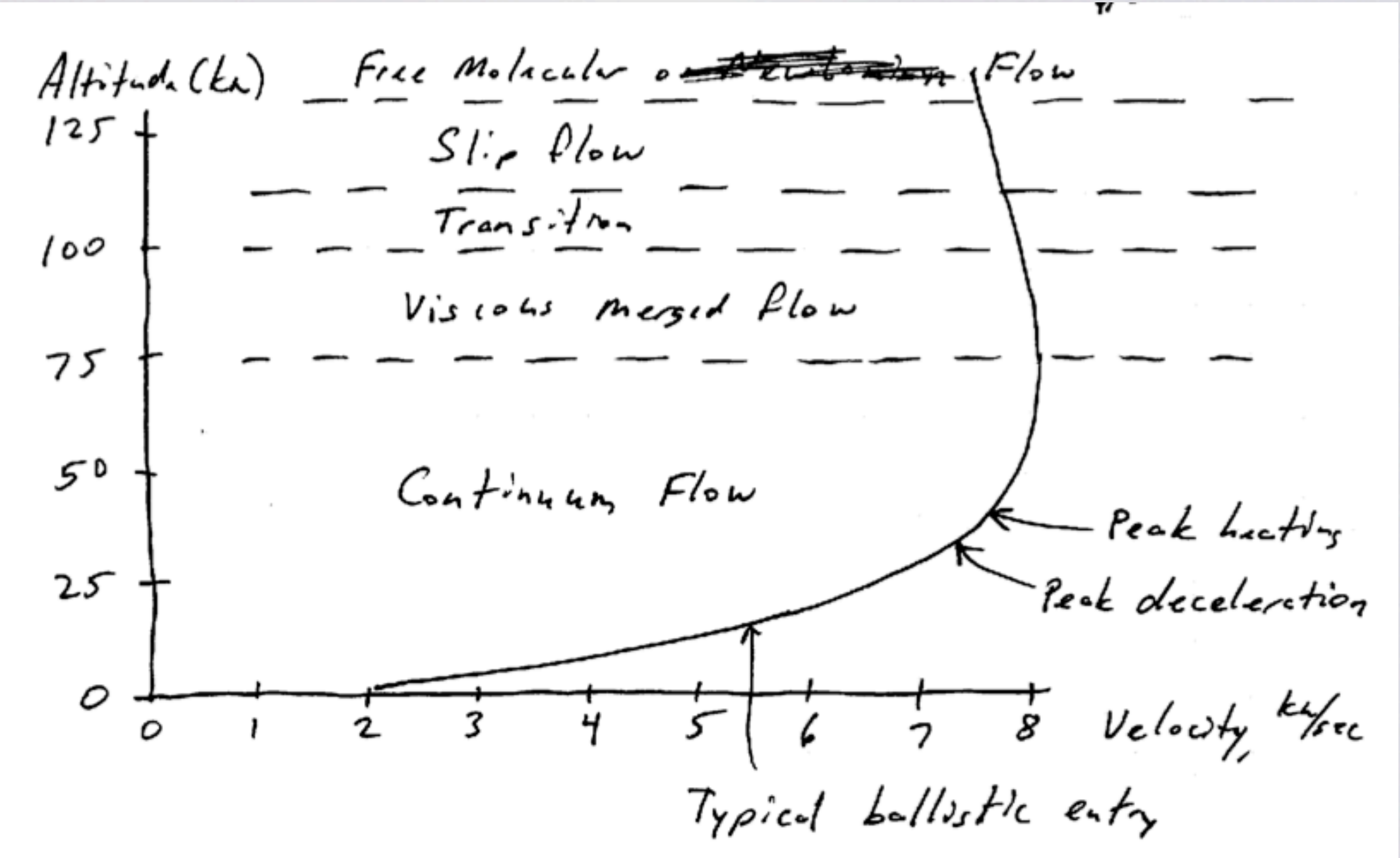
Newtonian Analysis of Rarified Flows

- Atmospheric Regimes on Entry
- Basic fluid parameters
- Definition of Mean Free Path
- Rarified gas Newtonian flow
- Continuum Newtonian flow (hypersonics)
- SphereConeAero software

Atmospheric Regimes on Entry



Basic Fluids Parameters

$$M \equiv \text{Mach Number} = \frac{v}{a}$$

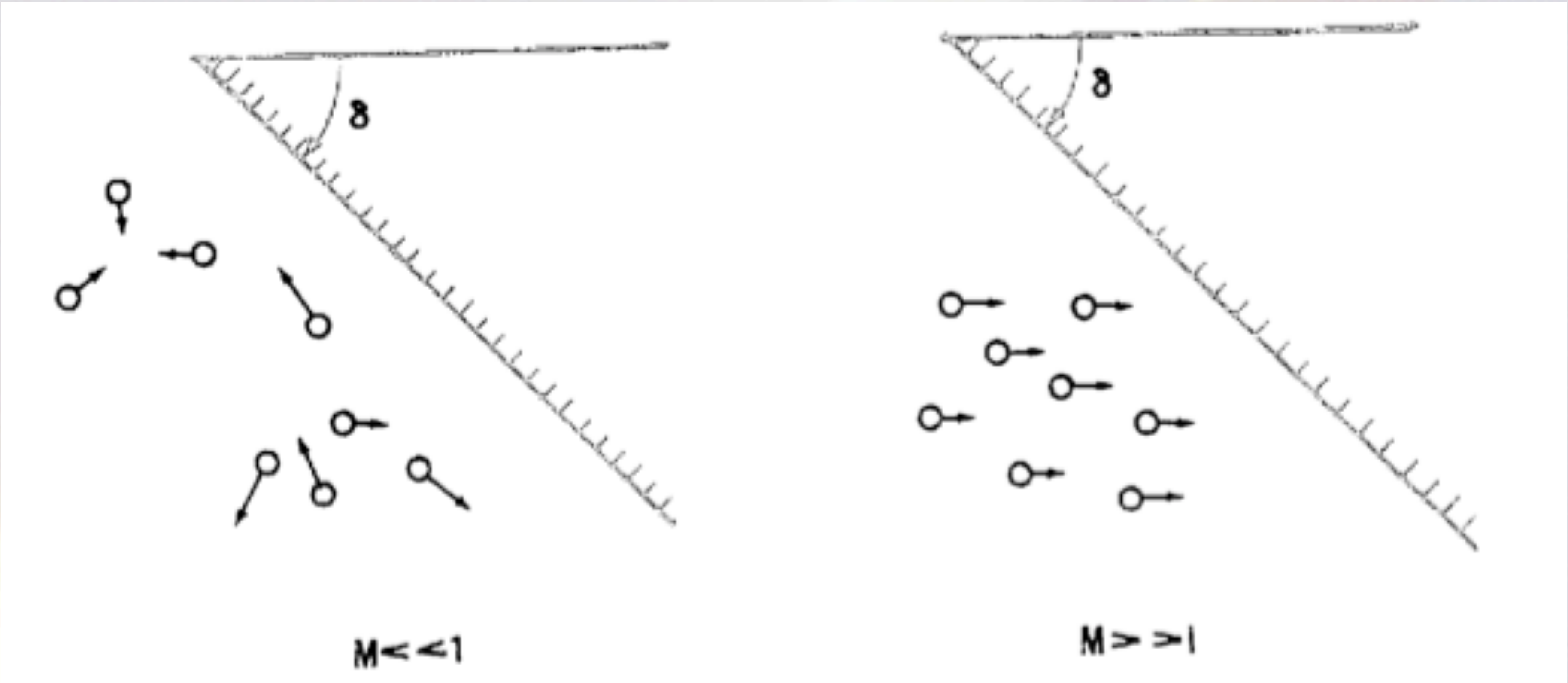
$$a \equiv \text{speed of sound} = \sqrt{\gamma R T} \qquad \left(R = \frac{\Re}{\bar{m}} \right)$$

