular orbit (r=a)

$$v_{circular} = \sqrt{\frac{\mu}{r}}$$

- Parabolic escape orbit (a tends to infinity)

$$v_{escape} = \sqrt{\frac{2\mu}{r}}$$

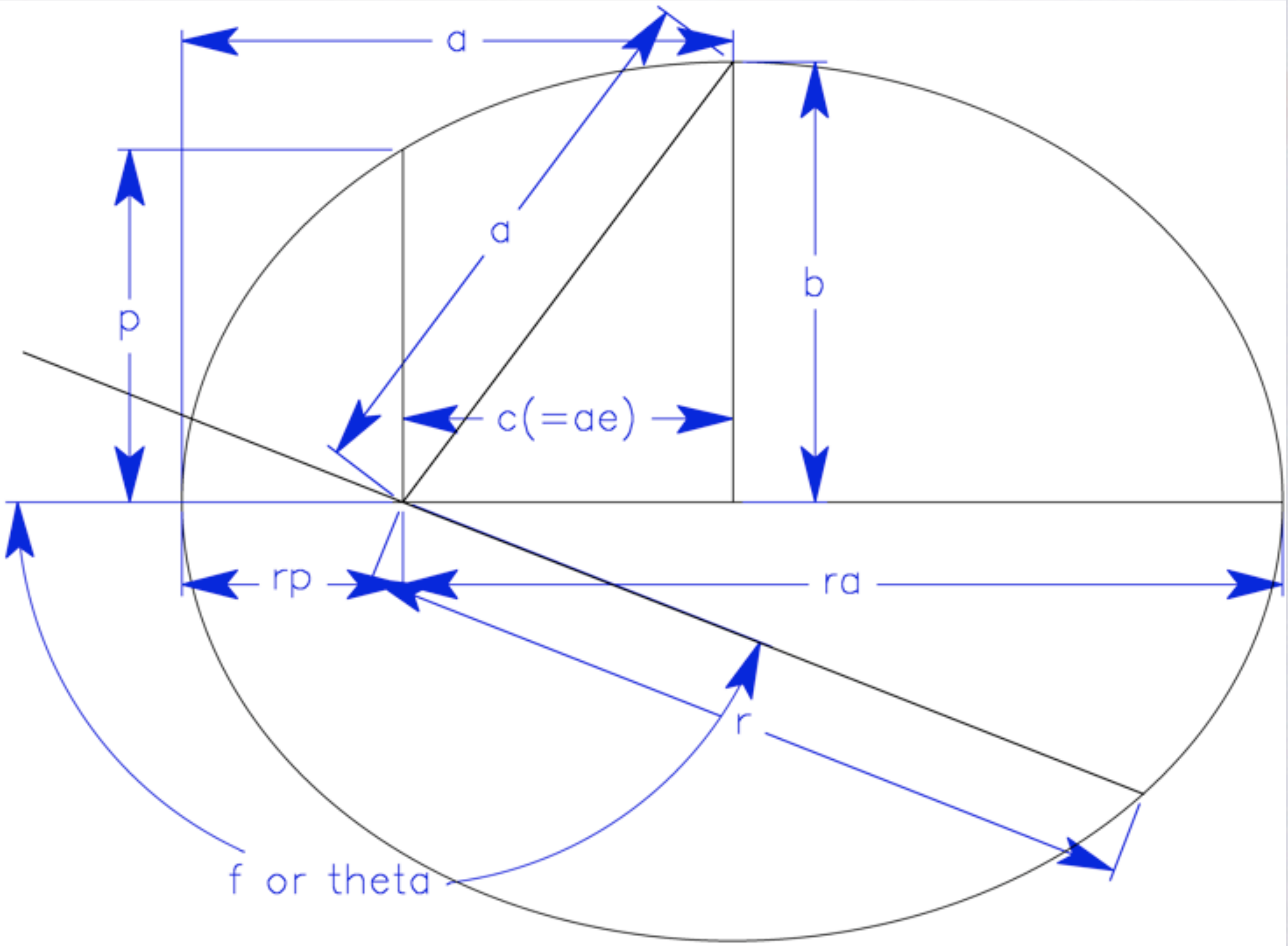
- Relationship between circular and parabolic orbits

$$v_{escape} = \sqrt{2}v_{circular}$$

Some Useful Constants

- Gravitation constant $\mu = GM$
  - Earth: 398,604 km^3/sec^2
  - Moon: 4667.9 km^3/sec^2
  - Mars: 42,970 km^3/sec^2
  - Sun: 1.327×10^{11} km^3/sec^2
- Planetary radii
  - $r_{Earth} = 6378$ km
  - $r_{Moon} = 1738$ km
  - $r_{Mars} = 3393$ km

