Course Overview/Orbital Mechanics

- Course Overview
  - Challenges of launch and entry
  - Course goals
  - Web-based Content
  - Syllabus
  - Policies
  - Project Content
- An overview of orbital mechanics at "point five past lightspeed"

Space Transportation System – NASA

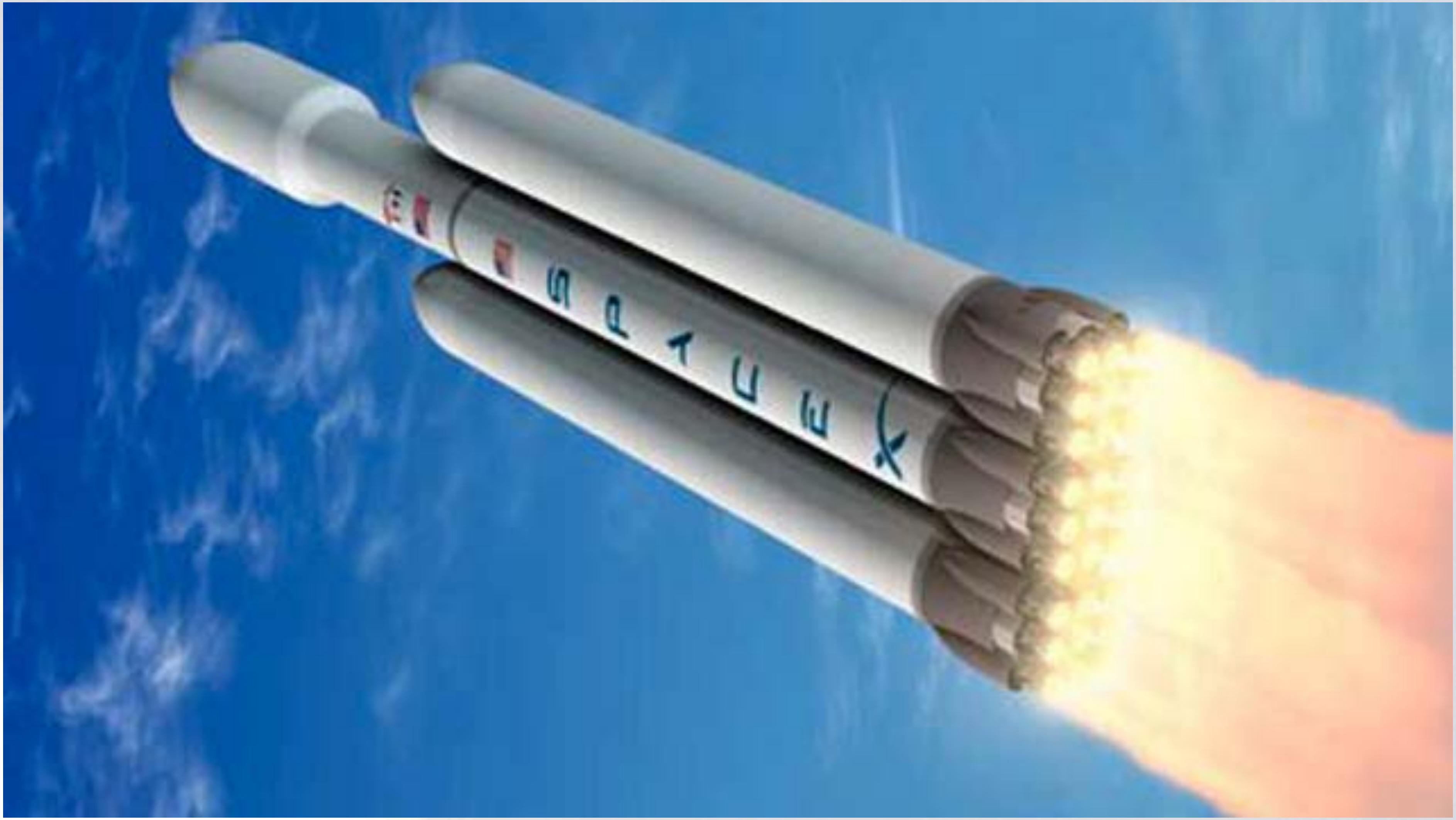
Space Launch System – NASA

Antares Launch Vehicle – Orbital

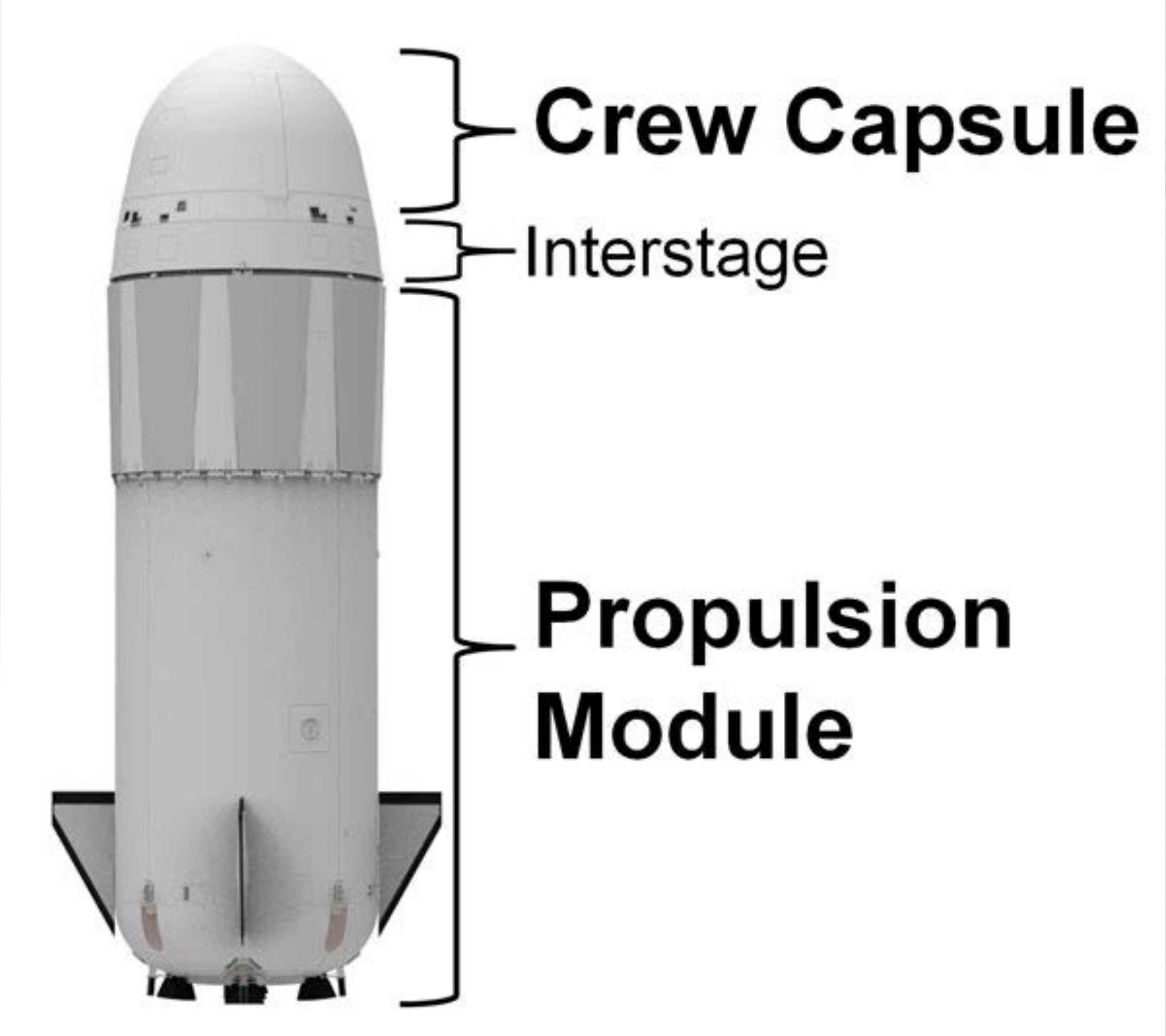
Falcon 9 v1.1 – SpaceX

Liberty Launch Vehicle – ATK