$$\frac{\text{ordered energy}}{\text{random energy}} = \frac{\frac{1}{2} m v^2}{\frac{1}{2} m \bar{v}_g^2} = \frac{v^2}{3RT} = \frac{\gamma}{3} \frac{v^2}{a^2} = \frac{\gamma}{3} M^2$$

$$Re \equiv \text{Reynold's number} = \frac{\text{inertial force}}{\text{viscous force}}$$

$$Re = \frac{\dot{m} v}{\tau A} = \frac{\rho A v^2}{\mu \frac{v}{L} A} = \frac{\rho v L}{\mu}$$

Random vs. Ordered Energy



More Fluid Parameters

$$K \equiv \text{Knudsen number}$$

$$K = \frac{\text{number of collisions with body}}{\text{number of collisions with other molecules}}$$

$$K = \frac{\lambda}{L}$$

$$\lambda \equiv \text{mean free path}$$

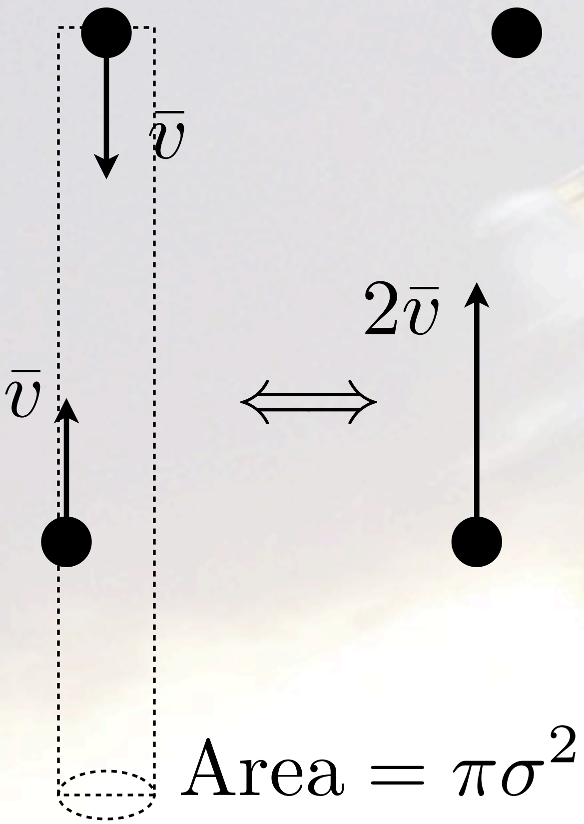
$$L \equiv \text{vehicle characteristic length}$$

Estimating Mean Free Path

Assume:

- All molecules are perfect rigid spheres
- Each has diameter σ, mass m, and velocity $\bar{v}$
- Consider a cube with side length L containing N molecules
- N/6 molecules are traveling in each direction
  - ±X
  - ±Y
  - ±Z

Consider Collisions in +Z Direction



- number of potential +Z collisions

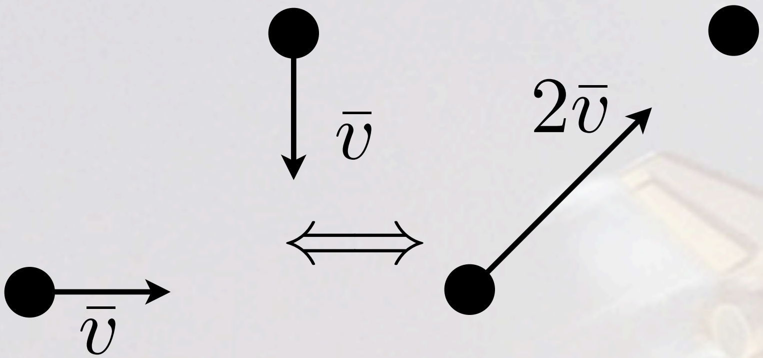
$$n(+Z) = \frac{1}{6} N \frac{\pi\sigma^2 L}{L^3} = \frac{1}{6} N \frac{\pi\sigma^2}{L^2}$$

frequency of +Z collisions

$$f(+Z) = \frac{n(+Z)}{\Delta t} = \frac{\frac{\pi}{6} N \frac{\sigma^2}{L^2}}{\frac{L}{2\bar{v}}}$$