Classical Parameters of Elliptical Orbits



Basic Orbital Parameters

- Semi-latus rectum (or parameter)

$$p = a(1 - e^2)$$

- Radial distance as function of orbital position

$$r = \frac{p}{1 + e\cos\theta}$$

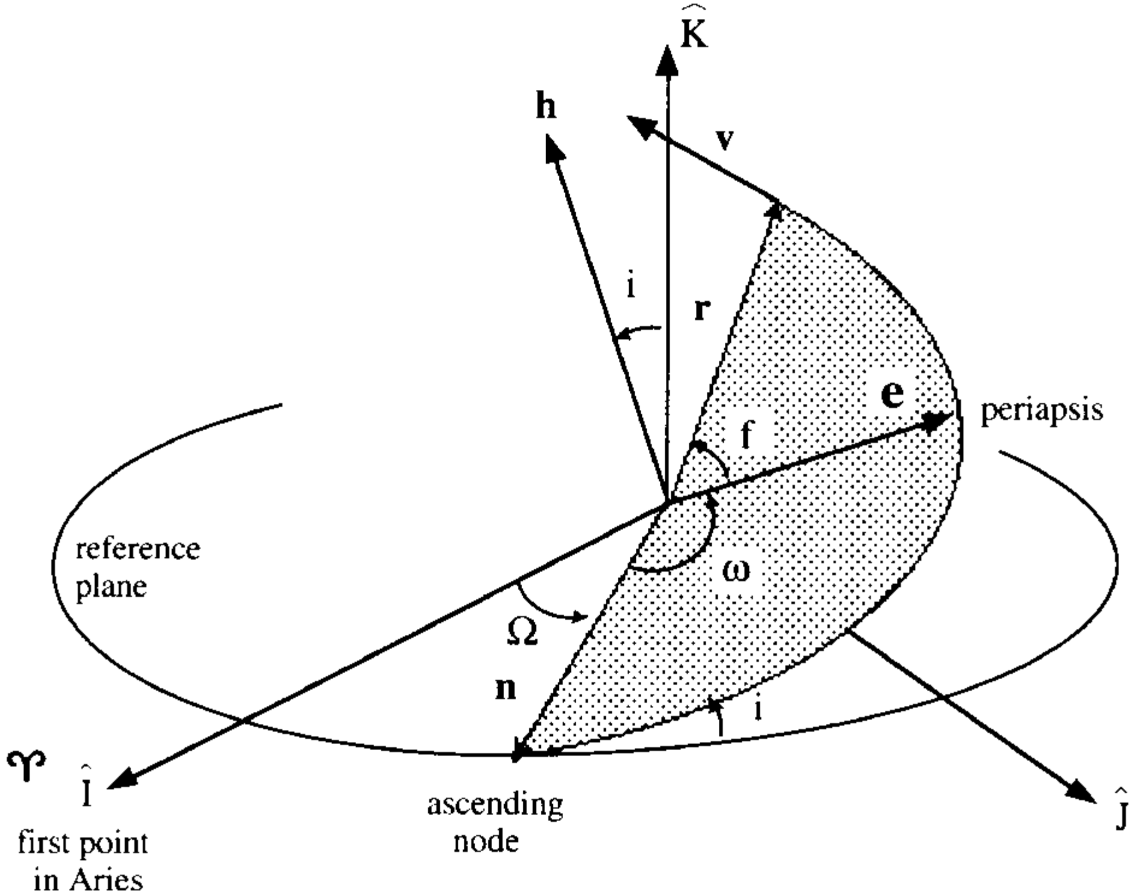
- Periapse and apoapse distances

$$r_p = a(1 - e) \qquad r_a = a(1 + e)$$

- Angular momentum

