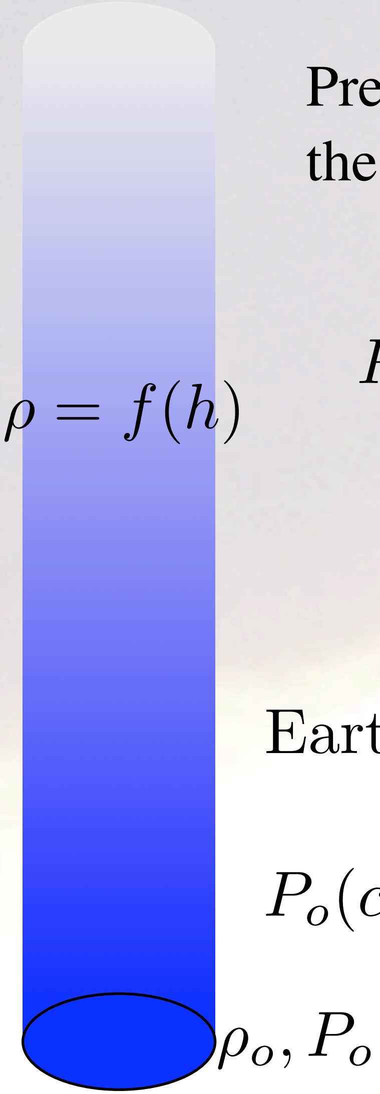
Ballistic Atmospheric Entry (Part II)

- Standard atmosphere revisited
- Straight-line (no gravity) ballistic entry based on altitude, rather than density
- Planetary entries (at least a start)

Atmospheric Density with Altitude



Pressure=the integral of the atmospheric density in the column above the reference area

$$P_o = \int_o^\infty \rho dh = \rho_o \int_o^\infty e^{-\frac{h}{h_s}} dh = -\rho_o h_s \left[e^{-\frac{h}{h_s}} \right]_o^\infty$$

$$= -\rho_o h_s [0 - 1]$$

$$\boxed{P_o = \rho_o h_s}$$

$$\text{Earth: } \rho_o = 1.226 \frac{kg}{m^3};\ h_s = 7524m;$$

$$P_o(calc) = 9224 \frac{kg}{m^2} = 90,400\ Pa;\ P_o(act) = 101,300\ Pa$$

Nondimensional Ballistic Coefficient

$$\frac{v}{v_e} = \exp\left(\frac{h_s \rho_o}{2\beta \sin\gamma} \frac{\rho}{\rho_o}\right) \quad = \exp\left(\frac{P_o}{2\beta \sin\gamma} \frac{\rho}{\rho_o}\right)$$