$$f(+Z) = \frac{\pi\rho\sigma^2\bar{v}}{3m}$$

Consider Collisions in +X Direction



frequency of +X collisions

$$f(+X) = \frac{n(+X)}{\Delta t} = \frac{\sqrt{2}\pi\rho\sigma^2\bar{v}}{6m}$$

$$f(-X) = f(+Y) = f(-Y) = f(+X) \qquad f(-Z) = 0$$

Total frequency of collisions

$$f = \frac{\pi}{3}(1 + 2\sqrt{2})\frac{\rho\sigma^2\bar{v}}{m}$$

Mean Free Path

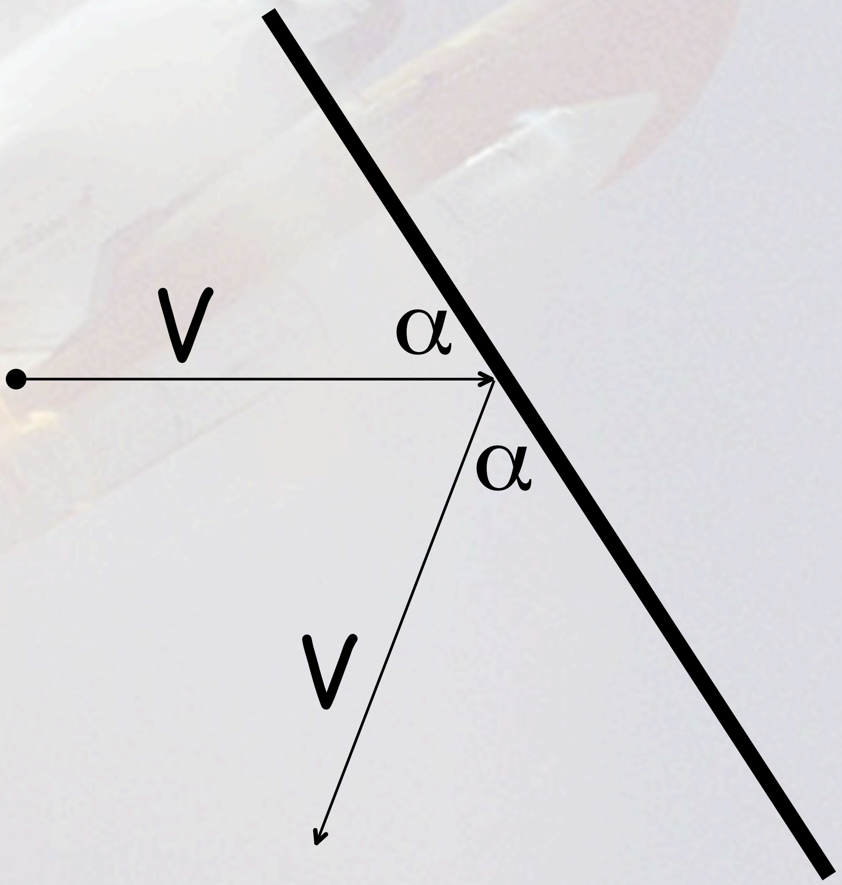
$$\lambda = \frac{\bar{v}}{f} = \frac{m/\sigma^2}{\frac{\pi}{3}(1+2\sqrt{2})\rho} \quad \left(\propto \frac{1}{\rho}\right)$$