$$\vec{h} = \vec{r} \times \vec{v} \qquad h = \sqrt{\mu p}$$

The Classical Orbital Elements



Ω : longitude of the ascending node

ω : argument of periapsis

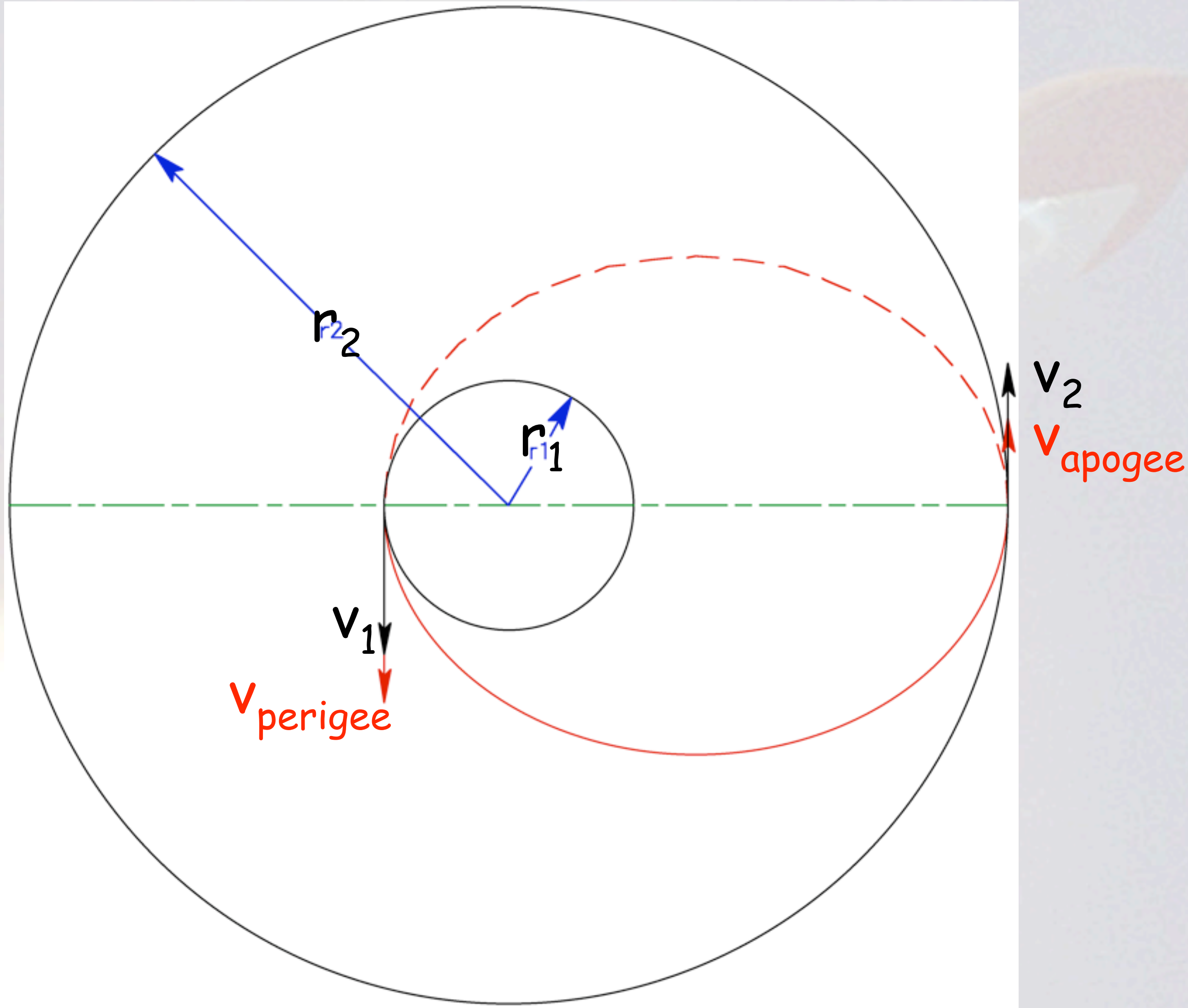
$\tilde{\omega} = \Omega + \omega$: longitude of periapsis

f : true anomaly

$L = \tilde{\omega} + f$: true longitude

Ref: J. E. Prussing and B. A. Conway, Orbital Mechanics Oxford University Press, 1993