Falcon Heavy – SpaceX



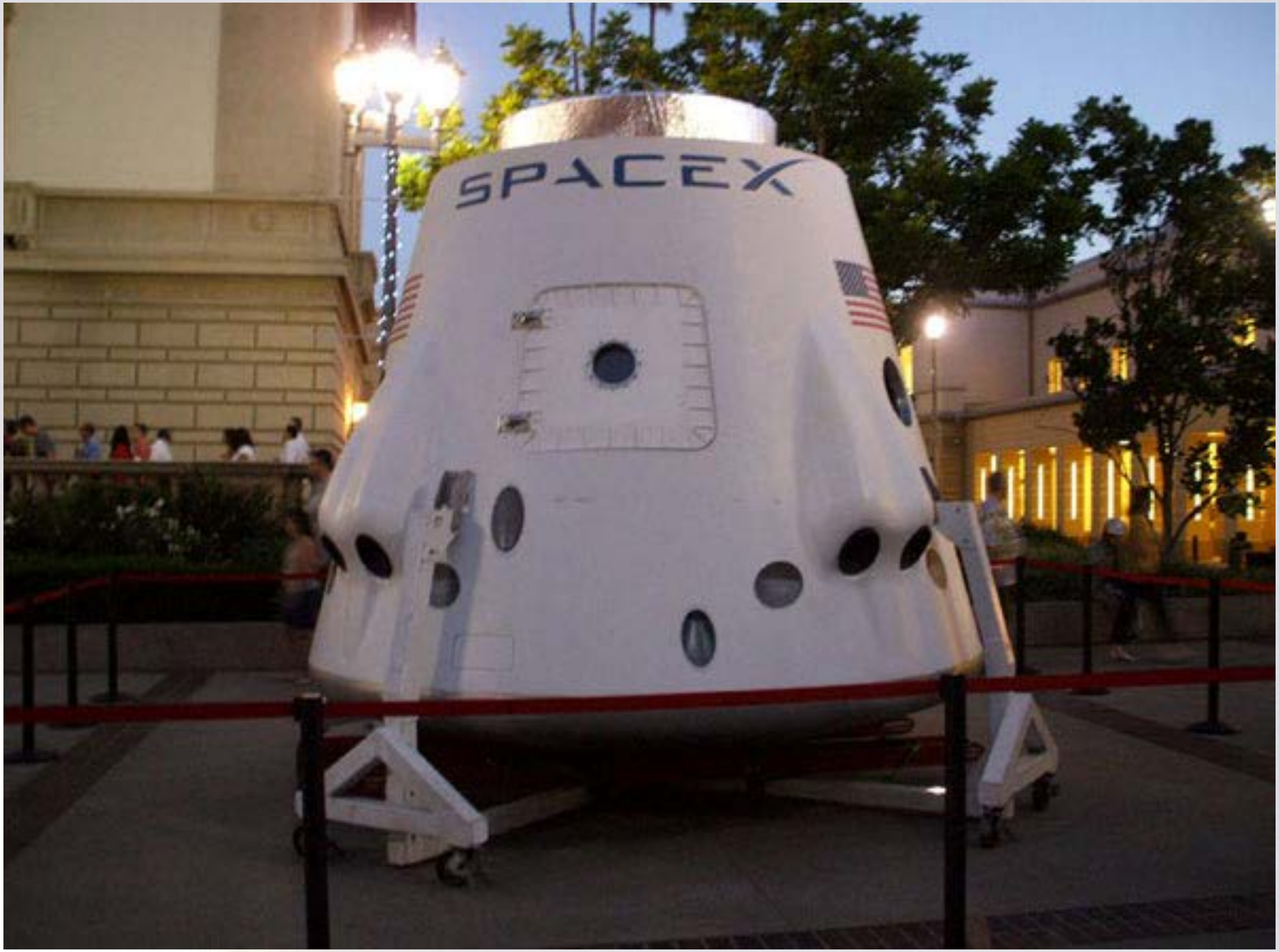
Blue Origins Launch Vehicle



Dragon Cargo Spacecraft – SpaceX

Orion Spacecraft – NASA

Dragon Rider Spacecraft – SpaceX



Dream Chaser – Sierra Nevada Corp.