$$\text{at sea level: } \lambda = 6.7 \times 10^{-8}\ m$$

$$\text{at 100 km: } \lambda = 0.3\ m \sim 1\ ft$$

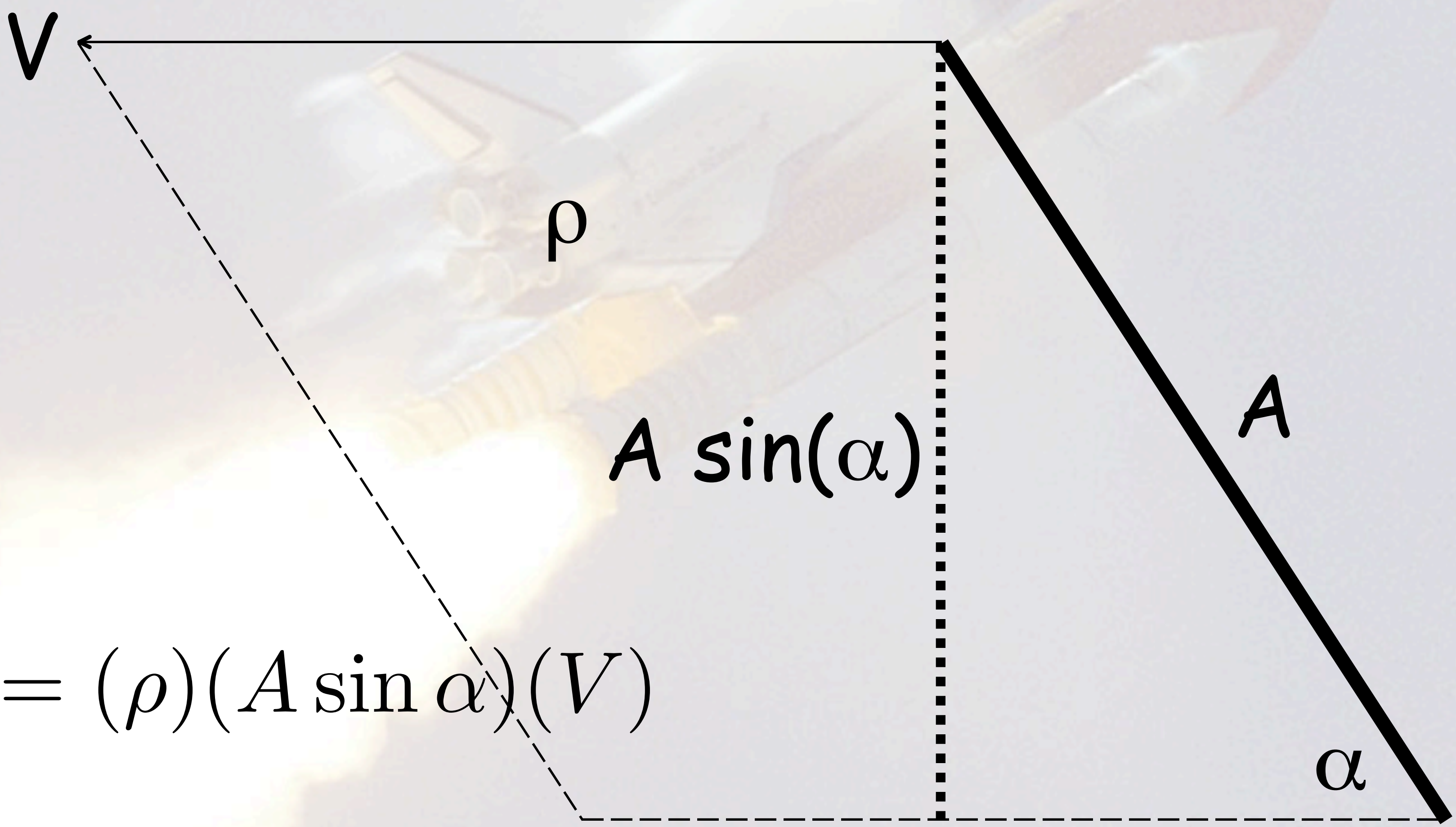
Newtonian Flow

- Mean free path of particles much larger than spacecraft --> no appreciable interaction of air molecules
- Model vehicle/ atmosphere interactions as independent perfectly elastic collisions



Newtonian Analysis

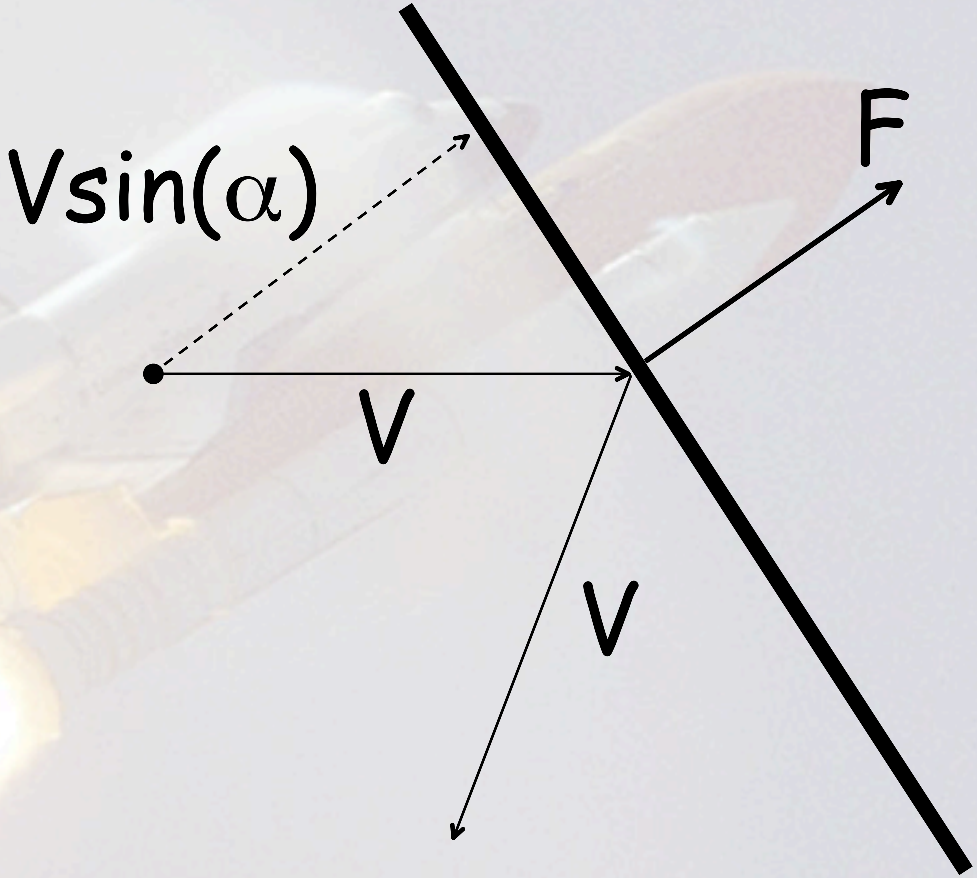
$$\text{mass flux} = (\text{density})(\text{swept area})(\text{velocity})$$



$$\frac{dm}{dt} = (\rho)(A \sin \alpha)(V)$$

Momentum Transfer

- Momentum perpendicular to wall is reversed at impact
- "Bounce" momentum is transferred to vehicle
- Momentum parallel to wall is unchanged