The Hohmann Transfer



First Maneuver Velocities

- Initial vehicle velocity

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

- Needed final velocity

$$v_{perigee} = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$

- Delta-V

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right)$$

Second Maneuver Velocities

- Initial vehicle velocity

$$v_{apogee} = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$$

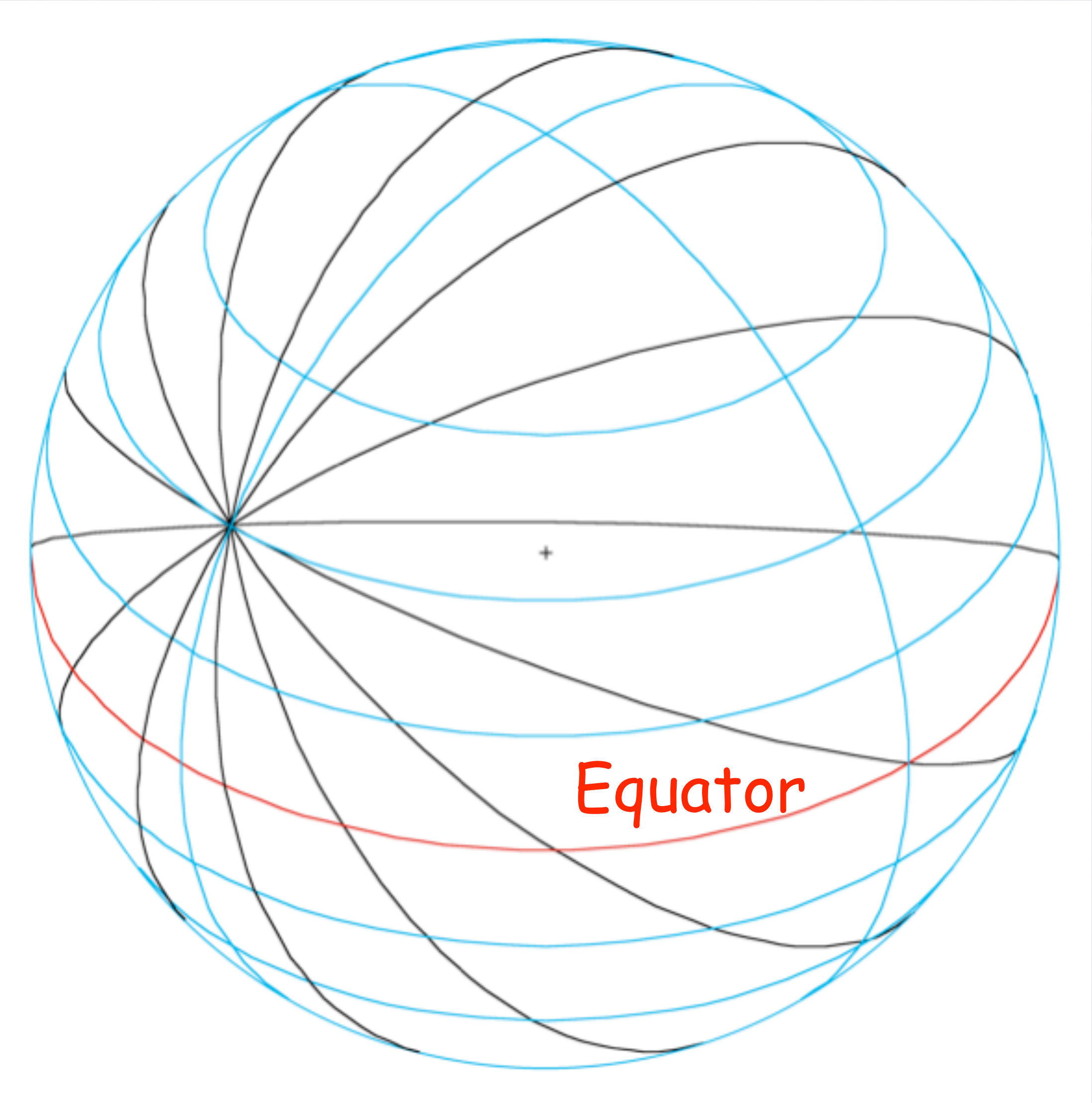
- Needed final velocity

$$v_2 = \sqrt{\frac{\mu}{r_2}}$$

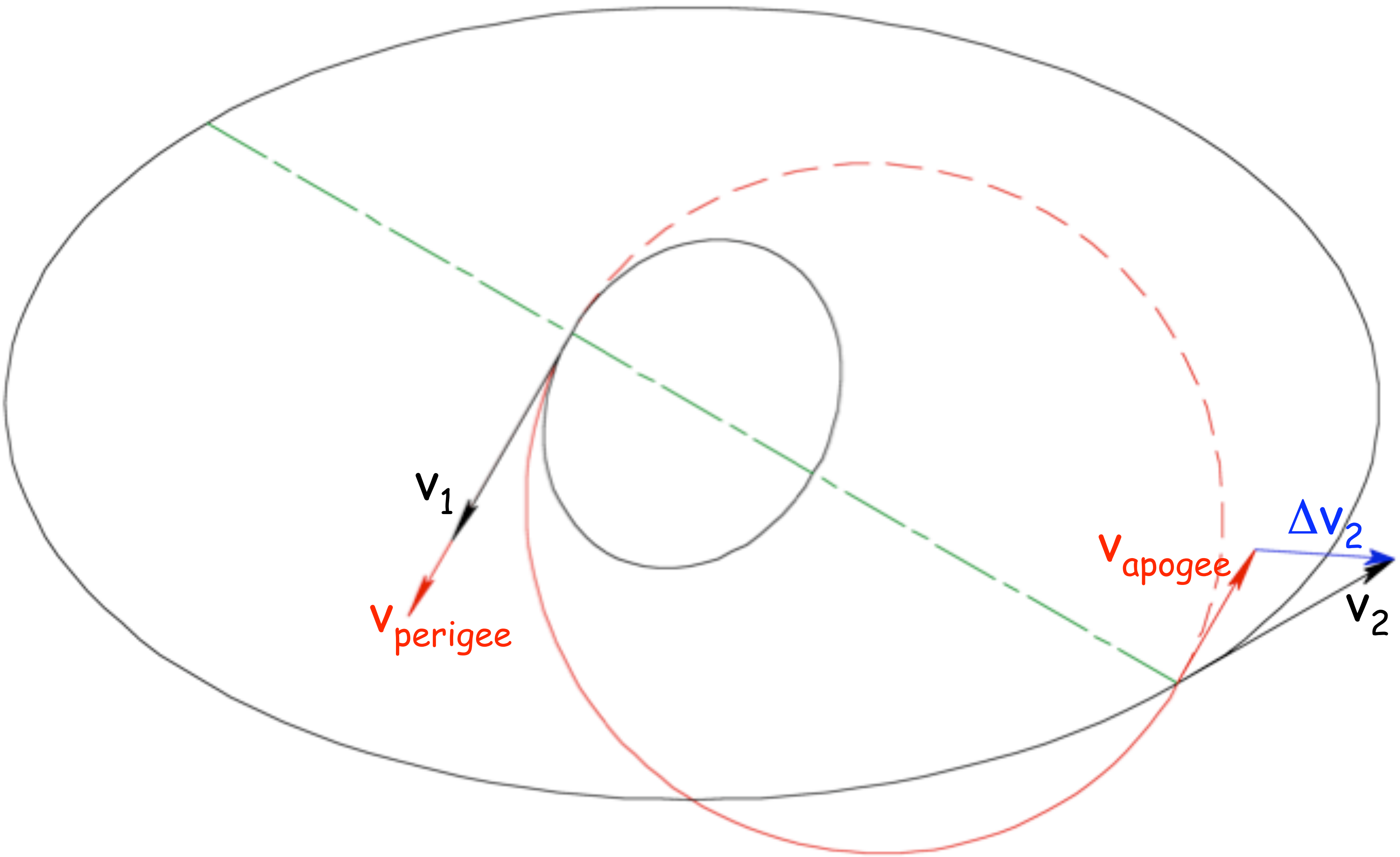
- Delta-V

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right)$$

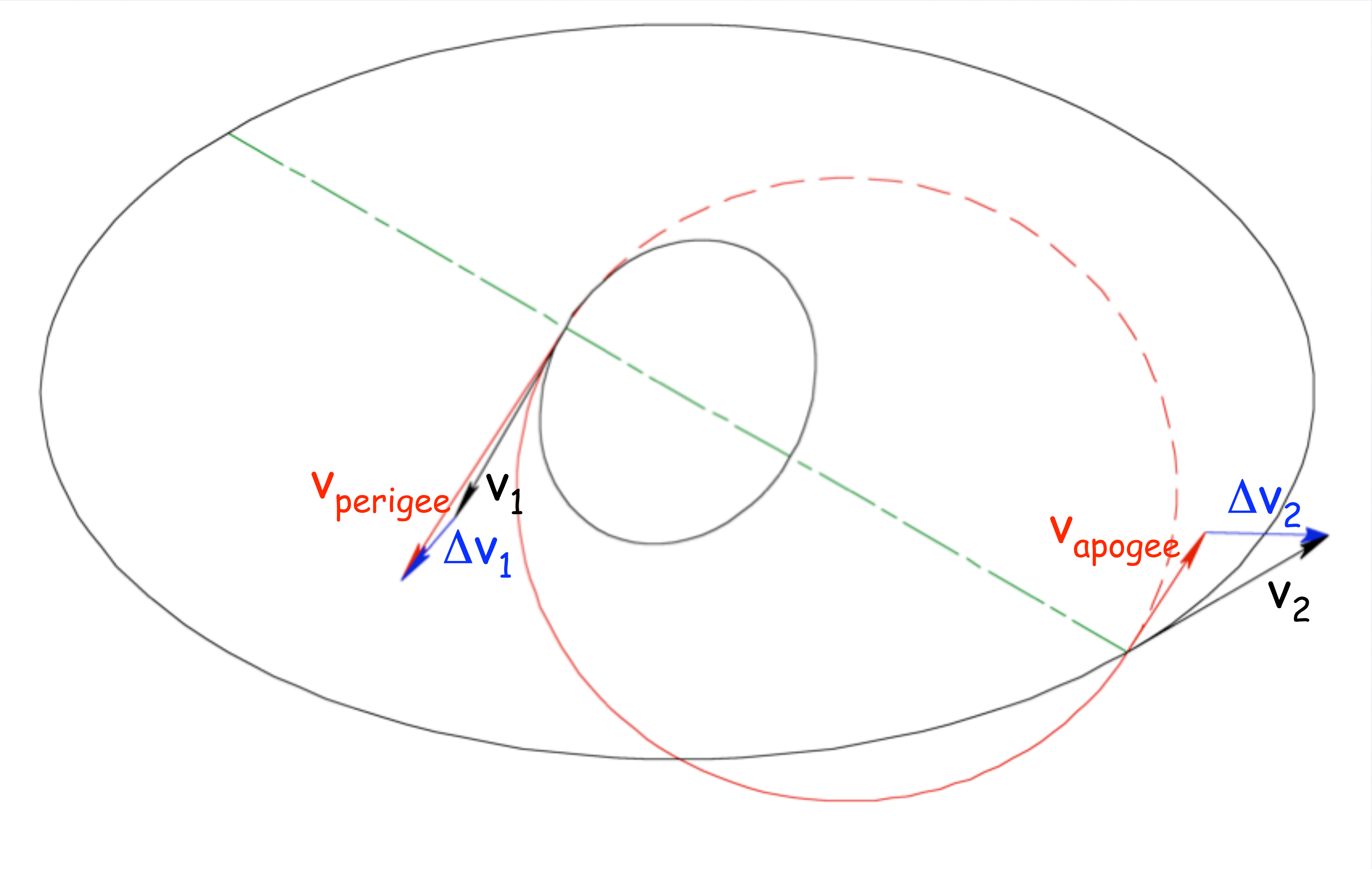
