

Ballistic/Lifting Atmospheric Entry

- Straight-line (no gravity) ballistic entry based on altitude, rather than density
- Planetary entries (at least a start)
- Basic equations of planar motion (with lift)
- Lifting equilibrium glide
-

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UNIVERSITY OF
MARYLAND

Ballistic and Lifting Atmospheric Entry
ENAE 791 - Launch and Entry Vehicle Design

Parametric Deceleration

$$\frac{dv}{dt} = v_e \exp\left(Be^{-\frac{h}{h_s}}\right) \frac{-B}{h_s} \left(e^{-\frac{h}{h_s}}\right) v_e \sin \gamma \exp\left(Be^{-\frac{h}{h_s}}\right)$$

$$\frac{dv}{dt} = \frac{-Bv_e^2}{h_s} \sin \gamma \left(e^{-\frac{h}{h_s}}\right) \exp\left(2Be^{-\frac{h}{h_s}}\right)$$

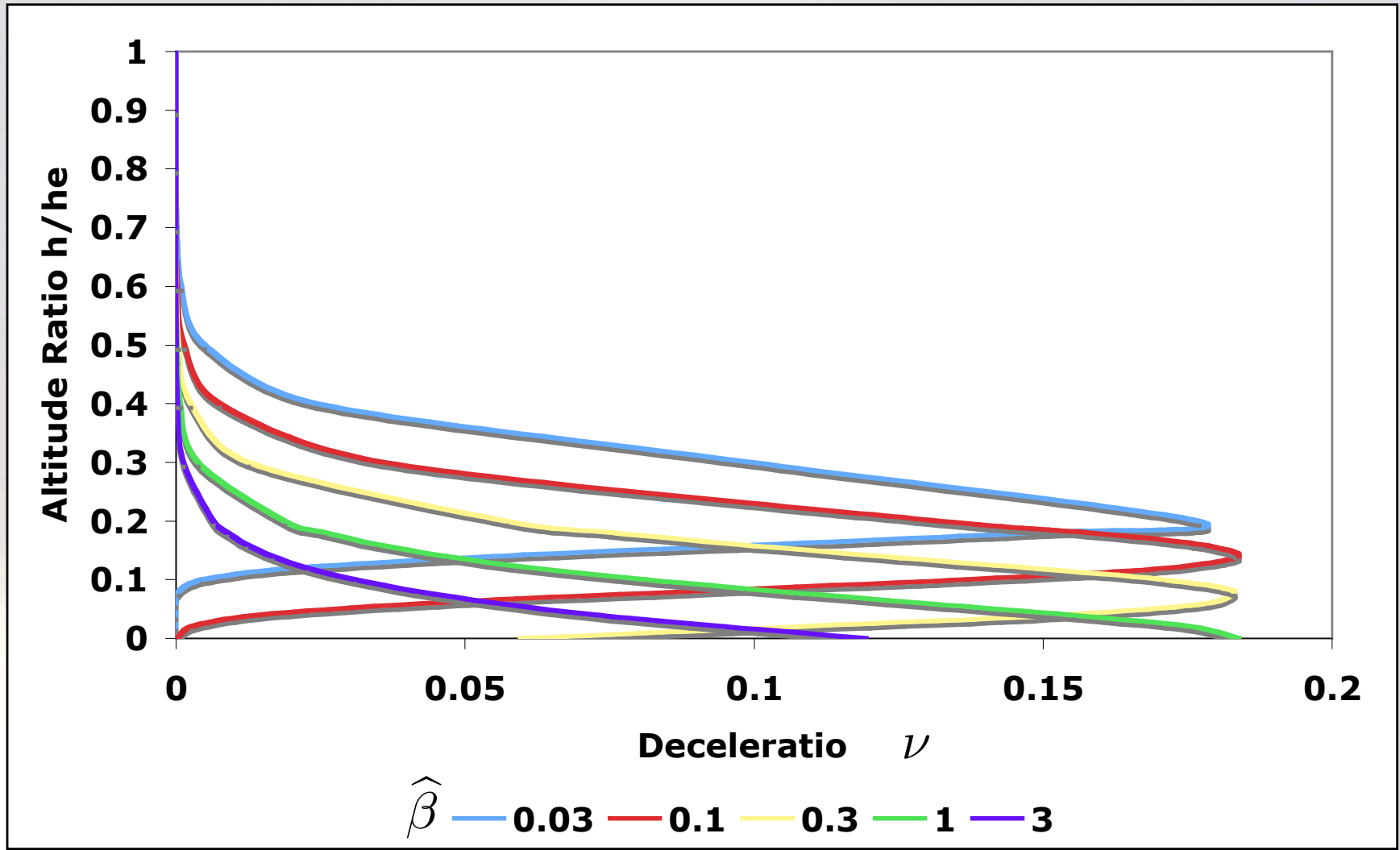
$$\frac{dv}{dt} = \frac{-v_e^2}{2h_s \hat{\beta}} \left(e^{-\frac{h}{h_s}}\right) \exp\left(\frac{1}{\hat{\beta} \sin \gamma} e^{-\frac{h}{h_s}}\right)$$

$$\text{Let } n_{ref} \equiv \frac{v_e^2}{h_s}, \nu \equiv \frac{dv/dt}{n_{ref}}, \varphi \equiv \frac{h_e}{h_s}$$

$$\nu = \frac{-1}{2\hat{\beta}} \left(e^{-\varphi \frac{h}{h_e}}\right) \exp\left(\frac{1}{\hat{\beta} \sin \gamma} e^{-\varphi \frac{h}{h_e}}\right)$$



Nondimensional Deceleration, $\gamma = -90^\circ$



Deceleration Equations

Nondimensional Form

$$v = \frac{-1}{2\hat{\beta}} \left(e^{-\varphi \frac{h}{h_e}} \right) \exp \left(\frac{1}{\hat{\beta} \sin \gamma} e^{-\varphi \frac{h}{h_e}} \right)$$

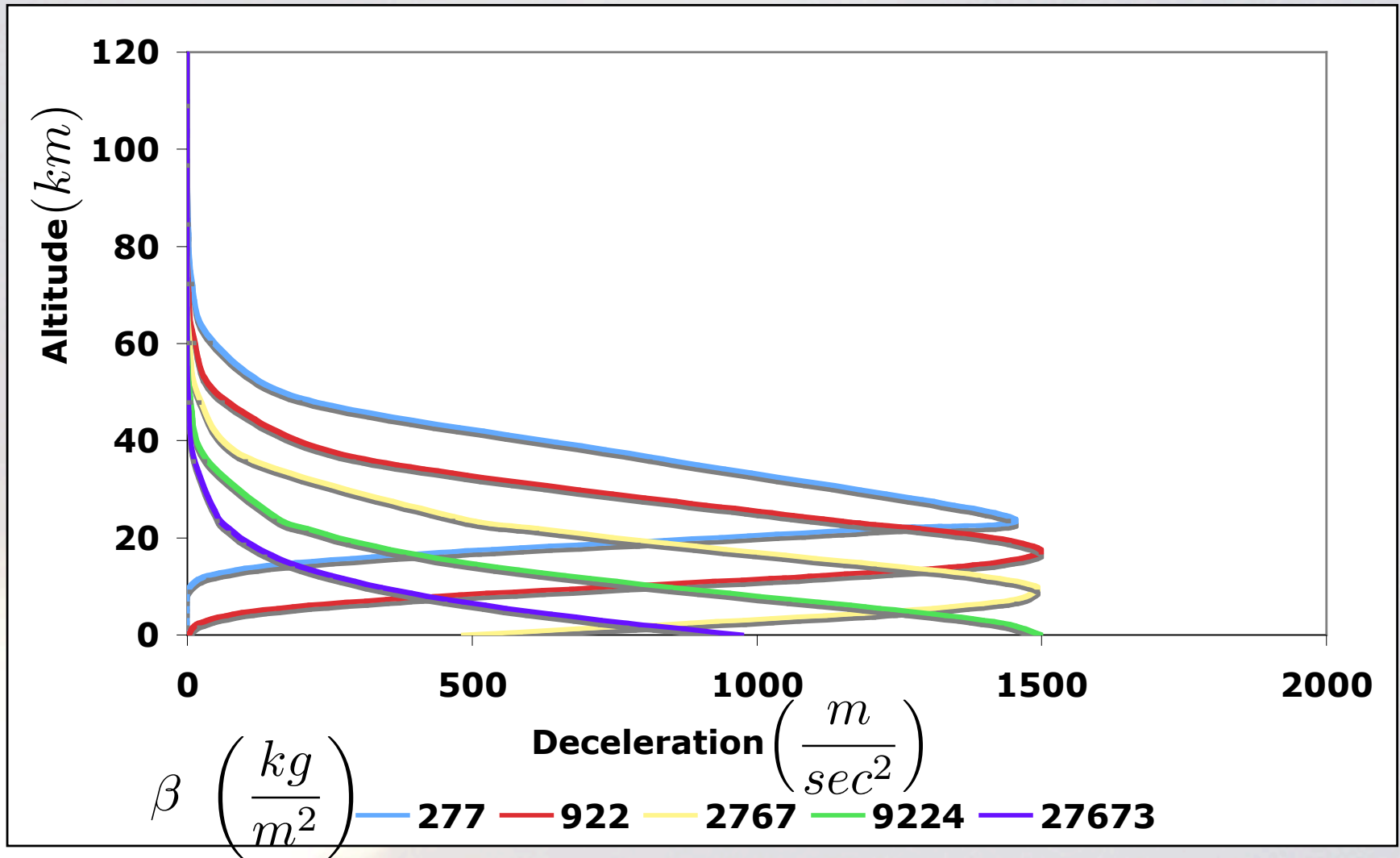
Dimensional Form

$$n = \frac{-\rho_o V_e^2}{2\beta} \left(e^{-\frac{h}{h_s}} \right) \exp \left(\frac{\rho_o h_s}{\beta \sin \gamma} e^{-\frac{h}{h_s}} \right)$$

Note that these equations result in values <0 (reflecting deceleration) - graphs are absolute values of deceleration for clarity.