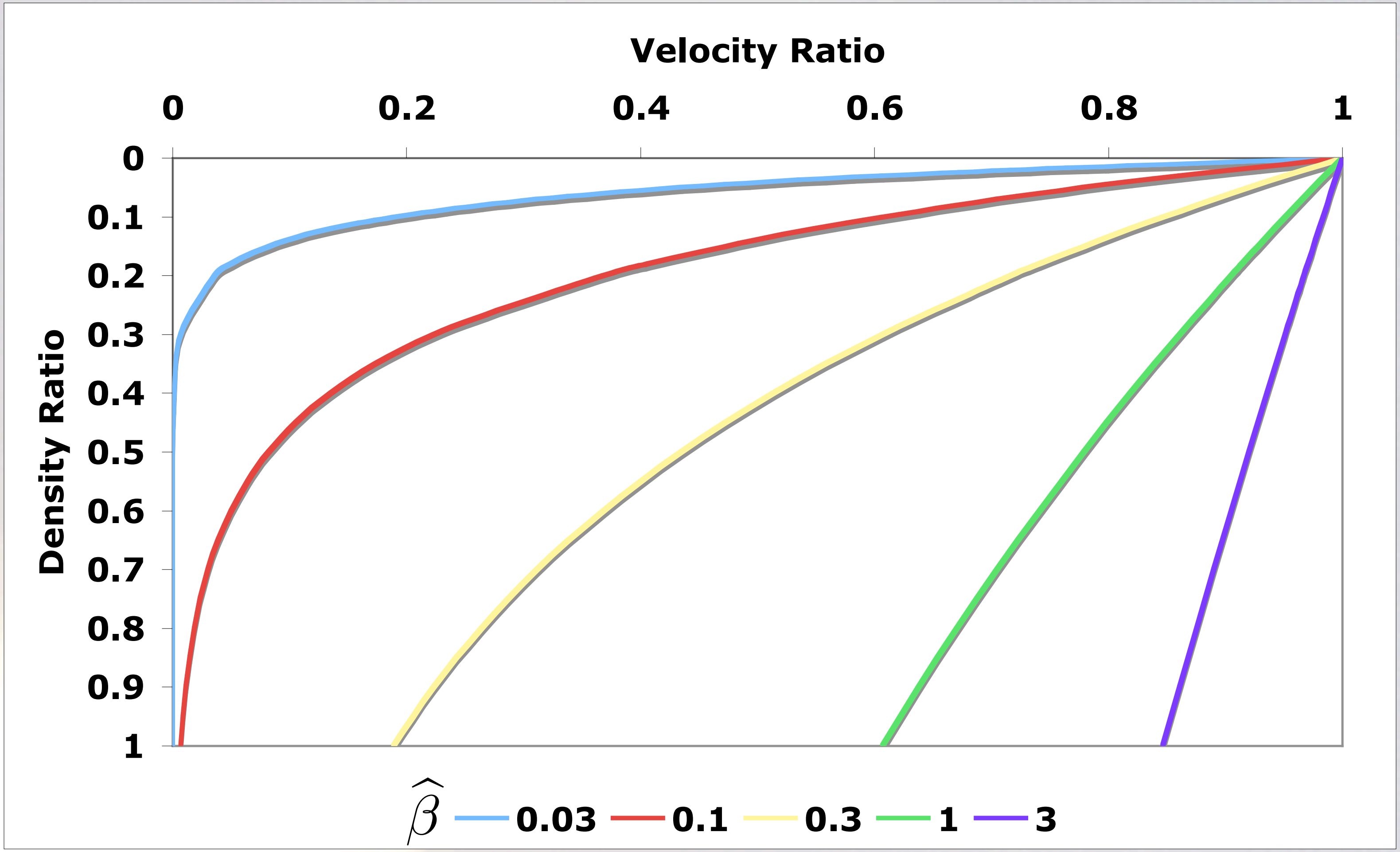
Let $\widehat{\beta} \equiv \dfrac{\beta}{P_o}$ (Nondimensional form of ballistic coefficient)

Note that we are using the estimated value of $P_o = \rho_o h_s$, not the actual surface pressure.

$$\frac{v}{v_e} = \exp\left(\frac{1}{2\widehat{\beta} \sin\gamma} \frac{\rho}{\rho_o}\right)$$

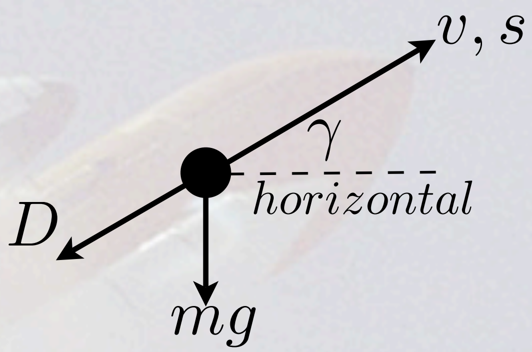
$$\beta_{crit} = -\frac{\rho_o h_s}{\sin\gamma} \qquad \widehat{\beta}_{crit} = -\frac{1}{\sin\gamma}$$

Entry Velocity Trends, γ=-90°



Ballistic Entry (no lift)

$s =$ distance along the flight path



$$\frac{dv}{dt} = -g\sin\gamma - \frac{D}{m}$$

Again assuming $D \gg g$,

$$\frac{dv}{dt} = -\frac{D}{m} \qquad \text{Drag } D \equiv \frac{1}{2}\rho v^2 A c_D$$

$$\frac{dv}{dt} = -\frac{\rho c_D A}{2m} v^2$$

Separating the variables,

$$\frac{dv}{v^2} = -\frac{\rho}{2\beta} dt$$

Calculating the Entry Velocity Profile

$$\frac{dh}{dt} = v\sin\gamma \Rightarrow dt = \frac{dh}{v\sin\gamma}$$

$$\frac{dv}{v^2} = -\frac{\rho}{2\beta v\sin\gamma}dh \quad \Rightarrow \frac{dv}{v} = -\frac{\rho}{2\beta\sin\gamma}dh$$

$$\frac{dv}{v} = -\frac{\rho_o}{2\beta\sin\gamma}e^{-\frac{h}{hs}}dh$$

$$\int_{v_e}^{v}\frac{dv}{v} = -\frac{\rho_o}{2\beta\sin\gamma}\int_{h_e}^{h}e^{-\frac{h}{hs}}dh$$

$$\ln\frac{v}{v_e} = \frac{\rho_o h_s}{2\beta\sin\gamma}\left[e^{-\frac{h}{hs}}\right]_{h_e}^{h} = \frac{1}{2\widehat{\beta}\sin\gamma}\left[e^{-\frac{h}{hs}} - e^{-\frac{he}{hs}}\right]$$