Limitations on Launch Inclinations



Simple Plane Change



Optimal Plane Change



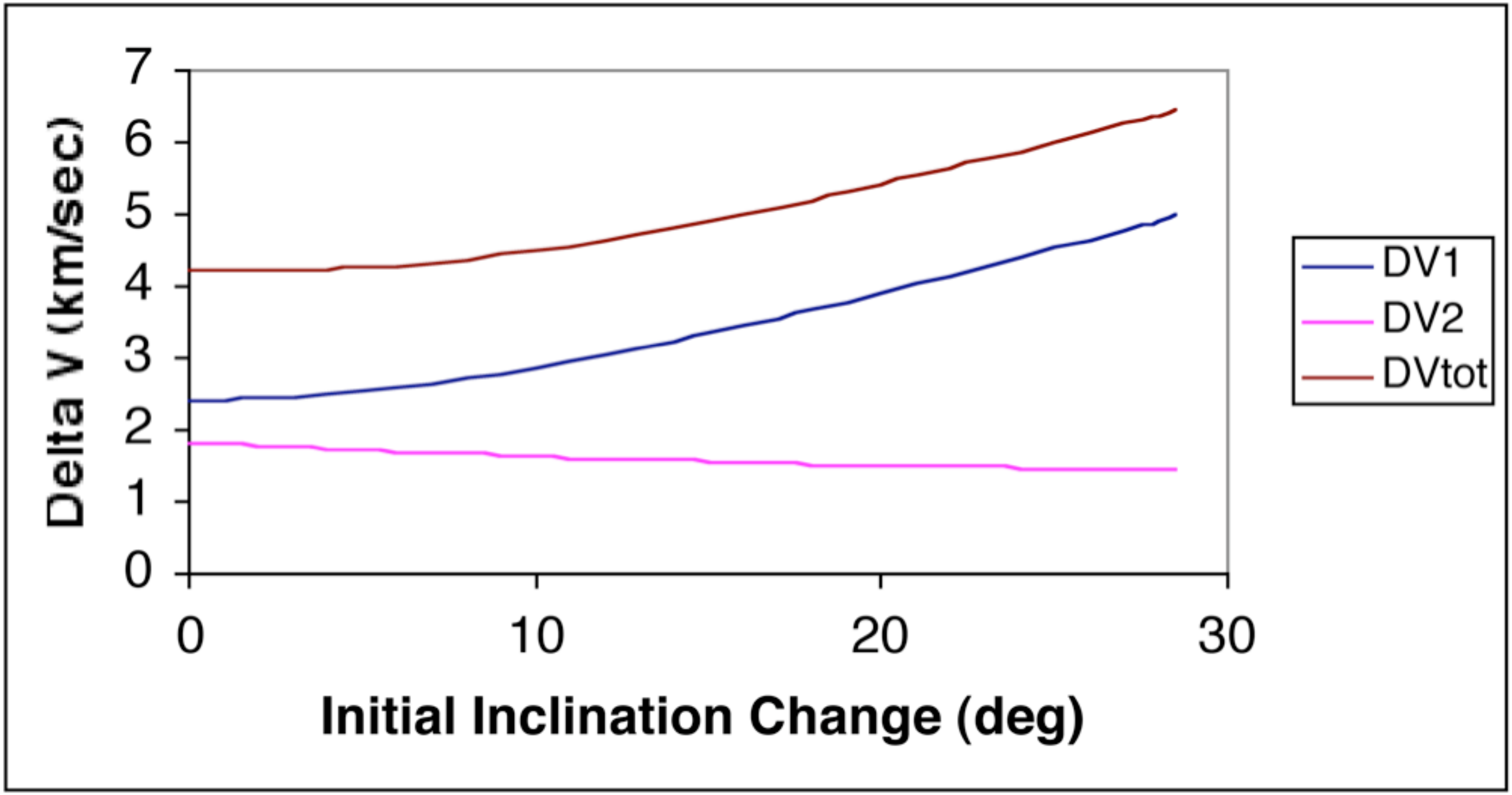
First Maneuver with Plane Change Δi_1

- Initial vehicle velocity $$v_1 = \sqrt{\frac{\mu}{r_1}}$$
- Needed final velocity $$v_p = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}}$$