CST-100 – Boeing

Blue Origins Biconic Spacecraft

Grasshopper – SpaceX

Spaceship One – Rutan Aircraft Factory

Spaceship Two - Virgin Galactic

Lynx Suborbital Vehicle – XCOR

Mars Colonial Transport – SpaceX

Space Launch - The Physics

- Minimum orbital altitude is ~200 km

$$\frac{Potential\ Energy}{kg\ in\ orbit} = -\frac{\mu}{r_{orbit}} + \frac{\mu}{r_E} = 1.9 \times 10^6\ \frac{J}{kg}$$

- Circular orbital velocity there is 7784 m/sec

$$\frac{Kinetic\ Energy}{kg\ in\ orbit} = \frac{1}{2}\frac{\mu}{r^2_{orbit}} = 30 \times 10^6\ \frac{J}{kg}$$

- Total energy per kg in orbit

$$\frac{Total\ Energy}{kg\ in\ orbit} = KE + PE = 32 \times 10^6\ \frac{J}{kg}$$

Theoretical Cost to Orbit

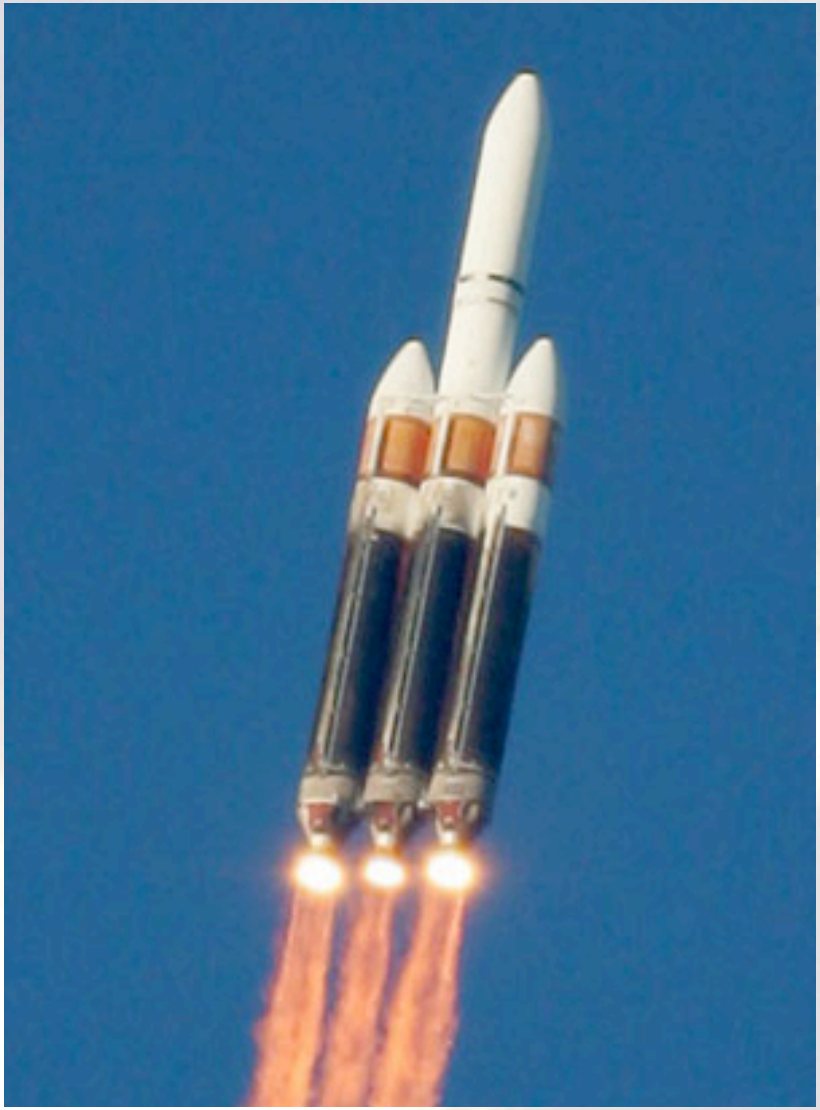
- Convert to usual energy units

$$\frac{Total\ Energy}{kg\ in\ orbit} = 32 \times 10^6\ \frac{J}{kg} = 8.9\ \frac{kWhrs}{kg}$$

- Domestic energy costs are ~\$0.09/kWhr

    - Theoretical cost to orbit \$0.99/kg

Actual Cost to Orbit



- Delta IV Heavy
  - 23,000 kg to LEO
  - $450 M per flight
- $19,570/kg of payload
- Factor of 19,800x higher than theoretical energy costs!