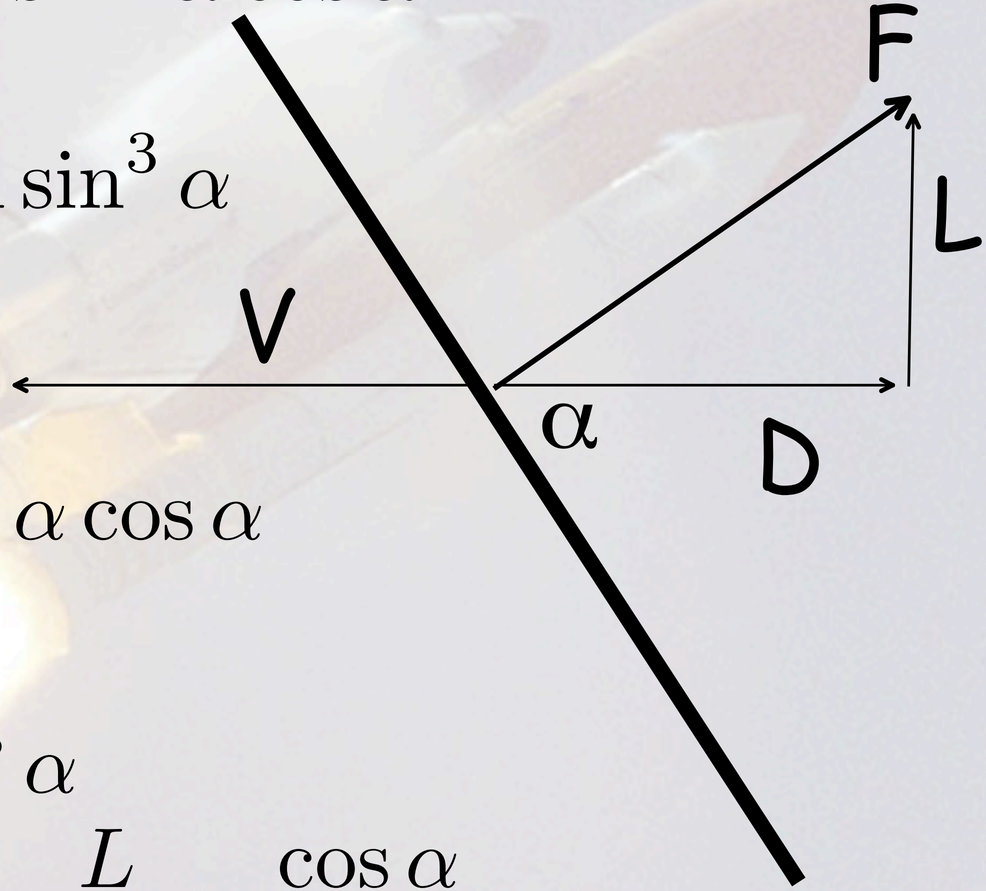


$$F = \frac{dm}{dt}\Delta V = \rho V A \sin\alpha(2V\sin\alpha) = 2\rho V^2 A \sin^2\alpha$$

Lift and Drag

$$L = F\cos\alpha = 2\rho V^2 A \sin^2\alpha\cos\alpha$$

$$D = F\sin\alpha = 2\rho V^2 A\sin^3\alpha$$

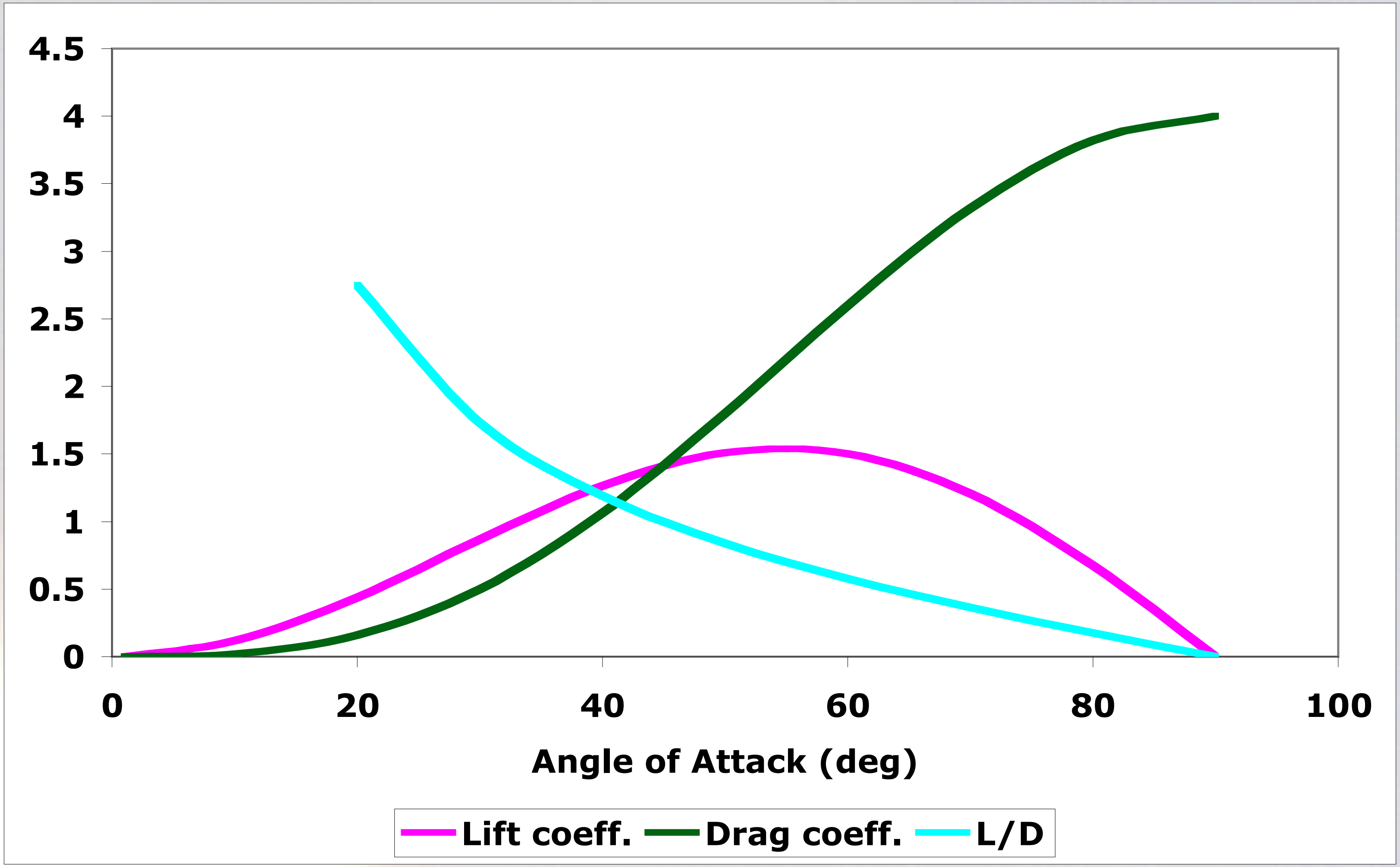


$$c_L = \frac{L}{\frac{1}{2}\rho V^2 A} = 4\sin^2\alpha\cos\alpha$$

$$c_D = \frac{D}{\frac{1}{2}\rho V^2 A} = 4\sin^3\alpha$$

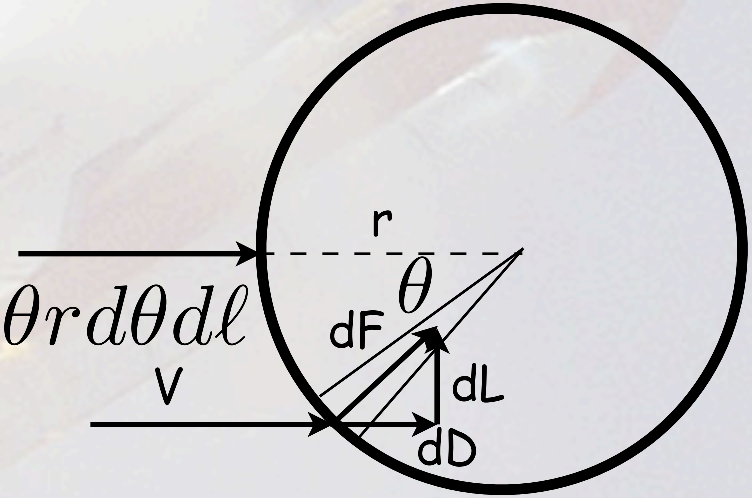
$$\frac{L}{D} = \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