- Delta-V $$\Delta v_1 = \sqrt{v_1^2 + v_p^2 - 2v_1 v_p \cos(\Delta i_1)}$$

Second Maneuver with Plane Change Δi_2

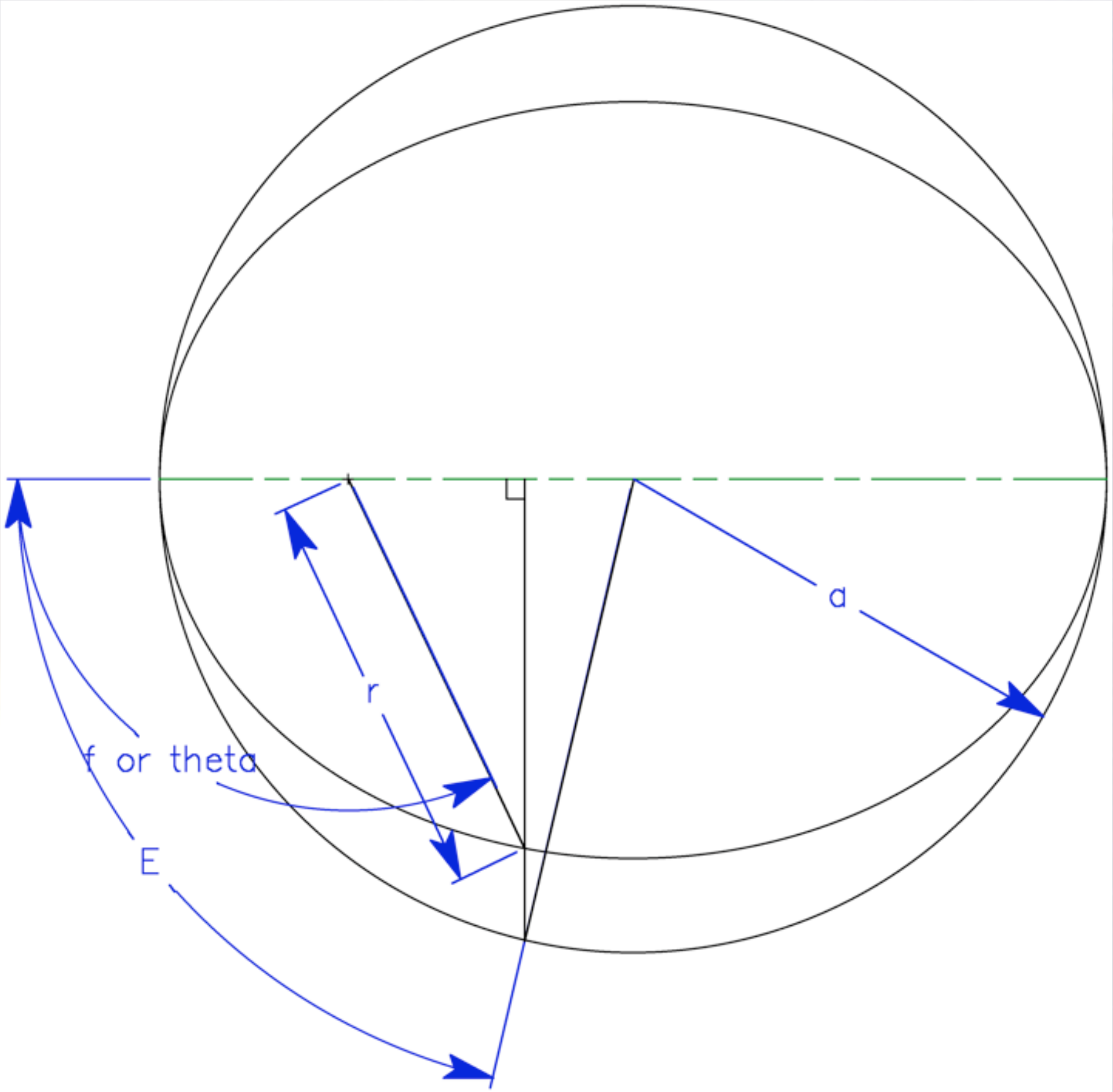
- Initial vehicle velocity $$v_a = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}}$$
- Needed final velocity $$v_2 = \sqrt{\frac{\mu}{r_2}}$$
- Delta-V $$\Delta v_2 = \sqrt{v_2^2 + v_a^2 - 2v_2 v_a \cos(\Delta i_2)}$$