What About Airplanes?

- For an aircraft in level flight,

$$\frac{\text{Weight}}{\text{Thrust}} = \frac{\text{Lift}}{\text{Drag}}, \text{ or } \frac{mg}{T} = \frac{L}{D}$$

- Energy = force x distance, so

$$\frac{\text{Total Energy}}{\text{kg}} = \frac{\text{thrust} \times \text{distance}}{\text{mass}} = \frac{Td}{m} = \frac{gd}{L/D}$$

- For an airliner (L/D=25) to equal orbital energy, d=81,000 km (2 roundtrips NY-Sydney)

Equivalent Airline Costs?

- Average economy ticket NY-Sydney round-round-trip (Travelocity 1/27/14) ~\$1500
- Average passenger (+ luggage) ~100 kg
- Two round trips = \$30/kg
  - Factor of 30x more than electrical energy costs
  - Factor of 660x less than current launch costs
- But...

  you get to refuel at each stop!

Equivalence to Air Transport

- 81,000 km ~ twice around the world
- Voyager - one of two aircraft to ever circle the world non-stop, non-refueled - ***once!***

Orbital Entry - The Physics

- 32 MJ/kg dissipated by friction with atmosphere over ~8 min = 66kW/kg
- Pure graphite (carbon) high-temperature material: c_p=709 J/kg°K
- Orbital energy would cause temperature gain of 45,000°K!
- Thus proving the comment about space travel, "It's utter bilge!" (Sir Richard Wooley, Astronomer Royal of Great Britain, 1956)

The Vision

"Once you make it to low Earth orbit, you're halfway to anywhere!"

- Robert A. Heinlein

Goals of ENAE 791

- Learn the underlying physics (orbital mechanics, flight mechanics, aerothermodynamics) which constrain and define launch and entry vehicles
- Develop the tools for preliminary design synthesis, including the fundamentals of systems analysis
- Provide an introduction to engineering economics, with a focus on the parameters affecting cost of launch and entry vehicles, such as reusability
- Examine specific challenges in the underlying design disciplines, such as thermal protection and structural dynamics

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Web-based Course Content

- Data web site at **http://spacecraft.ssl.umd.edu**
  - Course information
  - Syllabus
  - Lecture notes
  - Problems and solutions
- Interactive web site at **http://elms.umd.edu**
  - Communications for team projects (forums, wiki, blogs)
  - Surveys for course feedback
  - Videos of lectures

Syllabus Overview (1)

- Fundamentals of Launch and Entry Design
  - Orbital mechanics
  - Basic rocket performance
- Entry flight mechanics
  - Ballistic entry
  - Lifting entry
- Aerothermodynamics
- Thermal Protection System (TPS) analysis
- Entry, Descent, and Landing (EDL) systems

Syllabus Overview (2)

- Launch flight mechanics
  - Gravity turn
  - Targeted trajectories
  - Optimal trajectories
  - Airbreathing trajectories
- Launch vehicle systems
  - Propulsion systems
  - Structures and structural dynamics analysis
  - Avionics
  - Payload accommodations
  - Ground launch processing

Syllabus Overview (3)

- Systems Analysis
  - Cost estimation
  - Engineering economics
  - Reliability issues
  - Safety design concerns
  - Fleet resiliency
  - Multidisciplinary optimization
- Case studies
- Design project

Policies

- Grade Distribution
  - 25% Problems
  - 20% Midterm Exam
  - 25% Term Project
  - 30% Final Exam
- Late Policy
  - On time: Full credit
  - Before solutions: 70% credit
  - After solutions: 20% credit