Dimensional Deceleration, $\gamma = -90^\circ$



Altitude of Maximum Deceleration

Returning to shorthand notation for deceleration

$$\nu = -B \sin \gamma \left(e^{-\frac{h}{h_s}} \right) \exp \left(2B e^{-\frac{h}{h_s}} \right)$$

$$\text{Let } \eta \equiv \frac{h}{h_s}$$

$$\nu = -B \sin \gamma \left(e^{-\eta} \right) \exp \left(2B e^{-\eta} \right)$$

$$\frac{d\nu}{d\eta} = -B \sin \gamma \left[\frac{d}{d\eta} \left(e^{-\eta} \right) \exp \left(2B e^{-\eta} \right) + \left(e^{-\eta} \right) \frac{d}{d\eta} \exp \left(2B e^{-\eta} \right) \right]$$

$$\frac{d\nu}{d\eta} = -B \sin \gamma \left[- \left(e^{-\eta} \right) \exp \left(2B e^{-\eta} \right) + \left(e^{-\eta} \right) \left(-2B e^{-\eta} \right) \exp \left(2B e^{-\eta} \right) \right]$$

$$\frac{d\nu}{d\eta} = B \sin \gamma e^{-\eta} \exp \left(2B e^{-\eta} \right) \left[1 + \left(2B e^{-\eta} \right) \right] = 0$$



Altitude of Maximum Deceleration

$$1 + (2Be^{-\eta}) = 0 \Rightarrow e^{\eta} = -2B$$

$$\eta_{n_{max}} = \ln(-2B)$$

$$\eta_{n_{max}} = \ln\left(\frac{-1}{\hat{\beta} \sin \gamma}\right)$$

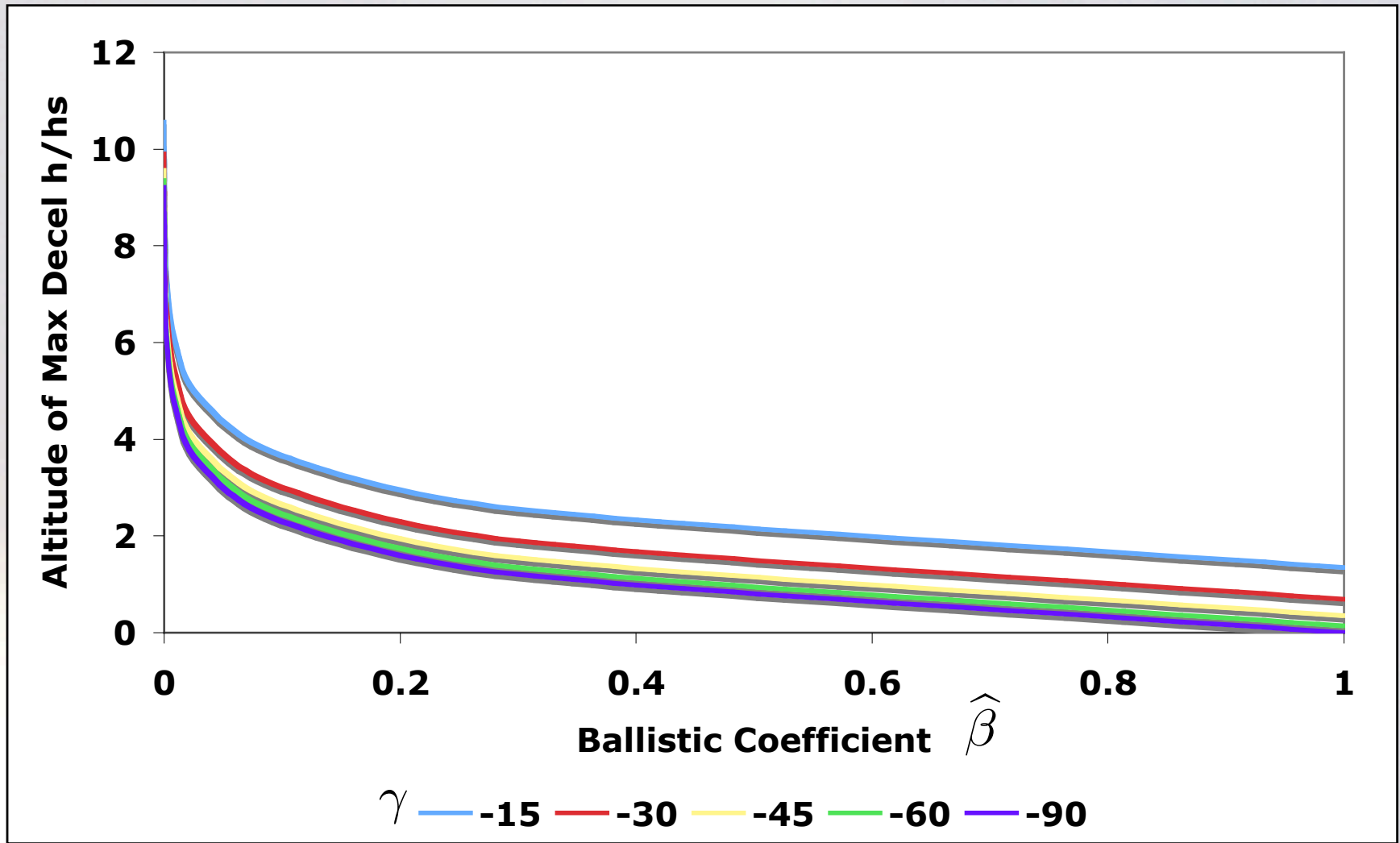
Converting from parametric to dimensional form gives

$$h_{n_{max}} = h_s \ln\left(\frac{-\rho_o h_s}{\beta \sin \gamma}\right)$$

Altitude of maximum deceleration is independent of entry velocity!



Altitude of Maximum Deceleration



Magnitude of Maximum Deceleration

Start with the equation for acceleration -

$$v = \frac{-1}{2\hat{\beta}} e^{-\eta} \exp\left(\frac{1}{\hat{\beta} \sin \gamma} e^{-\eta}\right)$$

and insert the value of η at the point of maximum deceleration

$$\eta_{n_{max}} = \ln\left(\frac{-1}{\hat{\beta} \sin \gamma}\right) \Rightarrow e^{-\eta} = -\hat{\beta} \sin \gamma$$