Remember that $e^{-\frac{h_e}{h_s}} = \frac{\rho_e}{\rho_o} \approx 0$

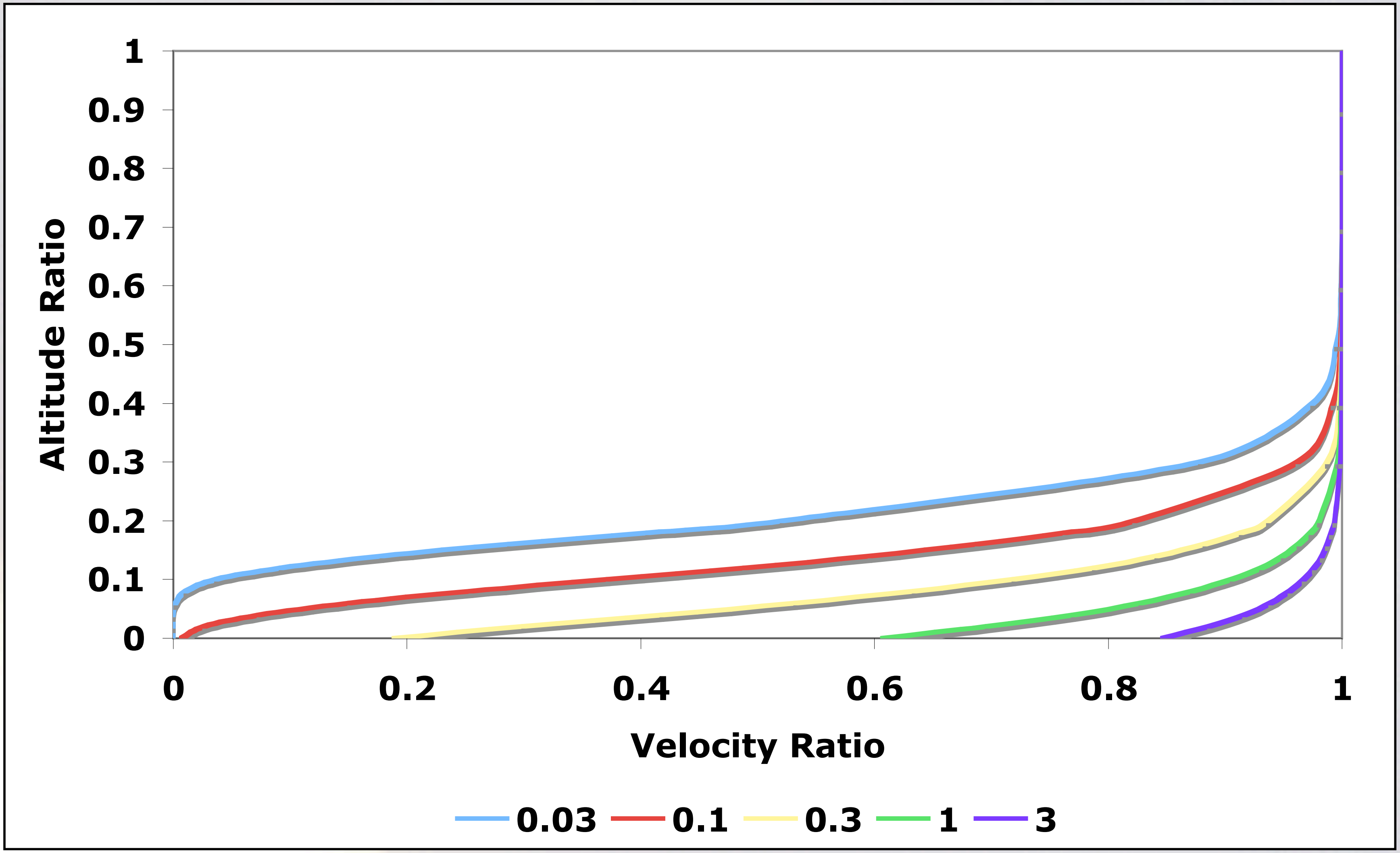
$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_s}}\right]$$

We have a parametric entry equation in terms of nondimensional velocity ratios, ballistic coefficient, and altitude. To bound the nondimensional altitude variable between 0 and 1, rewrite as

$$\boxed{\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_e}\frac{h_e}{h_s}}\right]}$$