Flat Plate Newtonian Aerodynamics



Example of Newtonian Flow Calculations

Consider a cylinder of length l, entering atmosphere transverse to flow



$$dA = r d\theta dl$$

$$d\dot{m} = \rho dA \cos\theta V = \rho V \cos\theta r d\theta d\ell$$

$$dF = d\dot{m}\Delta V = 2\rho V^2 \cos^2\theta r d\theta d\ell$$

$$dD = dF\cos\theta = 2\rho V^2 \cos^3\theta r d\theta d\ell$$

$$dL = dF\sin\theta = 2\rho V^2 \cos^2\theta \sin\theta r d\theta d\ell$$

Integration to Find Drag Coefficient

Integrate from $\theta = -\frac{\pi}{2} \rightarrow \frac{\pi}{2}$

$$D = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{\ell} dD = 2\rho V^2 r \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \int_0^{\ell} \cos^3\theta d\theta d\ell$$

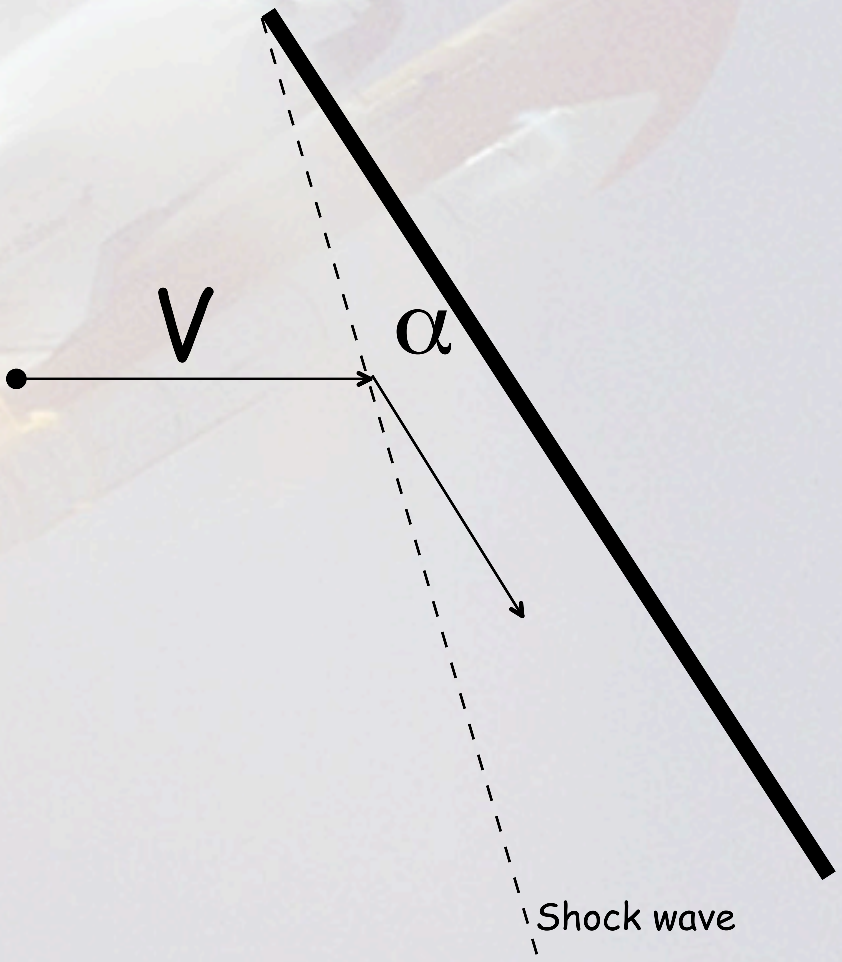
$$= 2\rho V^2 r\ell \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} \cos^3\theta d\theta = \boxed{\frac{8}{3}\rho V^2 r\ell}$$

By definition, $D = \frac{1}{2}\rho V^2 A c_D$ and, for a cylinder $A = 2r\ell$

$$\rho V^2 r\ell c_D = \frac{8}{3}\rho V^2 r\ell \Longrightarrow \boxed{c_D = \frac{8}{3}}$$

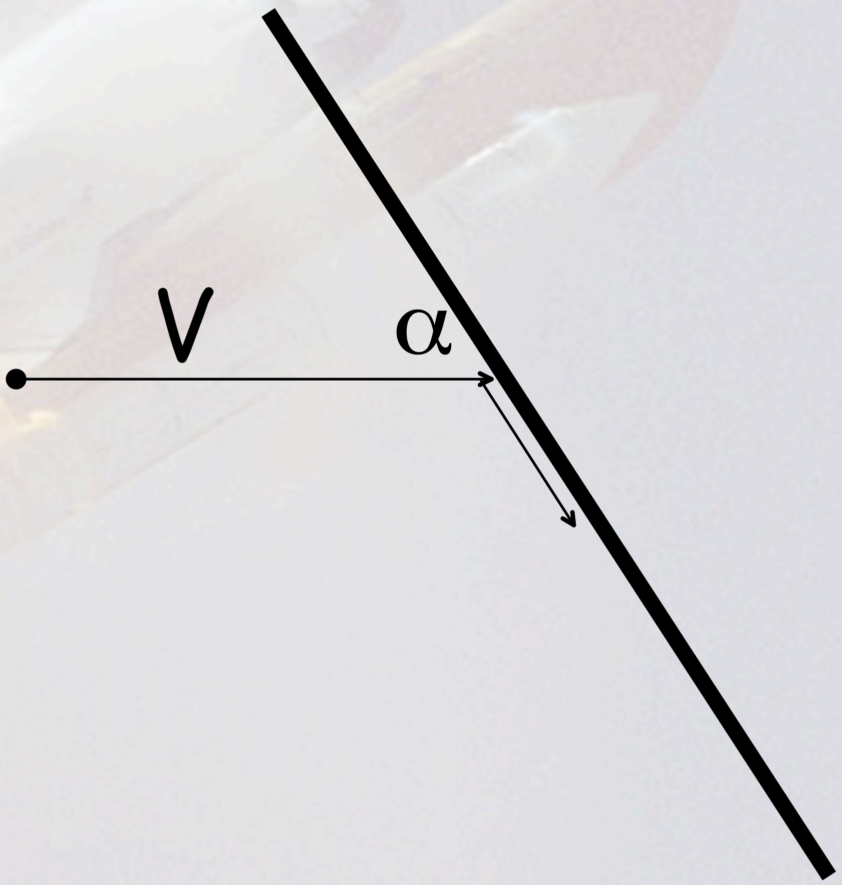
Continuum Newtonian Flow (Hypersonics)