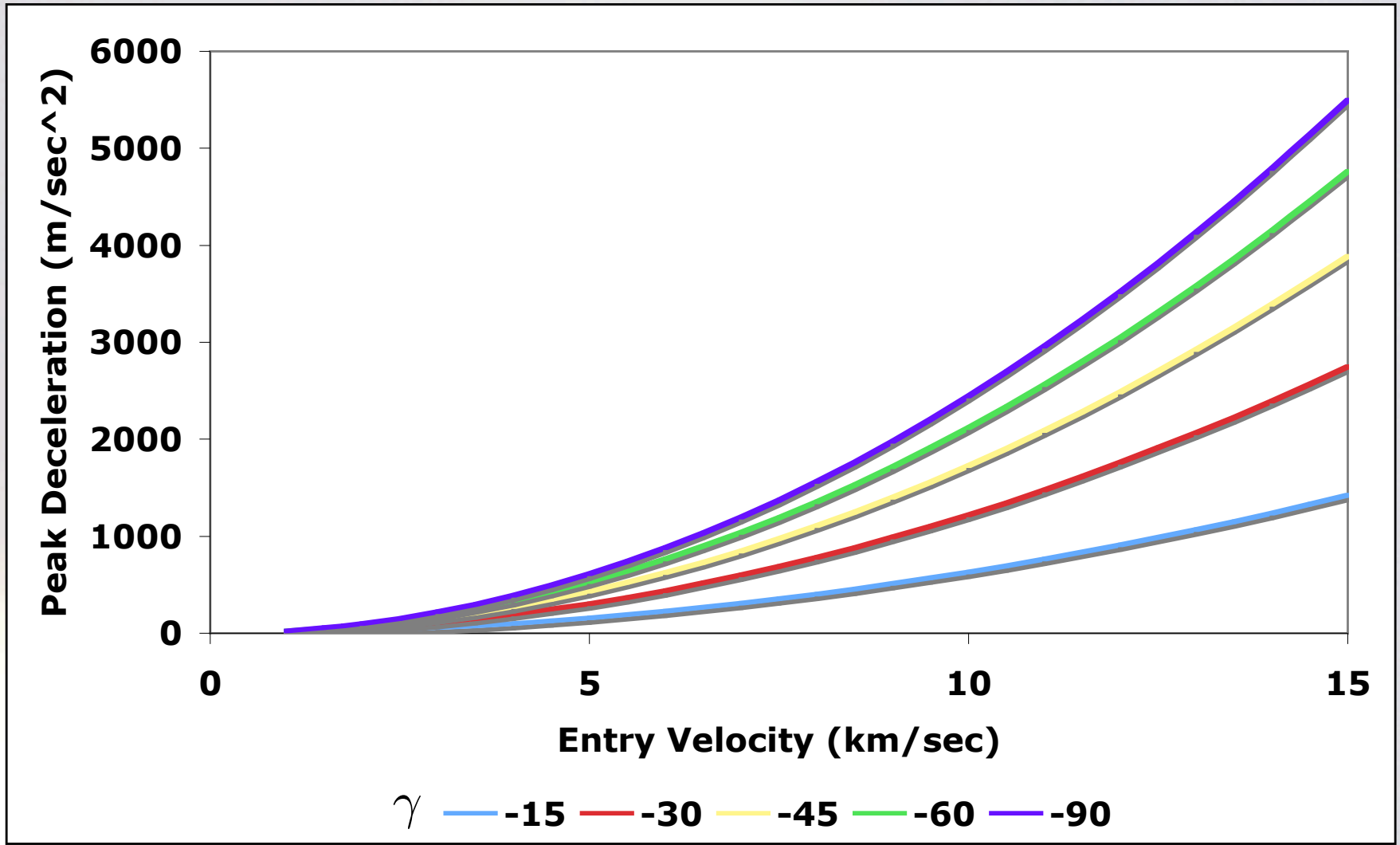
$$v_{n_{max}} = \frac{-1}{2\hat{\beta}} \left(-\hat{\beta} \sin \gamma\right) \exp\left(\frac{-\hat{\beta} \sin \gamma}{\hat{\beta} \sin \gamma}\right) \Rightarrow v_{n_{max}} = \frac{\sin \gamma}{2e}$$

$$n_{max} = \frac{v_e^2 \sin \gamma}{h_s 2e}$$

Maximum deceleration is not a function of ballistic coefficient!



Peak Ballistic Deceleration for Earth Entry



Velocity at Maximum Deceleration

Start with the equation for velocity

$$\frac{v}{v_e} = \exp \left[\frac{1}{2\hat{\beta} \sin \gamma} e^{-\eta} \right]$$

and insert the value of η at the point of maximum deceleration

$$\eta_{n_{max}} = \ln \left(\frac{-1}{\hat{\beta} \sin \gamma} \right) \Rightarrow e^{-\eta} = -\hat{\beta} \sin \gamma$$

$$\frac{v}{v_e} = \exp \left[\frac{-\hat{\beta} \sin \gamma}{2\hat{\beta} \sin \gamma} \right] \Rightarrow v_{n_{max}} = \frac{v_e}{\sqrt{e}} \cong 0.606v_e$$

Velocity at maximum deceleration is independent of everything except v_e

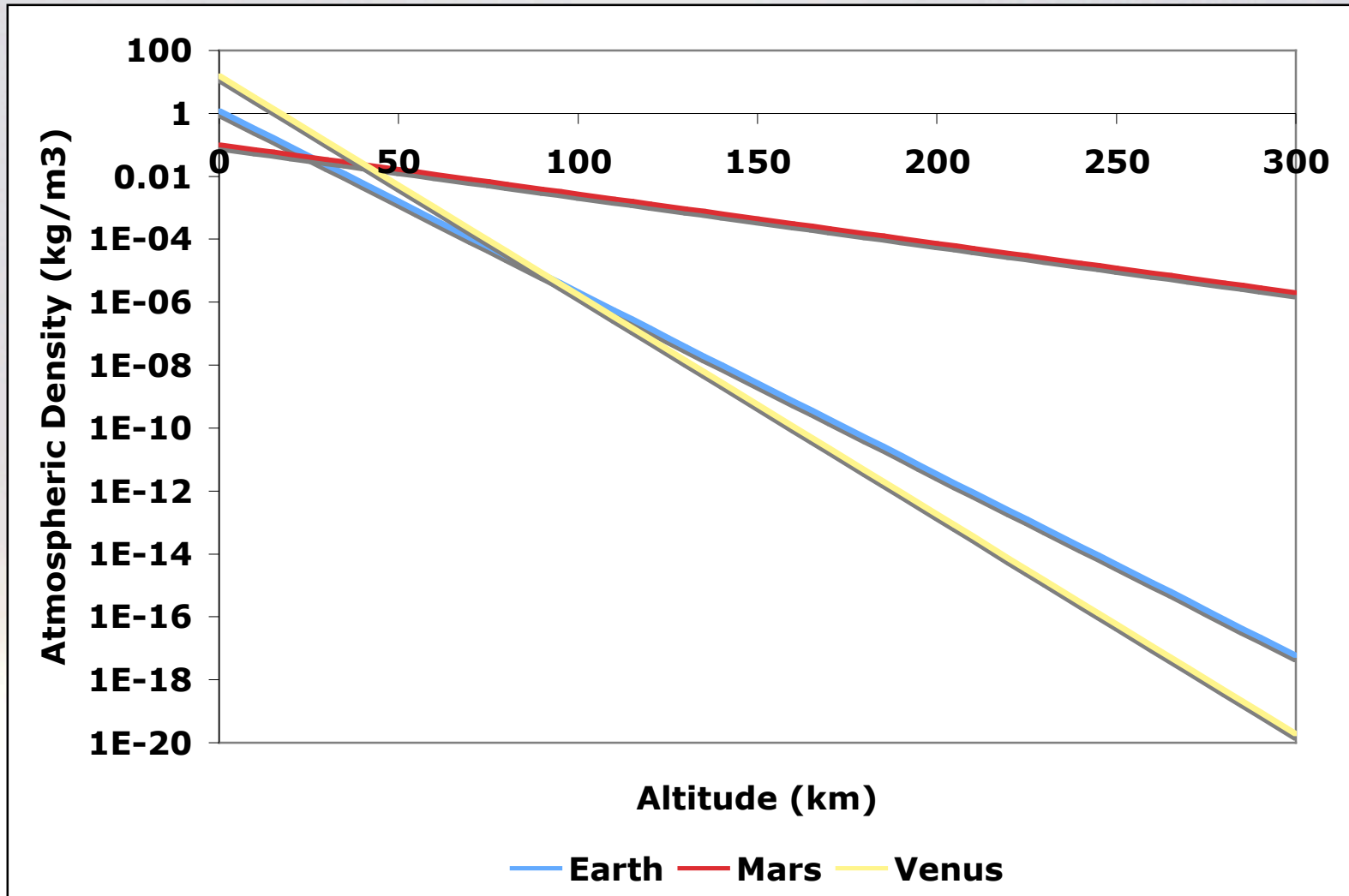


Planetary Entry - Physical Data

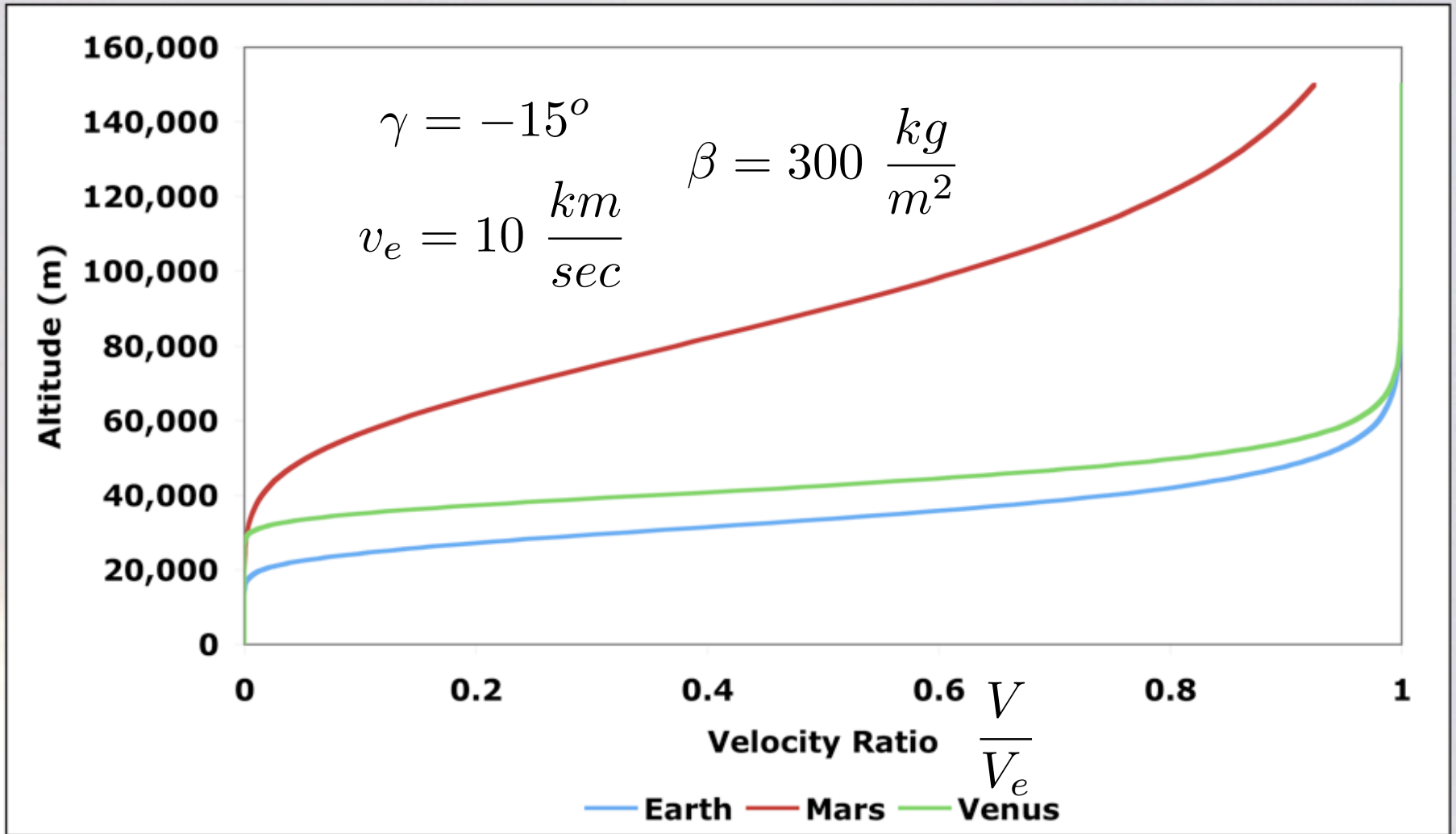
	Radius (km)	μ (km³/sec²)	ρ_0 (kg/m³)	h_s (km)	V_{esc} (km/sec)
Earth	6378	398,604	1.225	7.524	11.18
Mars	3393	42,840	0.0993	27.70	5.025
Venus	6052	325,600	16.02	6.227	10.37