$\frac{h_e}{h_s}$ and $\widehat{\beta}$ are the only variables that relate to a specific planet

Earth Entry, γ=-90°



Deceleration as a Function of Altitude

Start with

$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma} e^{-\frac{h}{he}\frac{h_e}{h_s}}\right]$$

Let $B \equiv \dfrac{1}{2\widehat{\beta}\sin\gamma}$

$$\frac{v}{v_e} = \exp\left(Be^{-\frac{h}{h_s}}\right)$$

$$\frac{d}{dt}\left(\frac{v}{v_e}\right) = \exp\left(Be^{-\frac{h}{h_s}}\right)\frac{d}{dt}\left(Be^{-\frac{h}{h_s}}\right)$$

$$\frac{dv}{dt} = v_e \exp\left(Be^{-\frac{h}{h_s}}\right)\frac{-B}{h_s}\left(e^{-\frac{h}{h_s}}\right)\frac{dh}{dt}$$

$$\frac{dh}{dt} = v\sin\gamma \quad = v_e \sin\gamma \exp Be^{-\frac{h}{h_s}}$$

Parametric Deceleration

$$\frac{dv}{dt} = v_e \exp\left(Be^{-\frac{h}{h_s}}\right) \frac{-B}{h_s} \left(e^{-\frac{h}{h_s}}\right) v_e \sin\gamma \exp\left(Be^{-\frac{h}{h_s}}\right)$$

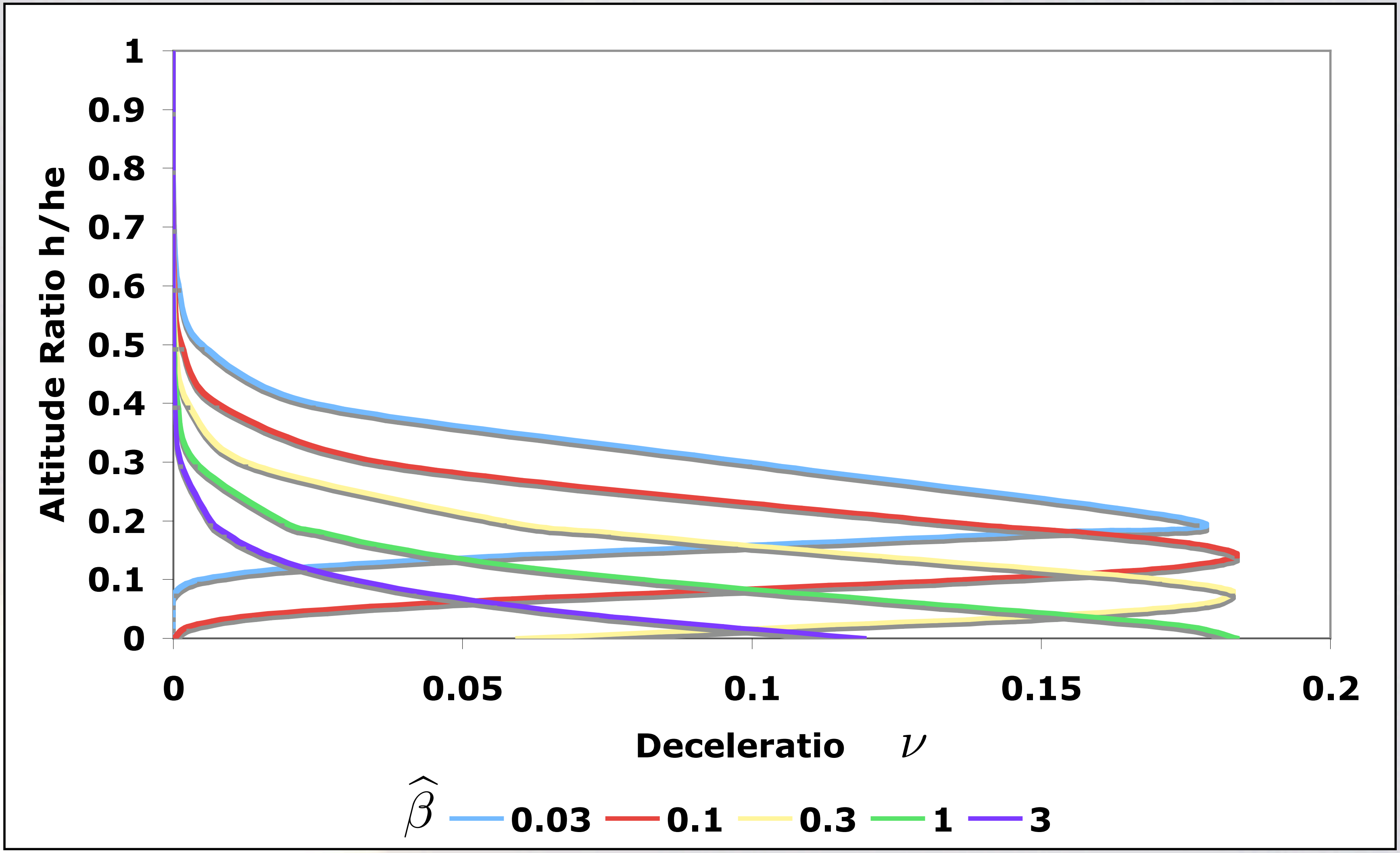
$$\frac{dv}{dt} = \frac{-Bv_e^2}{h_s} \sin\gamma \left(e^{-\frac{h}{h_s}}\right) \exp\left(2Be^{-\frac{h}{h_s}}\right)$$