- Air molecules predominately interact with shock waves
- Effect of shock wave passage is to decelerate flow and turn it parallel to vehicle surface



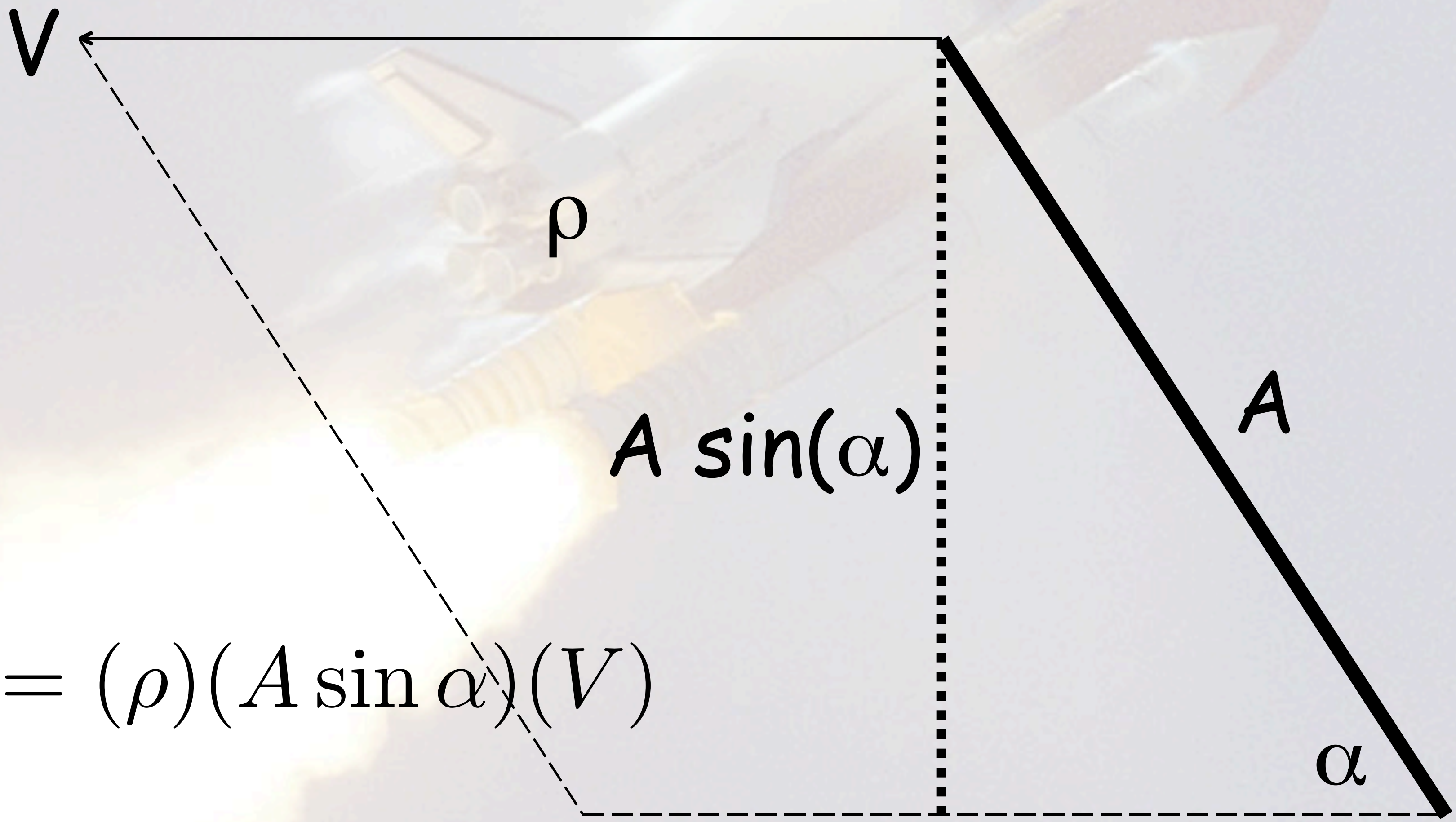
Continuum Newtonian Flow (Hypersonics)

- Treat hypersonic aerodynamics in manner similar to previous Newtonian flow analysis
- All momentum perpendicular to wall is absorbed by the wall



Mass Flux (unchanged)

