

ENAE 791 - Spring, 2008  
Problem Set 1

Due 2/14/08

1. A spacecraft in orbit around the Earth has the current position  $\vec{r}=(0, 7680 \text{ km}, 0)$  and velocity of  $\vec{v}=(-7.204 \text{ km/sec}, 1.441 \text{ km/sec}, 0)$ . At the current time, calculate
- Angular momentum  $\vec{h}$
  - Semi-major axis  $a$
  - Eccentricity  $e$
  - Parameter  $p$
  - Perigee radius  $r_p$
  - Apogee radius  $r_a$
  - Flight path angle  $\gamma$
  - True anomaly  $\theta$
  - Orbital period  $P$
  - Time since perigee passage  $t_p$

The current orbit will intersect the Earth's atmosphere at an altitude of 122 km. At that point, calculate

- True anomaly  $\theta$
  - Scalar velocity  $v$
  - Radial velocity  $v_r$
  - Tangential velocity  $v_\theta$
  - Flight path angle  $\gamma$
  - Time from the initial point specified above
2. Consider a two-stage vehicle for launching 50,000 kg of payload to Earth orbit ( $\Delta V=9200 \text{ m/sec}$ ), and three choices for propellants in each stage: LOX/LH2, storables, and solids. Using the mean values for  $\delta$  and  $V_e$  from the lecture notes, find the optimum  $\Delta V$  distribution for each possible propellant combination to find the one that minimizes  $\delta/\lambda$ . What is the gross vehicle mass and inert masses for each of the two stages for each case? (Hint: find numerical solutions, not analytic ones. Some form of mechanization (spreadsheet or Matlab) is highly recommended.)
3. The Falcon 1e launch vehicle by SpaceX has the following properties:
- 1<sup>st</sup> stage empty mass - 4000 lb
  - 1<sup>st</sup> stage propellant mass - 69,000 lb
  - 1<sup>st</sup> stage exhaust velocity - 2979 m/sec
  - 1<sup>st</sup> stage nominal burn time - 169 sec
  - 2<sup>nd</sup> stage empty mass - 1125 lb
  - 2<sup>nd</sup> stage propellant mass - 8881 lb
  - 2<sup>nd</sup> stage exhaust velocity - 3205 m/sec

Payload mass - 1529 lb

Payload fairing mass - 300 lb

- a) Calculate the  $\Delta V$  contributions of each of the stages. Assume the payload fairing is jettisoned at the same time as the first stage
- b) Calculate the change in total  $\Delta V$  if the payload fairing is not jettisoned, but stays with the payload all the way to orbit
- c) How much payload would you have to give up to achieve the same total  $\Delta V$  as in (a) with the payload fairing retained throughout the flight?
- d) Find the three trade-off ratios for both the first and second stage, assuming the fairing is jettisoned with the first stage.
- e) You wish to augment the payload by adding strap-on solid rocket motors to the first stage. You choose the Castor IV-A solid rocket motor, which has a total mass of 25,800 lb, a propellant mass of 22,300 lbs, an exhaust velocity of 2325 m/sec, and a burning time of 56.2 seconds. Two of these motors will be added to the first stage and ignited at lift-off in parallel with the Falcon 1e first stage engine. Find the payload capacity of this system in order to reach the same  $\Delta V$  as in (a).
- f) You need to put 5800 kg of payload into orbit [same  $\Delta V$  as (a)]. You plan to mass-produce Falcon 1e first stages to build a modular launch vehicle to carry at least this much payload into orbit. Find the configuration (modules/stage and number of stages) which will accomplish this with the minimum total number of modules. You may use up to four stages. (To simplify this calculation, assume the payload fairing is carried all the way to orbit.)