$$\frac{dv}{dt} = \frac{-v_e^2}{2h_s\widehat{\beta}} \left(e^{-\frac{h}{h_s}}\right) \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\frac{h}{h_s}}\right)$$

$$\text{Let } n_{ref} \equiv \frac{v_e^2}{h_s},\ \nu \equiv \frac{dv/dt}{n_{ref}},\ \varphi \equiv \frac{h_e}{h_s}$$

$$\nu = \frac{-1}{2\widehat{\beta}} \left(e^{-\varphi\frac{h}{h_e}}\right) \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\varphi\frac{h}{h_e}}\right)$$

Nondimensional Deceleration, γ=-90°



Deceleration Equations

Nondimensional Form

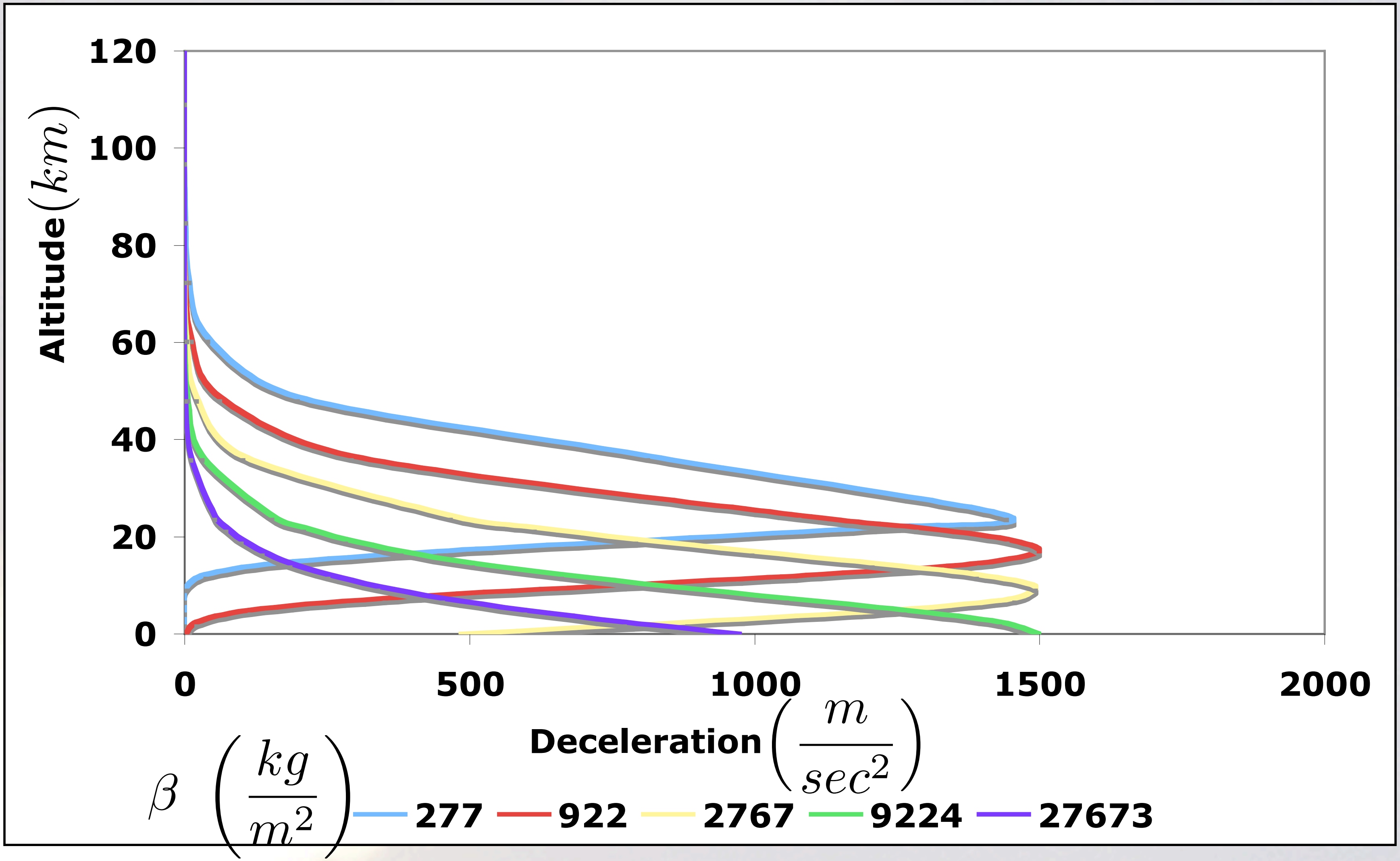
$$\nu = \frac{-1}{2\widehat{\beta}}\left(e^{-\varphi\frac{h}{h_e}}\right)\exp\left(\frac{1}{\widehat{\beta}\sin\gamma}e^{-\varphi\frac{h}{h_e}}\right)$$