$$\text{mass flux} = (\text{density})(\text{swept area})(\text{velocity})$$



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Momentum Transfer

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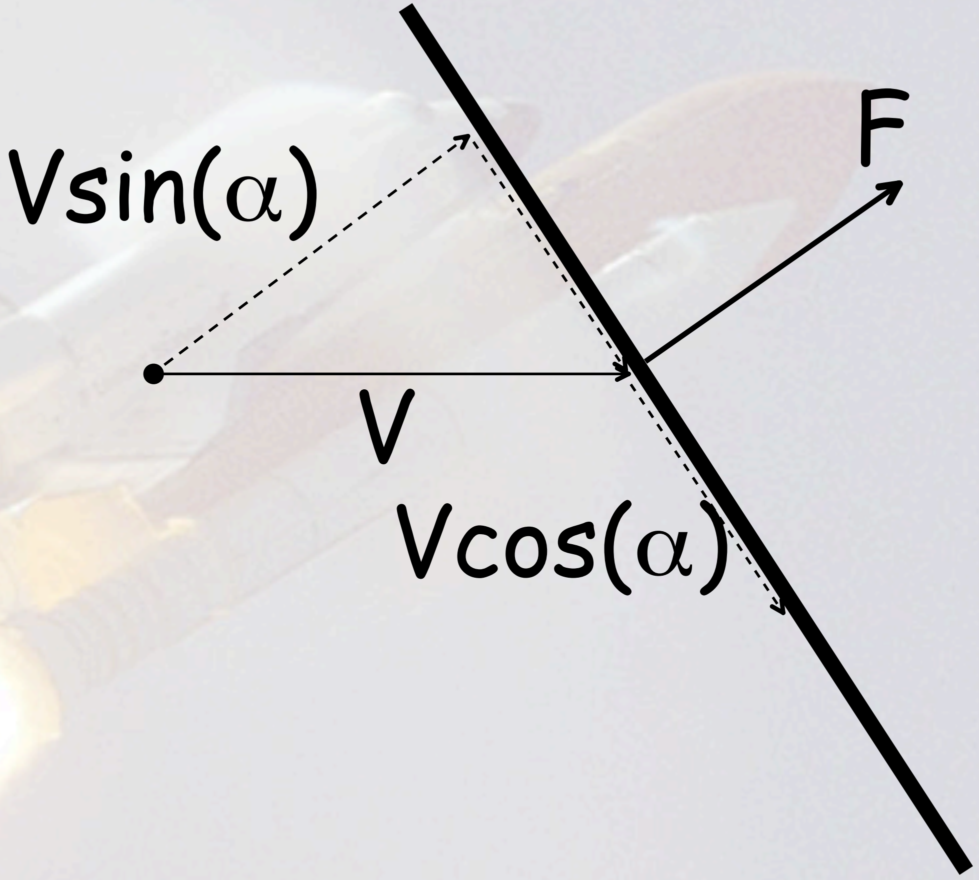

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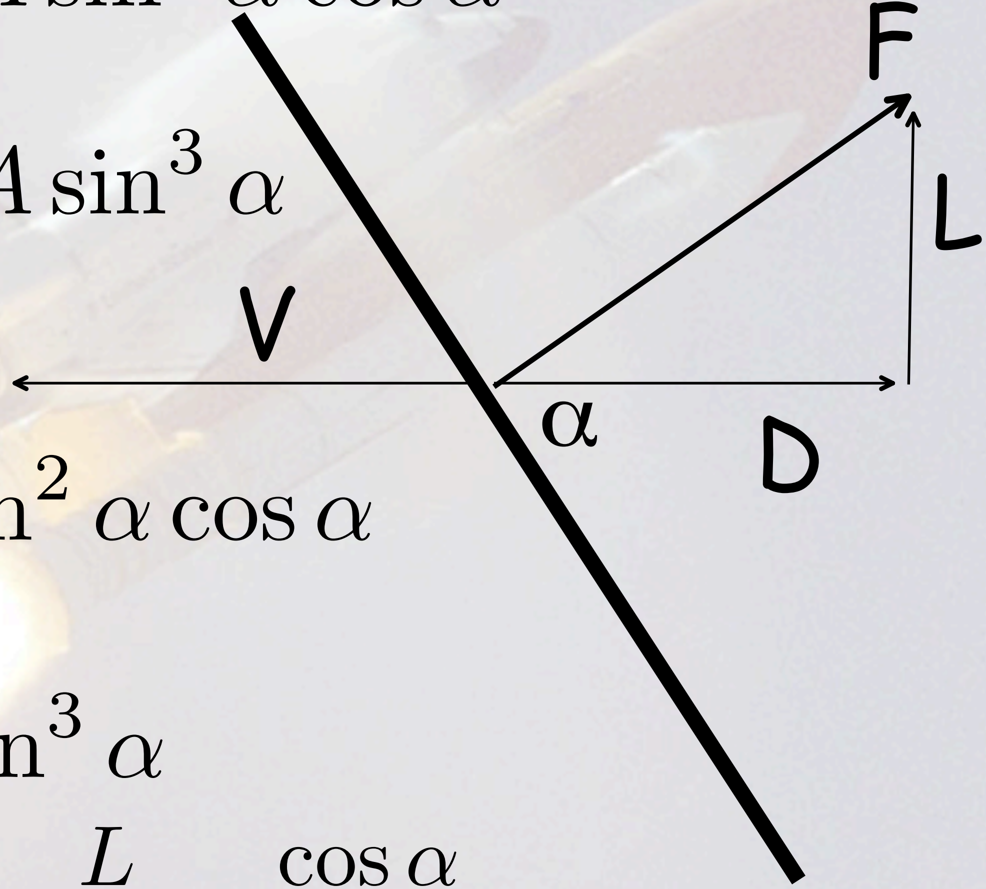

$$c_L = \frac{L}{\frac{1}{2}\rho V^2 A} = 2\sin^2\alpha\cos\alpha$$

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$$\frac{L}{D} = \frac{\cos\alpha}{\sin\alpha} = \cot\alpha$$

Modified Newtonian Flow

- Coefficient of pressure in "classical" Newtonian flow

$$c_p = 2\sin^2(\alpha)$$

- Coefficient of pressure in modified Newtonian flow

$$c_p = c_{p_{max}} \sin^2(\alpha)$$

- Cp(max) is the pressure coefficient behind a normal shock at flight conditions

$$c_{p_{max}} = \frac{P_{shock} - P_\infty}{\frac{1}{2}\rho_\infty v_\infty^2}$$

Maximum Coefficient of Pressure

$$c_{p_{max}} = \frac{2}{\gamma M_\infty^2} \left\{ \left[\frac{(\gamma+1)^2 M_\infty^2}{4\gamma M_\infty^2 - 2(\gamma-1)} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_\infty^2}{\gamma+1} \right] - 1 \right\}$$

as $M \longrightarrow \infty$

$$c_{p_{max}} \longrightarrow \left[\frac{(\gamma+1)^2}{4\gamma} \right]^{\frac{\gamma}{\gamma-1}} \left[\frac{4}{\gamma+1} \right]$$

$$c_{p_{max}} \longrightarrow 1.839 \text{ for } \gamma = 1.4$$

$$c_{p_{max}} \longrightarrow 2 \text{ for } \gamma = 1$$