

ENAE 791 - Spring, 2008  
Problem Set 1 Solutions

(Currently only problem 2 posted)

1. A spacecraft in orbit around the Earth has the current position  $\vec{r}=(0, 7680 \text{ km}, 0)$  and velocity of  $\vec{v}=(-7.204 \text{ km/sec}, 1.441 \text{ km/sec}, 0)$ . At the current time, calculate
- Angular momentum  $\vec{h}$
  - Semi-major axis  $a$
  - Eccentricity  $e$
  - Parameter  $p$
  - Perigee radius  $r_p$
  - Apogee radius  $r_a$
  - Flight path angle  $\gamma$
  - True anomaly  $\theta$
  - Orbital period  $P$
  - Time since perigee passage  $t_p$

The current orbit will intersect the Earth's atmosphere at an altitude of 122 km. At that point, calculate

- True anomaly  $\theta$
- Scalar velocity  $v$
- Radial velocity  $v_r$
- Tangential velocity  $v_\theta$
- Flight path angle  $\gamma$
- Time from the initial point specified above

2. Consider a two-stage vehicle for launching 50,000 kg of payload to Earth orbit ( $\Delta V=9200$  m/sec), and three choices for propellants in each stage: LOX/LH2, storables, and solids. Using the mean values for  $\delta$  and  $V_e$  from the lecture notes, find the optimum  $\Delta V$  distribution for each possible propellant combination to find the one that minimizes  $\delta/\lambda$ . What is the gross vehicle mass and inert masses for each of the two stages for each case? (Hint: find numerical solutions, not analytic ones. Some form of mechanization (spreadsheet or Matlab) is highly recommended.)

Using the parameters for  $V_e$  and  $\delta$  from page 14 of the Lecture #02 notes, and choosing an arbitrary initial value for  $\Delta v_2$ , I set up the following equations in a spreadsheet:

$$\begin{aligned}
 r_1 &= e^{-\frac{\Delta v_1}{v_e}} & r_2 &= e^{-\frac{\Delta v_2}{v_e}} \\
 \lambda_1 &= r_1 - \delta_1 & \lambda_2 &= r_2 - \delta_2 \\
 M_{o2} &= \frac{M_{pl}}{\lambda_2} & M_o &= \frac{M_{o2}}{\lambda_1} \\
 M_{in1} &= \delta_1 M_o & M_{in2} &= \delta_2 M_{o2} \\
 \delta_o &= \frac{M_{in1} + M_{in2}}{M_o} & \lambda_o &= \frac{M_{pl}}{M_o}
 \end{aligned}$$

and used Solver in Excel to find the value of  $\Delta v_2$  that minimizes  $\delta_o/\lambda_o$ . By repeating this for each of the nine cases, I generated the data in the attached table. The preferred system is one where both stages are LOX/LH2, and the  $\Delta V$  split is 5824/3376 m/sec.

	First Stage	Second Stage	First Stage	Second Stage	First Stage	Second Stage
<b>Propellants</b>	Solid	Solid	Solid	Storables	Solid	Cryo
$\Delta v$	4747	4453	3881	5319	2546	6654
$\gamma$	0.1805	0.2007	0.2467	0.1756	0.3992	0.2107
$\delta_{stage}$	0.087	0.087	0.087	0.061	0.087	0.075
$\lambda_{stage}$	0.0935	0.1137	0.1597	0.1146	0.3122	0.1357
$M_o$	4,700,932	439,692	2,731,243	436,129	1,179,842	368,341
$M_{inert}$	408,981	38,253	237,618	26,604	102,646	27,626
$\delta_0$		0.0951		0.0967		0.1104
$\lambda_0$		0.0106		0.0183		0.0424
$\delta_0/\lambda_0$		8.945		5.284		2.605
<b>Propellants</b>	Storables	Solids	Storables	Storables	Storables	Cryo
$\Delta v$	5908	3292	5084	4116	3736	5464
$\gamma$	0.1449	0.3051	0.1897	0.2603	0.2947	0.2784
$\delta_{stage}$	0.061	0.087	0.061	0.061	0.061	0.075
$\lambda_{stage}$	0.0839	0.2181	0.1287	0.1993	0.2337	0.2034
$M_o$	2,733,865	229,294	1,950,087	250,893	1,051,814	245,819
$M_{inert}$	166,766	19,949	118,955	15,304	64,161	18,436
$\delta_0$		0.0683		0.0688		0.0785
$\lambda_0$		0.0183		0.0256		0.0475
$\delta_0/\lambda_0$		3.734		2.685		1.652
<b>Propellants</b>	Cryo	Solids	Cryo	Storables	Cryo	Cryo
$\Delta v$	7984	1216	7314	1886	5824	3376
$\gamma$	0.1544	0.645	0.1805	0.5398	0.2559	0.4538
$\delta_{stage}$	0.075	0.087	0.075	0.061	0.075	0.075
$\lambda_{stage}$	0.0794	0.558	0.1055	0.4788	0.1809	0.3788
$M_o$	1,129,141	89,602	989,470	104,435	729,665	131,978
$M_{inert}$	84,686	7,795	74,210	6,371	54,725	9,898
$\delta_0$		0.0819		0.0814		0.0886
$\lambda_0$		0.0443		0.0505		0.0685
$\delta_0/\lambda_0$		1.85		1.612		1.292

3. The Falcon 1e launch vehicle by SpaceX has the following properties:
- 1<sup>st</sup> stage empty mass - 4000 lb
  - 1<sup>st</sup> stage propellant mass - 69,000 lb
  - 1<sup>st</sup> stage exhaust velocity - 2979 m/sec
  - 1<sup>st</sup> stage nominal burn time - 169 sec
  - 2<sup>nd</sup> stage empty mass - 1125 lb
  - 2<sup>nd</sup> stage propellant mass - 8881 lb
  - 2<sup>nd</sup> stage exhaust velocity - 3205 m/sec
  - Payload mass - 1529 lb
  - Payload fairing mass - 300 lb
- a) Calculate the  $\Delta V$  contributions of each of the stages. Assume the payload fairing is jettisoned at the same time as the first stage
  - b) Calculate the change in total  $\Delta V$  if the payload fairing is not jettisoned, but stays with the payload all the way to orbit
  - c) How much payload would you have to give up to achieve the same total  $\Delta V$  as in (a) with the payload fairing retained throughout the flight?
  - d) Find the three trade-off ratios for both the first and second stage, assuming the fairing is jettisoned with the first stage.
  - e) You wish to augment the payload by adding strap-on solid rocket motors to the first stage. You choose the Castor IV-A solid rocket motor, which has a total mass of 25,800 lb, a propellant mass of 22,300 lbs, an exhaust velocity of 2325 m/sec, and a burning time of 56.2 seconds. Two of these motors will be added to the first stage and ignited at lift-off in parallel with the Falcon 1e first stage engine. Find the payload capacity of this system in order to reach the same  $\Delta V$  as in (a).
  - f) You need to put 5800 kg of payload into orbit [same  $\Delta V$  as (a)]. You plan to mass-produce Falcon 1e first stages to build a modular launch vehicle to carry at least this much payload into orbit. Find the configuration (modules/stage and number of stages) which will accomplish this with the minimum total number of modules. You may use up to four stages. (To simplify this calculation, assume the payload fairing is carried all the way to orbit.)