Dimensional Form

$$n = \frac{-\rho_o V_e^2}{2\beta}\left(e^{-\frac{h}{h_s}}\right)\exp\left(\frac{\rho_o h_s}{\beta\sin\gamma}e^{-\frac{h}{h_s}}\right)$$

Note that these equations result in values <0 (reflecting deceleration) - graphs are absolute values of deceleration for clarity.

Dimensional Deceleration, γ=-90°



Altitude of Maximum Deceleration

Returning to shorthand notation for deceleration

$$\nu = -B\sin\gamma\left(e^{-\frac{h}{h_s}}\right)\exp\left(2Be^{-\frac{h}{h_s}}\right)$$

$$\text{Let } \eta \equiv \frac{h}{h_s}$$

$$\nu = -B\sin\gamma\left(e^{-\eta}\right)\exp\left(2Be^{-\eta}\right)$$

$$\frac{d\nu}{d\eta} = -B\sin\gamma\left[\frac{d}{d\eta}\left(e^{-\eta}\right)\exp\left(2Be^{-\eta}\right) + \left(e^{-\eta}\right)\frac{d}{d\eta}\exp\left(2Be^{-\eta}\right)\right]$$

$$\frac{d\nu}{d\eta} = -B\sin\gamma\left[-\left(e^{-\eta}\right)\exp\left(2Be^{-\eta}\right) + \left(e^{-\eta}\right)\left(-2Be^{-\eta}\right)\exp\left(2Be^{-\eta}\right)\right]$$

$$\frac{d\nu}{d\eta} = B\sin\gamma e^{-\eta}\exp\left(2Be^{-\eta}\right)\left[1 + \left(2Be^{-\eta}\right)\right] = 0$$