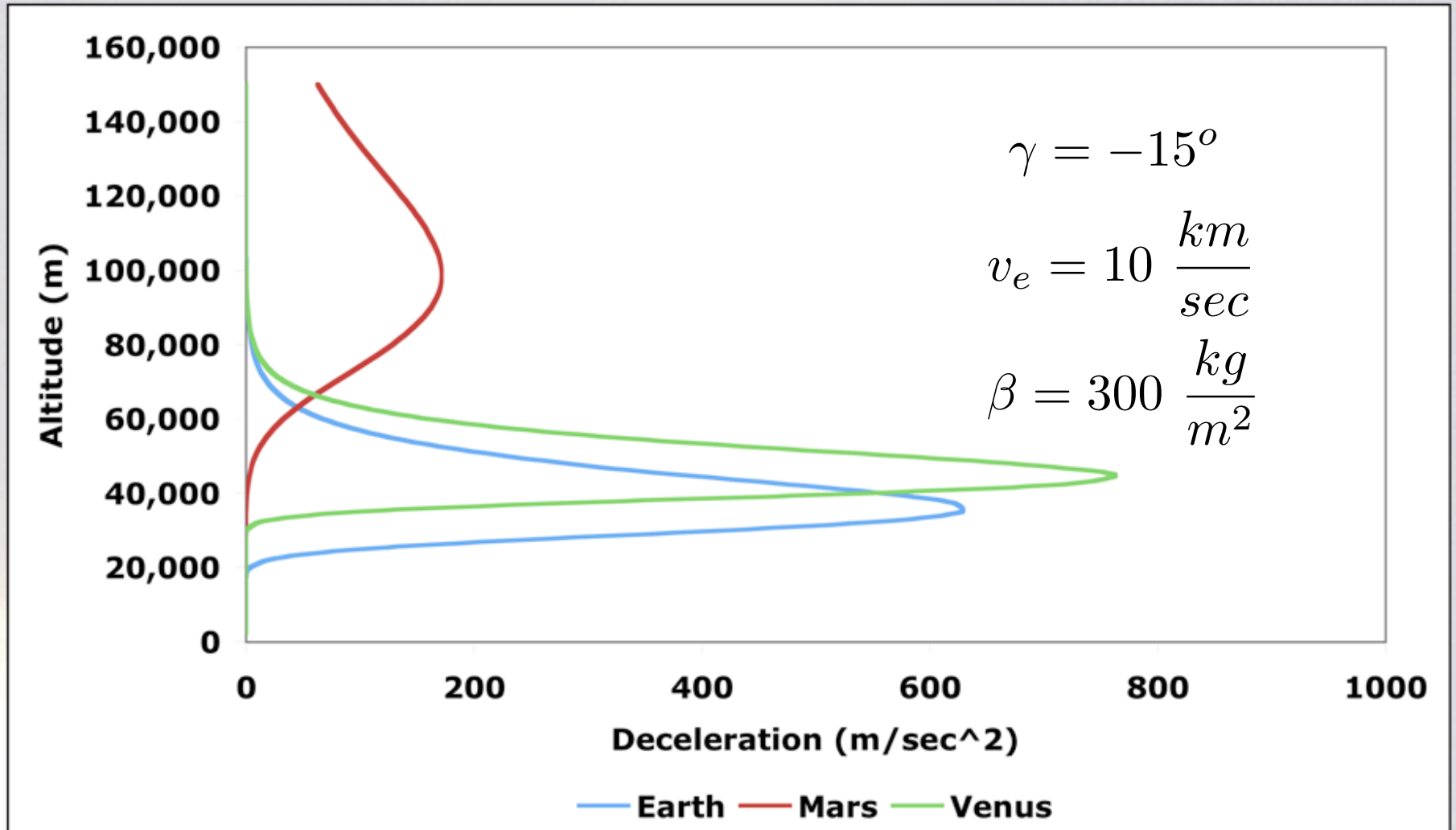
Comparison of Planetary Atmospheres



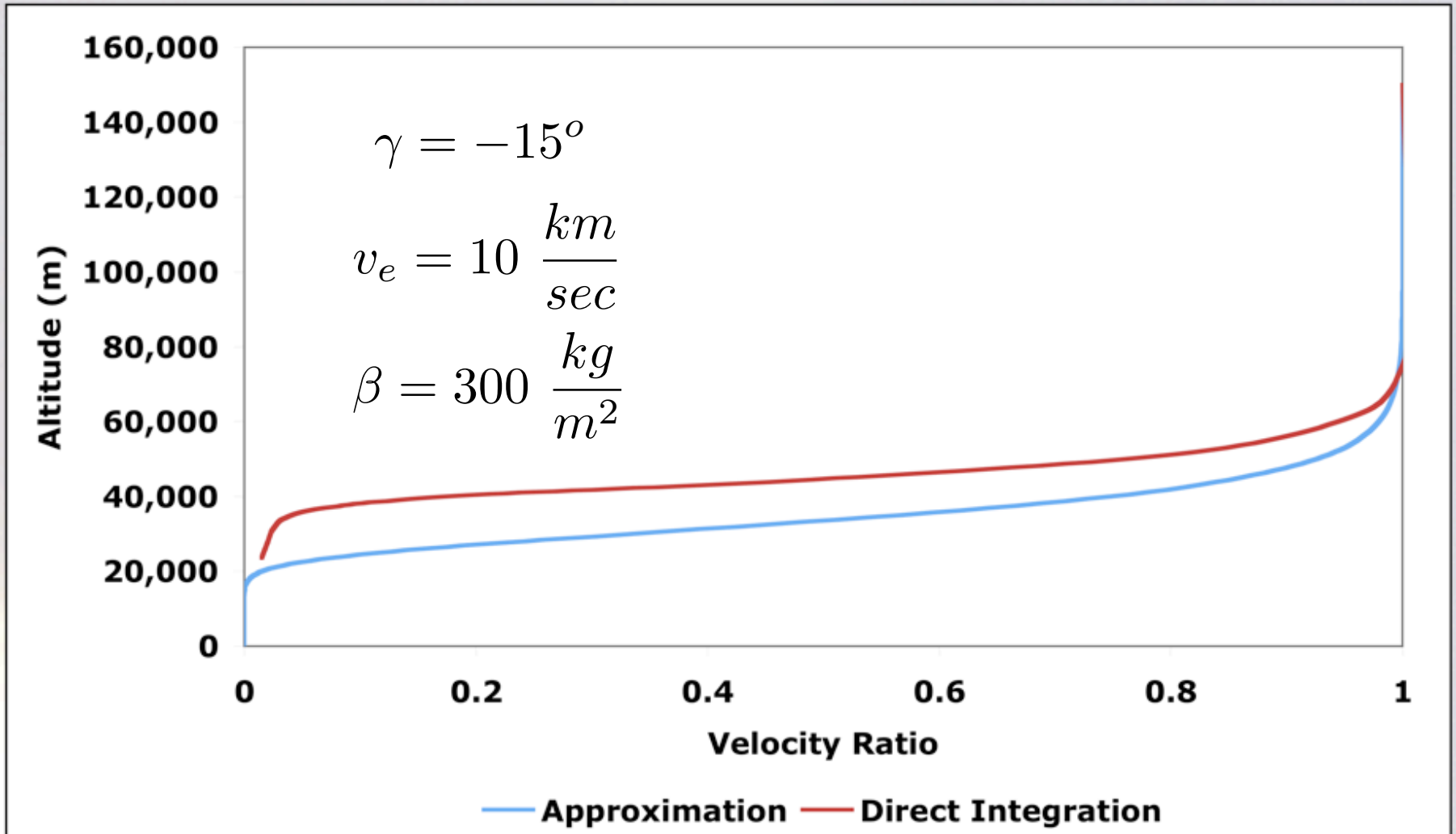
Planetary Entry Profiles



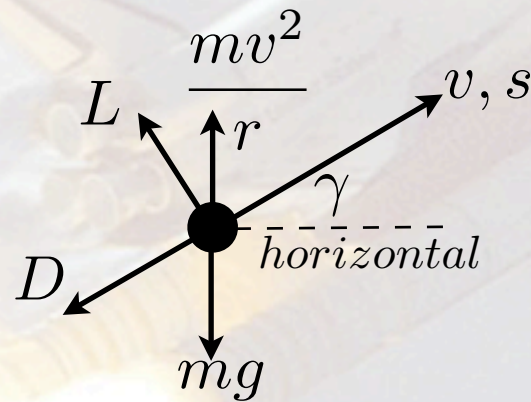
Planetary Entry Deceleration Comparison



Check on Approximation Formulas



Free-Body Diagram



Dynamics of Lifting Entry

Along the velocity vector,

$$m \frac{dv}{dt} = m \frac{v^2}{r} \sin \gamma - mg \sin \gamma - D$$

Perpendicular to the velocity vector,

$$mv \frac{d\gamma}{dt} = L + m \frac{v^2}{r} \cos \gamma - mg \cos \gamma$$

(unbalanced lift rotates flight path angle)



Equations of Planar Lifting Entry

$$\frac{dv}{dt} = \left(\frac{v^2}{r} - g \right) \sin \gamma - \frac{D}{m}$$

$$v \frac{d\gamma}{dt} = \frac{L}{m} + \left(\frac{v^2}{r} - g \right) \cos \gamma$$



Equilibrium Glide

- Forces perpendicular to velocity vector are balanced

$$\implies \frac{d\gamma}{dt} = 0; \gamma = \text{constant}$$

- Typically very shallow glide

$$\implies \text{assume } \gamma \rightarrow 0; \sin(\gamma) \rightarrow 0; \cos(\gamma) \rightarrow 1$$



Equilibrium Glide Equations

$$\frac{dv}{dt} = -\frac{D}{m}$$

$$D = \frac{1}{2} \rho v^2 c_D A$$

$$\rho = \rho_o e^{-\frac{h}{h_s}}$$

$$\frac{dv}{dt} = -\frac{1}{2} \rho_o v^2 \frac{c_D A}{m} e^{-\frac{h}{h_s}}$$