Sample Plane Change Maneuver



Optimum initial plane change = 2.20°

Calculating Time in Orbit



Time in Orbit

- Period of an orbit

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

- Mean motion (average angular velocity)

$$n = \sqrt{\frac{\mu}{a^3}}$$

- Time since pericenter passage

$$M = nt = E - e\sin E$$

M=mean anomaly

Dealing with the Eccentric Anomaly

- Relationship to orbit

$$r = a(1 - e\cos E)$$

- Relationship to true anomaly

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}$$

- Calculating M from time interval: iterate

$$E_{i+1} = nt + e\sin E_i$$

until it converges

Example: Time in Orbit

- Hohmann transfer from LEO to GEO
  - h_1=300 km --> r_1=6378+300=6678 km
  - r_2=42240 km
- Time of transit (1/2 orbital period)

$$a = \frac{1}{2}(r_1 + r_2) = 24,459km$$

$$t_{\mathrm{transit}} = \frac{P}{2} = \pi\sqrt{\frac{a^3}{\mu}} = 19,034sec = 5h17m14s$$

Example: Time-based Position

Find the spacecraft position 3 hours after perigee

$$n = \sqrt{\frac{\mu}{a^3}} = 1.650 \times 10^{-4} \frac{rad}{sec}$$

$$e = 1 - \frac{r_p}{a} = 0.7270$$

$$E_{j+1} = nt + e \sin E_j = 1.783 + 0.7270 \sin E_j$$

E=0; 1.783; 2.494; 2.222; 2.361; 2.294; 2.328; 2.311; 2.320; 2.316; 2.318; 2.317; 2.317; 2.317

Example: Time-based Position (continued)

$$E = 2.317$$

$$r = a(1 + e\cos E) = 12,387km$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2} \Longrightarrow \theta = 160\,\mathrm{deg}$$

Have to be sure to get the position in the proper quadrant - since the time is less than 1/2 the period, the spacecraft has yet to reach apogee --> $0° < \theta < 180°$

Velocity Components in Orbit

$$r = \frac{p}{1 + e\cos\theta}$$

$$v_r = \frac{dr}{dt} = \frac{d}{dt}\left(\frac{p}{1 + e\cos\theta}\right) = \frac{-p\left(-e\sin\theta\,\frac{d\theta}{dt}\right)}{\left(1 + e\cos\theta\right)^2}$$

$$v_r = \frac{pe\sin\theta}{\left(1 + e\cos\theta\right)^2}\frac{d\theta}{dt}$$

$$1 + e\cos\theta = \frac{p}{r} \Longrightarrow v_r = \frac{r^2\frac{d\theta}{dt}e\sin\theta}{p}$$

$$\vec{h} = \vec{r} \times \vec{v}$$

Velocity Components in Orbit (continued)

$$\vec{h} = \vec{r} \times \vec{v}$$

$$h = rv\cos\gamma = r\left(r\frac{d\theta}{dt}\right) = r^2\frac{d\theta}{dt}$$

$$v_r = \frac{r^2\frac{d\theta}{dt}e\sin\theta}{p} = \frac{he\sin\theta}{p} = \frac{\sqrt{p\mu}}{p}e\sin\theta$$

$$v_r = \sqrt{\frac{\mu}{p}}e\sin\theta$$

$$v_\theta = r\frac{d\theta}{dt} = r\frac{h}{r^2} = \frac{h}{r} = \frac{\sqrt{p\mu}}{r}$$

$$v_\theta = \sqrt{\frac{\mu}{p}}(1+e\cos\theta)$$

$$\tan\gamma = \frac{v_r}{v_\theta} = \frac{e\sin\theta}{1+e\cos\theta}$$