Altitude of Maximum Deceleration

$$1 + \left(2Be^{-\eta}\right) = 0 \quad \Rightarrow e^{\eta} = -2B$$

$$\eta_{n_{max}} = \ln\left(-2B\right)$$

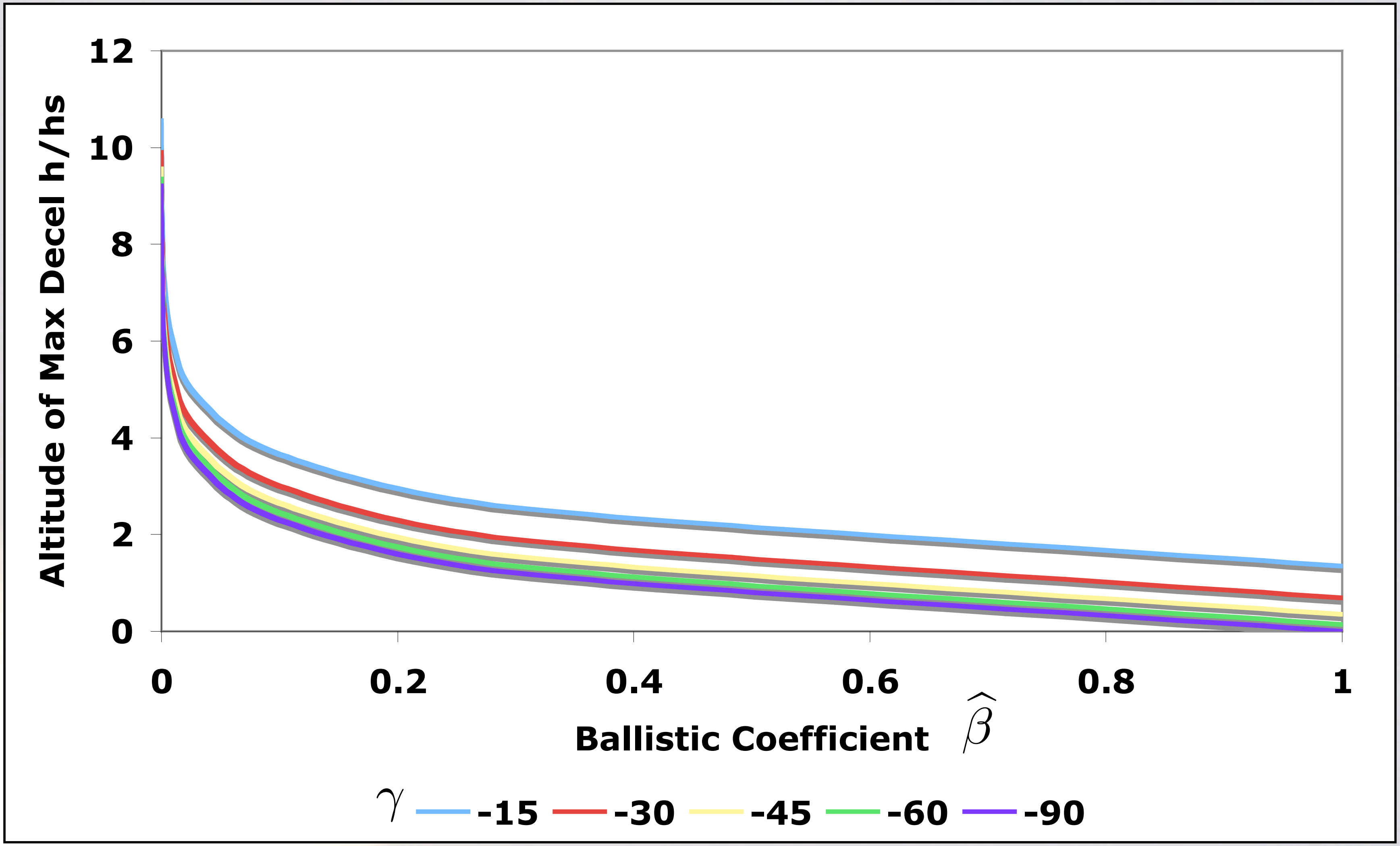
$$\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right)$$

Converting from parametric to dimensional form gives

$$h_{n_{max}} = h_s \ln\left(\frac{-\rho_o h_s}{\beta\sin\gamma}\right)$$

Altitude of maximum deceleration is independent of entry velocity!

Altitude of Maximum Deceleration



Magnitude of Maximum Deceleration

Start with the equation for acceleration -

$$\nu = \frac{-1}{2\widehat{\beta}} e^{-\eta} \exp\left(\frac{1}{\widehat{\beta}\sin\gamma} e^{-\eta}\right)$$