$$0 = \frac{L}{m} + \left(\frac{v^2}{r} - g \right)$$

$$\frac{v^2}{r} - g = -\frac{1}{2} \rho_o v^2 \frac{c_L A}{m} e^{-\frac{h}{h_s}}$$



Dynamics Perpendicular to Velocity

$$\frac{c_L}{c_D} = \frac{L}{D}$$

L/D set by vehicle aerodynamics, flight velocity, and angle of attack (assumed constant)

$$\frac{v^2}{r} - g = -\frac{1}{2}\rho_0 v^2 \frac{L}{D} \frac{c_D A}{m} e^{-\frac{h}{h_s}}$$

$$\frac{v^2}{r} = -\frac{1}{2}\rho_0 v^2 \frac{L/D}{\beta} e^{-\frac{h}{h_s}} + g$$

$$v^2 = -\frac{1}{2}\rho_0 r v^2 \frac{L/D}{\beta} e^{-\frac{h}{h_s}} + gr$$



More Lift-Direction Dynamics

Let $e^{-\frac{h}{h_s}} \equiv \sigma$ $\left(= \text{density ratio} \equiv \frac{\rho}{\rho_o} \right)$

$$v^2 + \frac{1}{2} \rho_o r v^2 \frac{L/D}{\beta} \sigma = gr$$

$$v^2 \left(1 + \frac{1}{2} \rho_o r \frac{L/D}{\beta} \sigma \right) = gr$$

$$v = \sqrt{\frac{gr}{1 + \frac{\rho_o r \sigma (L/D)}{2\beta}}}$$



Velocity during Entry

$$v_{c_o} = \sqrt{\frac{\mu}{r_o}} = \sqrt{g_o r_o} \quad (\mu = g_o r_o^2)$$

$$\frac{v}{v_{c_o}} = \sqrt{\frac{1}{1 + \frac{\rho_o r_o \sigma (L/D)}{2\beta}}}$$

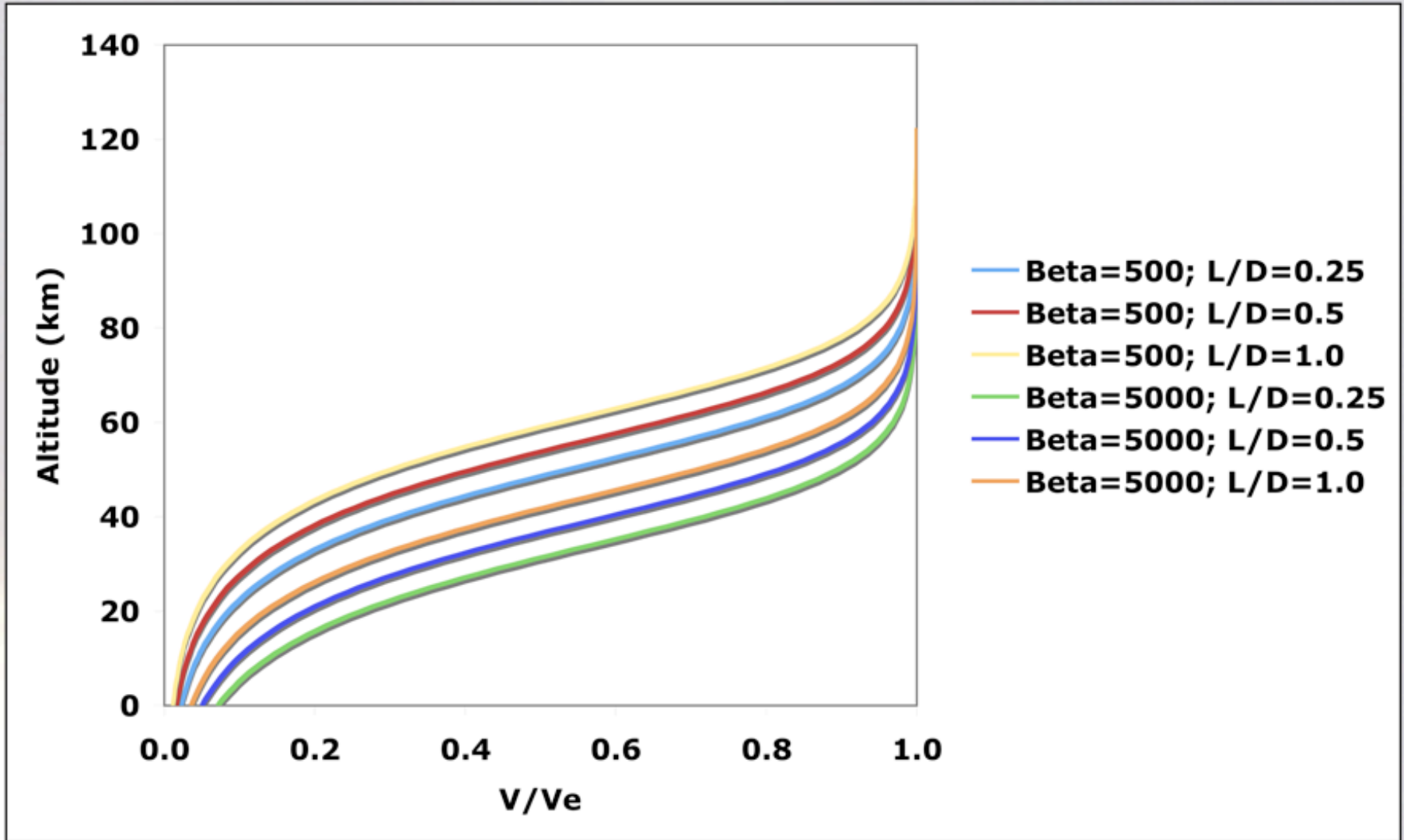
$$v_{c_o} \cong v_e \quad (\text{within } 1\text{-}2\% \text{ for Earth})$$

$$\frac{v}{v_e} = \left[1 + \frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}} \right]^{-\frac{1}{2}}$$