and insert the value of η at the point of maximum deceleration

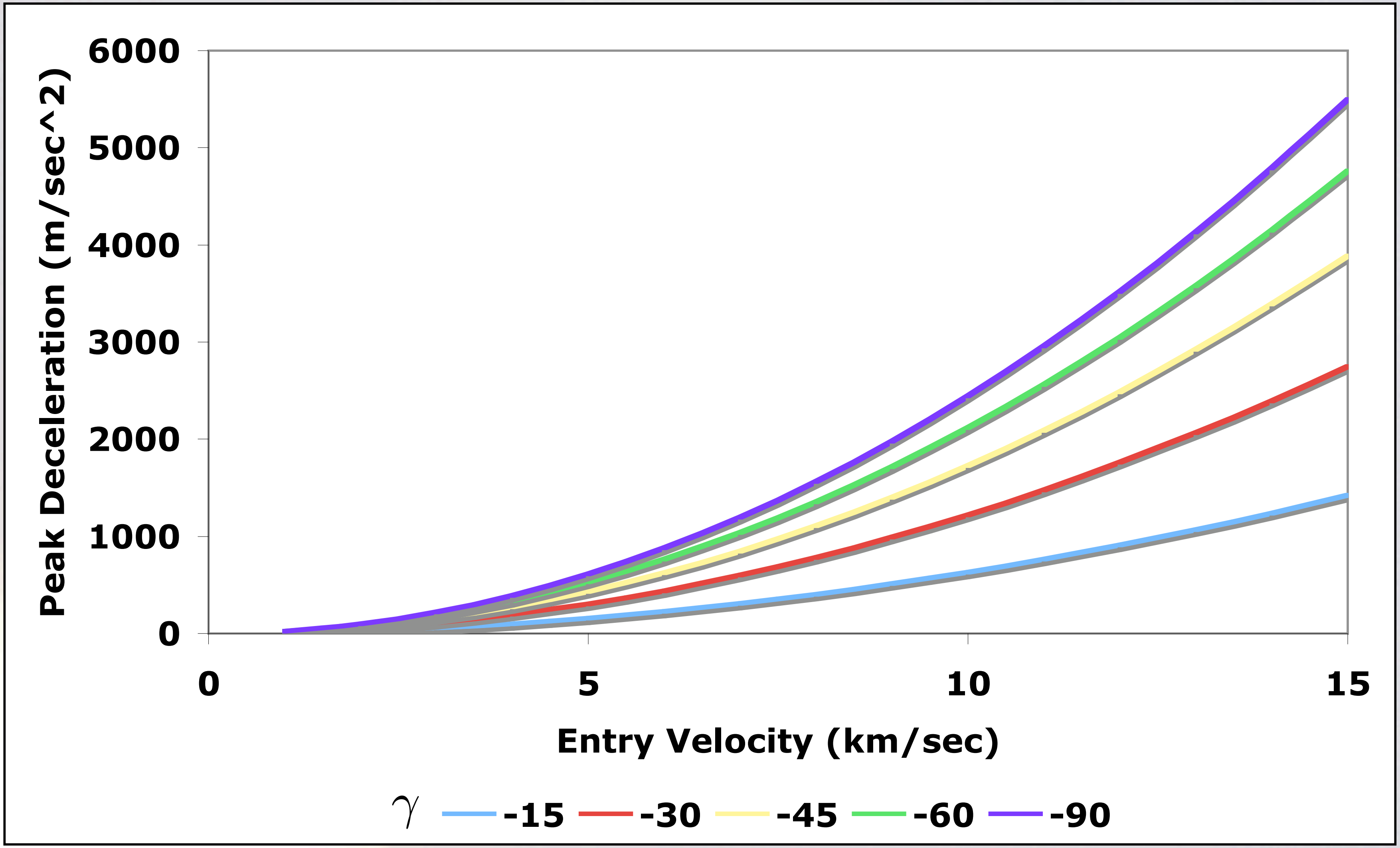
$$\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right) \Rightarrow e^{-\eta} = -\widehat{\beta}\sin\gamma$$

$$\nu_{n_{max}} = \frac{-1}{2\widehat{\beta}}\left(-\widehat{\beta}\sin\gamma\right)\exp\left(\frac{-\widehat{\beta}\sin\gamma}{\widehat{\beta}\sin\gamma}\right) \Rightarrow \nu_{n_{max}} = \frac{\sin\gamma}{2e}$$

$$n_{max} = \frac{v_e^2}{h_s}\frac{\sin\gamma}{2e}$$

Maximum deceleration is not a function of ballistic coefficient!

Peak Ballistic Deceleration for Earth Entry



Velocity at Maximum Deceleration

Start with the equation for velocity

$$\frac{v}{v_e} = \exp\left[\frac{1}{2\widehat{\beta}\sin\gamma}e^{-\eta}\right]$$

and insert the value of η at the point of maximum deceleration

$$\eta_{n_{max}} = \ln\left(\frac{-1}{\widehat{\beta}\sin\gamma}\right) \Rightarrow e^{-\eta} = -\widehat{\beta}\sin\gamma$$

$$\frac{v}{v_e} = \exp\left[\frac{-\widehat{\beta}\sin\gamma}{2\widehat{\beta}\sin\gamma}\right] \Rightarrow v_{n_{max}} = \frac{v_e}{\sqrt{e}} \cong 0.606 v_e$$

Velocity at maximum deceleration is independent of everything except v_e

Planetary Entry - Physical Data

	Radius (km)	μ (km³/sec²)	ρ_o (kg/m³)	h_s (km)	v_{esc} (km/sec)
Earth	6378	398,604	1.225	7.524	11.18
Mars	3393	42,840	0.0993	27.70	5.025
Venus	6052	325,600	16.02	6.227	10.37

Comparison of Planetary Atmospheres