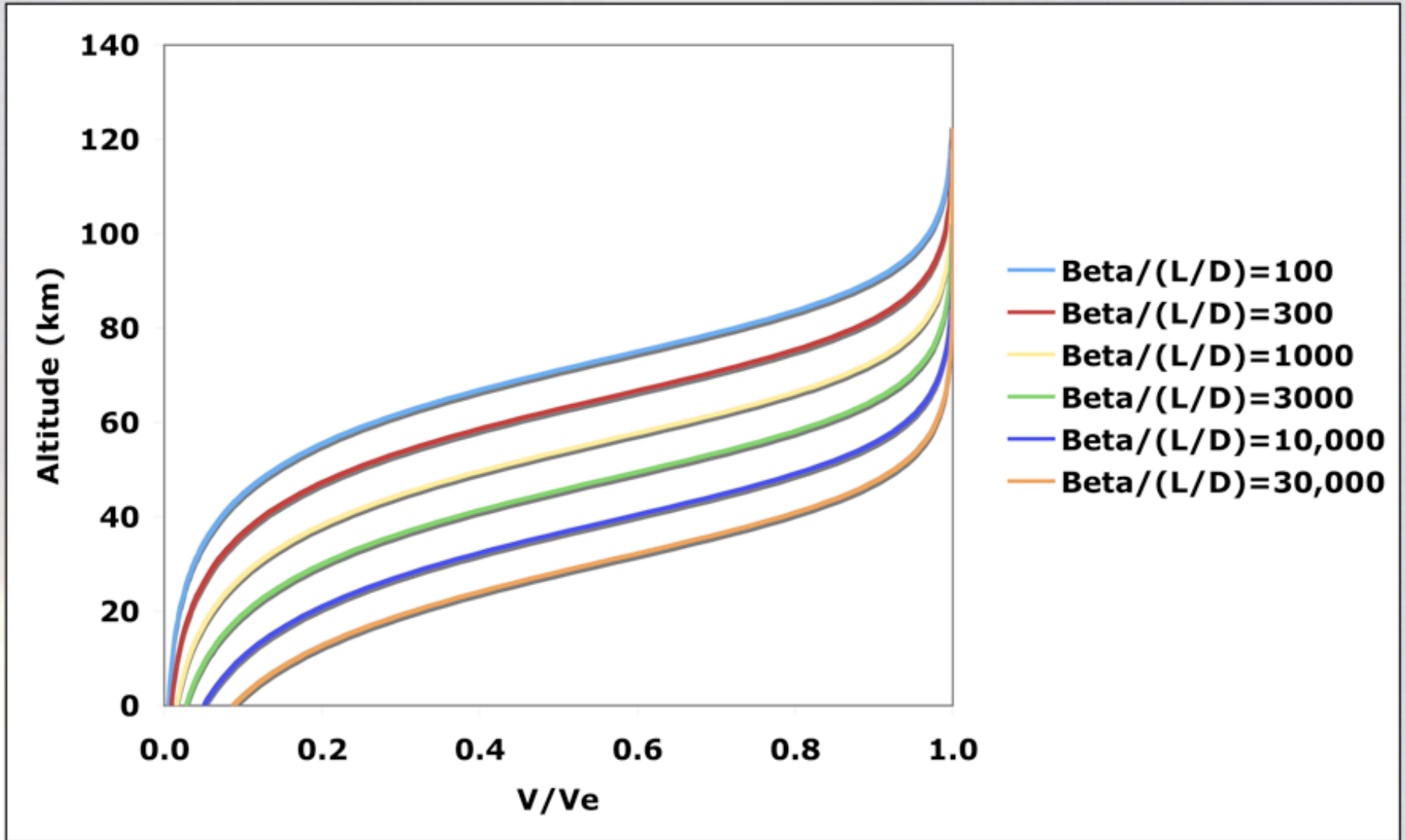
Entry trajectory as a function of altitude, ratio $\left(\frac{\beta}{L/D} \right)$



Equilibrium Glide Velocity Trends



Trends with “Lifting Ballistic Coefficient”



Nondimensional Form of Equation

$$\frac{v}{v_e} = \left[1 + \frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}} \right]^{-\frac{1}{2}}$$

Remember $\hat{\beta} \equiv \frac{\beta}{\rho_o h_s}$

$$\frac{v}{v_e} = \left[1 + \frac{\rho_o h_s}{2\beta} \frac{r_o}{h_s} \frac{L}{D} e^{-\frac{h}{h_s}} \right]^{-\frac{1}{2}}$$

$$\frac{v}{v_e} = \left[1 + \frac{1}{2\hat{\beta}} \frac{r_o}{h_s} \frac{L}{D} e^{-\frac{h}{h_e} \frac{h_e}{h_s}} \right]^{-\frac{1}{2}}$$



Deceleration

$$\frac{L}{m} = g - \frac{v^2}{r} = g - \frac{gv^2}{gr} = g \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$

$$\frac{dv}{dt} = -\frac{D}{m} = -\frac{L}{L/D} \frac{1}{m} = -\frac{1}{L/D} \frac{L}{m}$$

$$\frac{dv}{dt} = -\frac{g}{L/D} \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$



Deceleration

Let $n \equiv \frac{1}{g} \frac{dv}{dt}$ (deceleration in g's)

$$n = -\frac{1}{L/D} \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$

$$n = -\frac{1}{L/D} \left[1 - \frac{1}{1 + \frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}}} \right]$$

$$\left(\text{Note: } 1 - \frac{1}{1 + K} = \frac{1 + K}{1 + K} - \frac{1}{1 + K} = \frac{K}{1 + K} \right)$$



Deceleration

$$n = -\frac{1}{L/D} \left[\frac{\frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}}}{1 + \frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}}} \right]$$

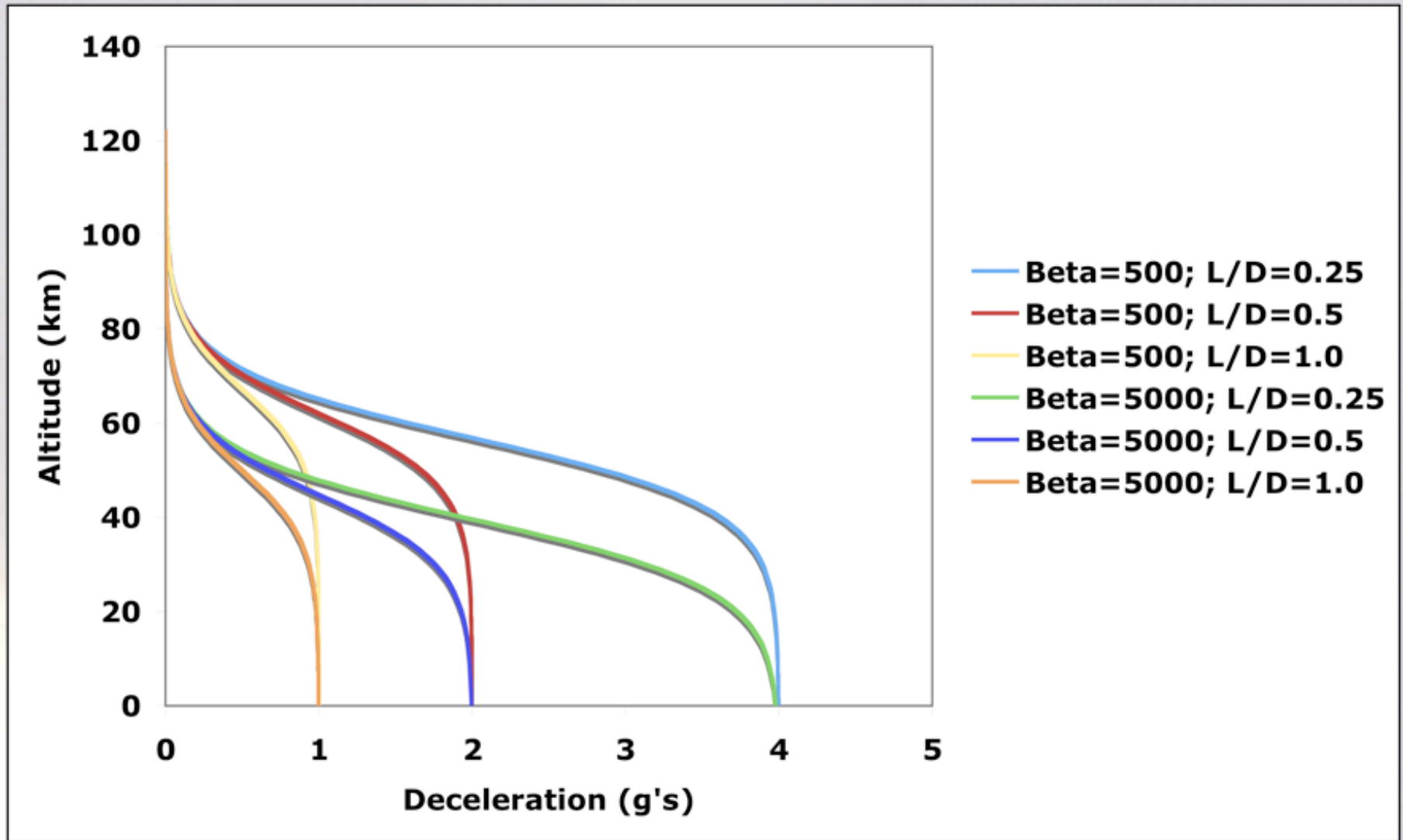
$$n = -\frac{\frac{\rho_o r_o}{2\beta} e^{-\frac{h}{h_s}}}{1 + \frac{\rho_o r_o}{2\beta} \frac{L}{D} e^{-\frac{h}{h_s}}}$$

$$n = \frac{-1}{\frac{2\beta}{\rho_o r_o} e^{+\frac{h}{h_s}} + \frac{L}{D}}$$

n monotonically increases with decreasing altitude



Deceleration Trends with Altitude



Limiting Deceleration

$$n_{limit} = \frac{-1}{\frac{2\beta}{\rho_o r_o} + \frac{L}{D}}$$

$$\beta \sim O(10^3); \rho_o \sim O(1); r_o \sim O(10^6) \implies \frac{2\beta}{\rho_o r_o} \sim O(10^{-3})$$

$$n_{limit} \approx \frac{-1}{L/D}$$

Lift significantly moderates peak g's on entry ($L/D=0.25 \rightarrow n_{limit}=4 \text{ g's}$)



Time for Entry

$$\frac{dv}{dt} = -\frac{g}{L/D} \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$

$$dt = \frac{-(L/D)dv}{g \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]}$$

$$\int_0^t dt = \int_{v_e}^0 \frac{-(L/D)dv}{g \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]}$$



Time for Entry

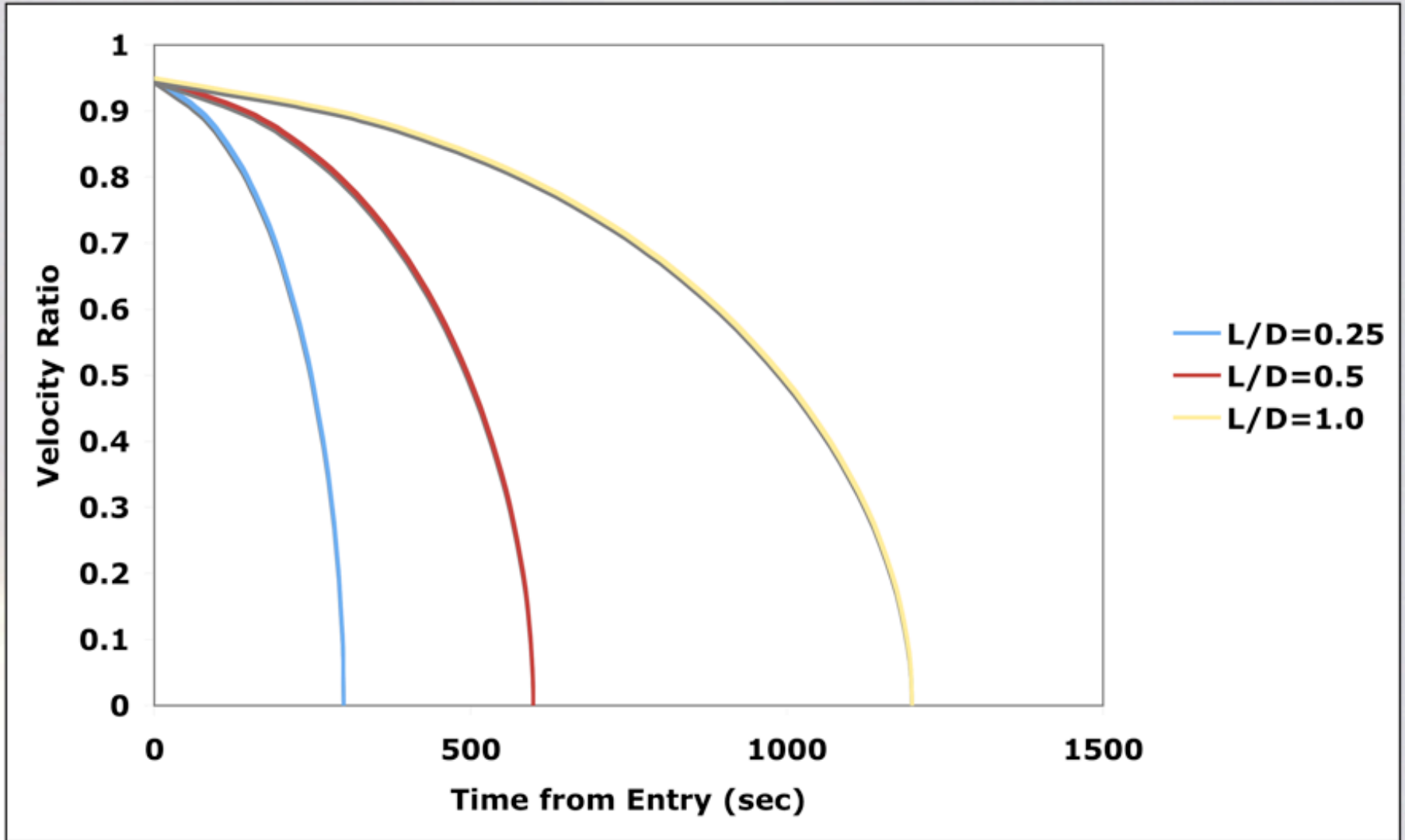
$$\Delta t = \frac{1}{2} \sqrt{\frac{r_o}{g_o}} \frac{L}{D} \ln \frac{1 + \left(\frac{v}{v_{co}}\right)^2}{1 - \left(\frac{v}{v_{co}}\right)^2}$$

Time for entry $\propto \frac{L}{D}$

Not a function of β



Time From Entry Interface



Distance Along Flight Path

For shallow entry, $\frac{ds}{dt} \cong v$ $ds = v dt$

$$ds = \frac{-(L/D)v dv}{g \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]}$$

Let $u \equiv 1 - \left(\frac{v}{v_{c_o}} \right)^2$ $du = \frac{-2v}{v_{c_o}^2} dv$ $v dv = \frac{-v_{c_o}^2}{2} du$

$$\int ds = \frac{L/D}{2g} \int \frac{v_{c_o}^2 du}{u}$$



Distance Along Flight Path

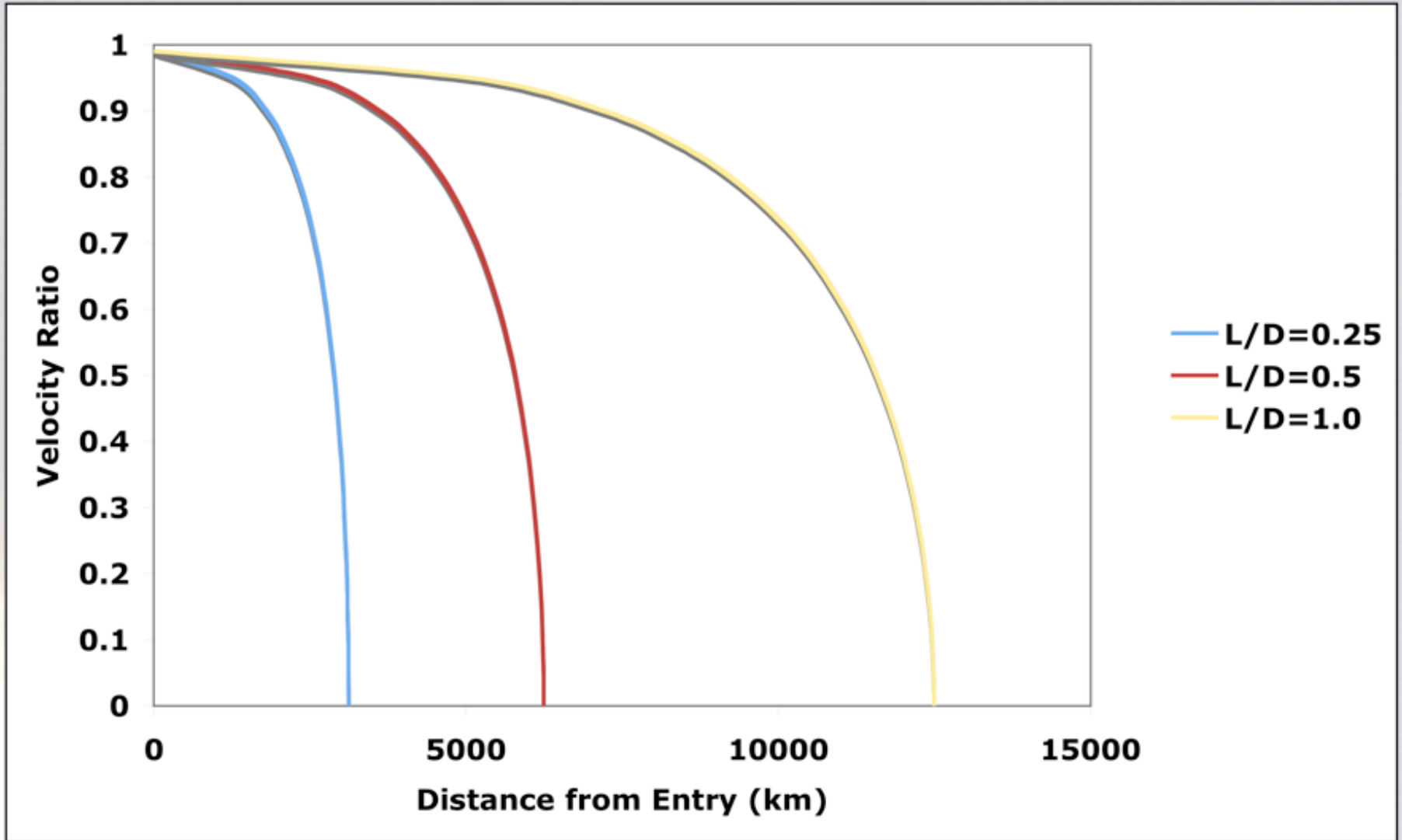
$$\Delta s = \frac{(L/D)v_{c_o}^2}{2g} \ln u = \frac{(L/D)g_o r_o}{2g_o} \ln \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$

$$\Delta s = \frac{r_o}{2} \frac{L}{D} \ln \left[1 - \left(\frac{v}{v_{c_o}} \right)^2 \right]$$

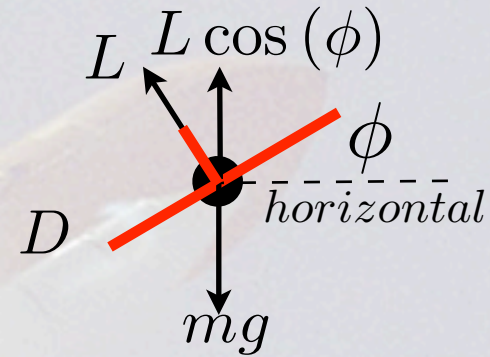
Not a function of β or g



Distance from Entry



Bank Angle



$\phi \equiv$ Bank Angle

Level turn: $L \cos \phi = mg$

$$\frac{L}{mg} = \frac{1}{\cos \phi} \equiv n_{bank}$$

(g's you pull in the banked turn)



Crossrange (without derivations)

- Use of lift to deviate laterally from planar groundtrack
- Optimum bank angle to maximize crossrange

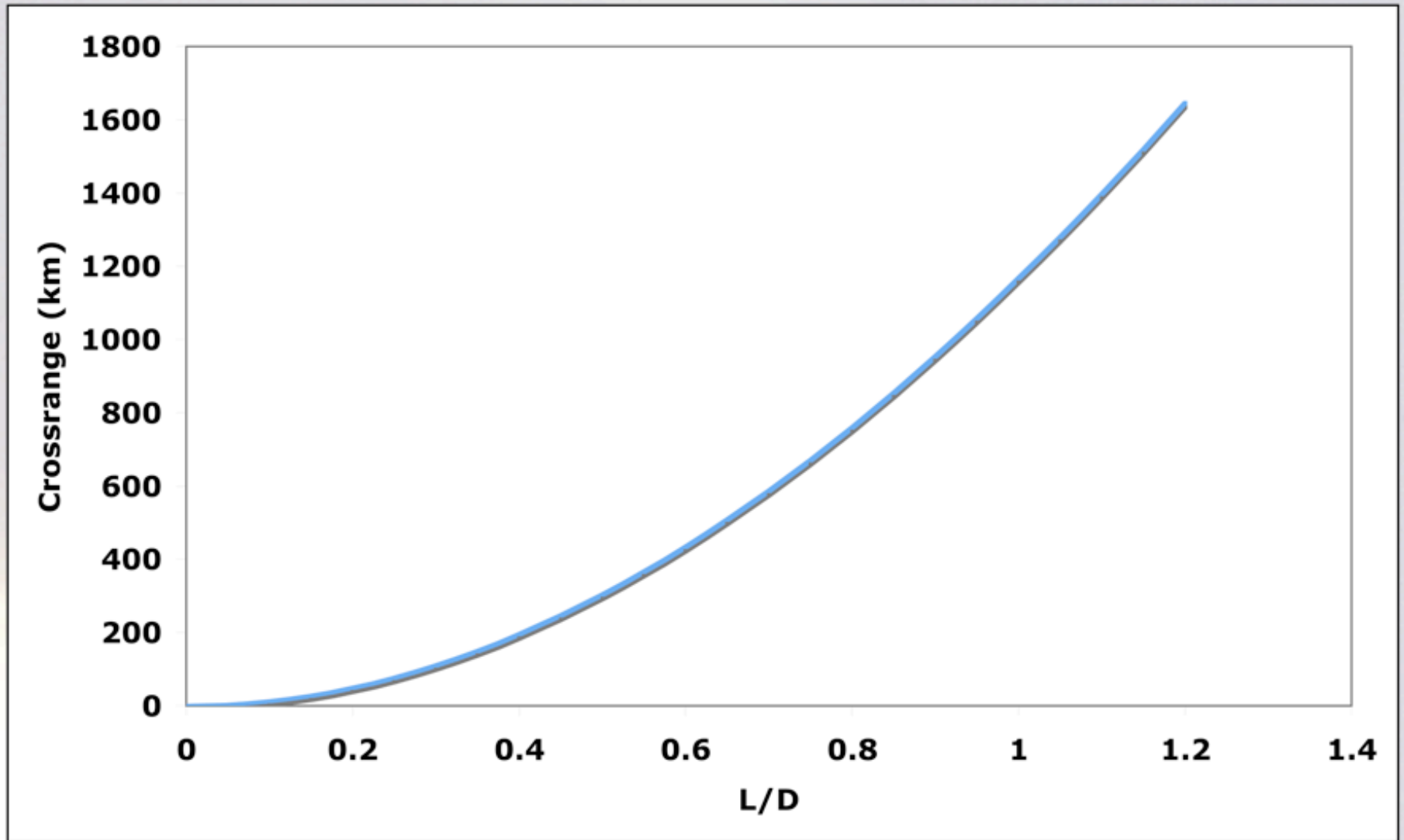
$$\phi_{opt} \cong \cot^{-1} \sqrt{1 + 0.106 \left(\frac{L}{D}\right)^2}$$

- Maximum achievable crossrange

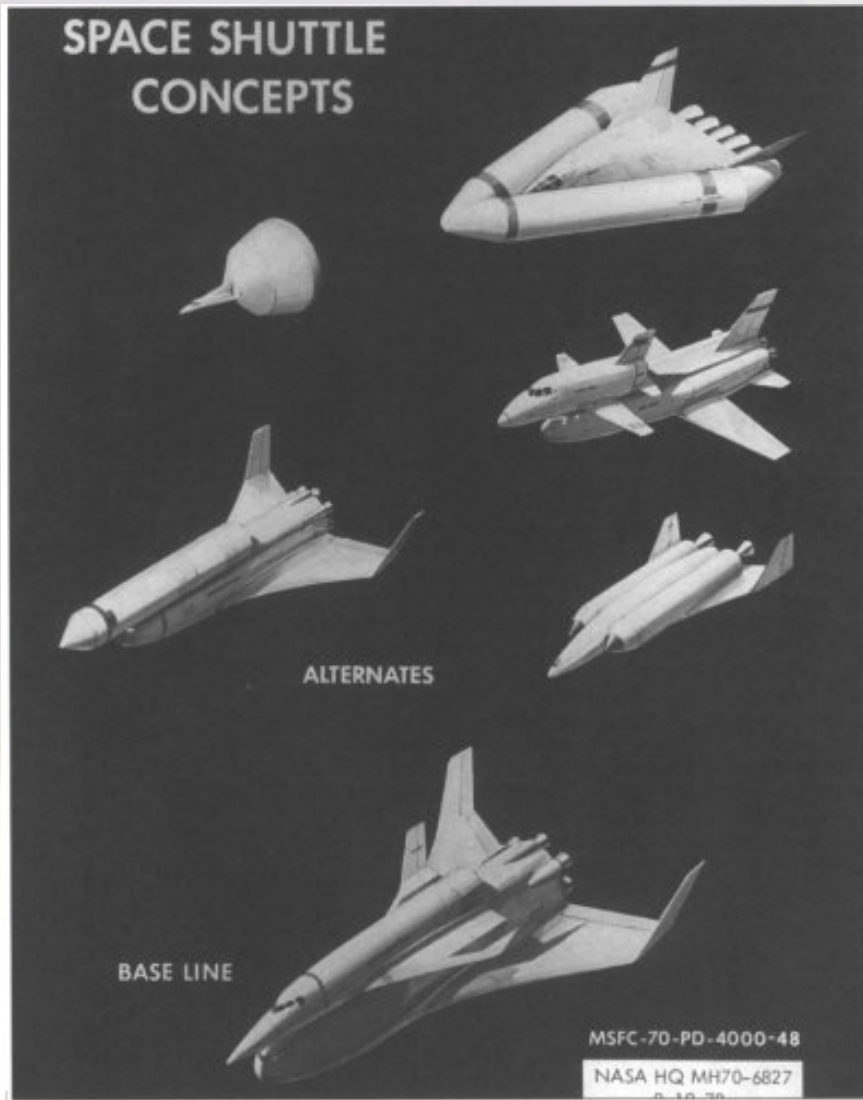
$$y_{max} \cong \frac{r_o}{5.2} \left(\frac{L}{D}\right)^2 \frac{1}{\sqrt{1 + 0.106 \left(\frac{L}{D}\right)^2}}$$



Crossrange Based on L/D



Early Shuttle Design Configurations



- Delta wing configuration
 - High hypersonic lift
 - High landing velocity
 - High crossrange
- Straight-wing configuration
 - High subsonic lift
 - Low(er) landing velocity
 - Low crossrange