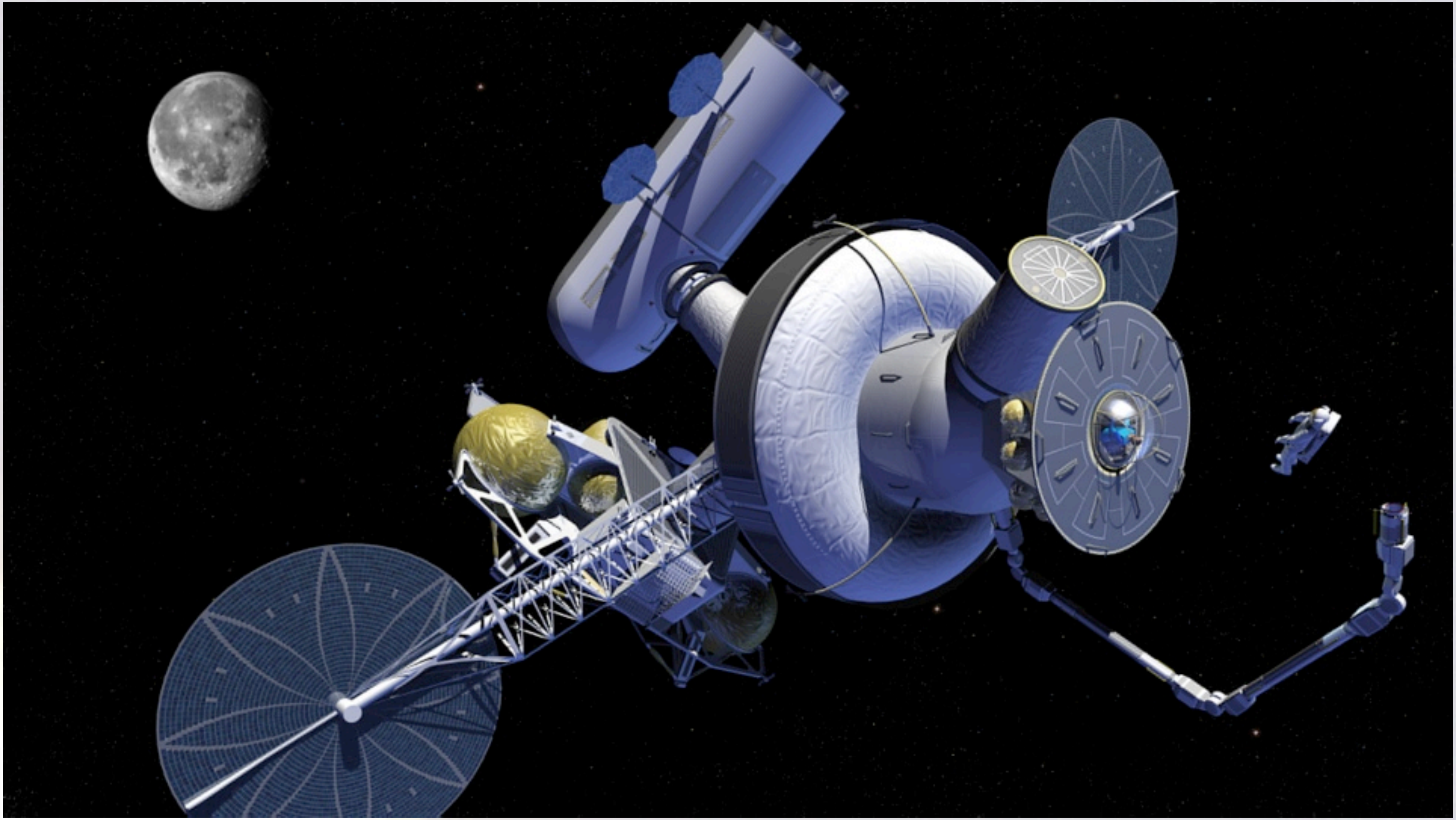
A Word on Homework Submissions...

- Good methods of handing in homework
  - Hard copy in class (best!)
  - Electronic or scanned copies via e-mail (please put "ENAE791"in the subject line)
- Methods that don't work so well
  - Leaving it in my mailbox (particularly in EGR)
  - Leaving it in my office
  - Uncommented spreadsheets or .m files
  - Handing it to me in random locations
  - Handing it to Dr. Bowden

A Word about Homework Grading

- Homework is graded via a discrete filter
  - ✓ for homework problems which are essentially correct (10 pts)
  - ✓- for homework with significant problems (7 pts)
  - ✓-- for homework with major problems (4 pts)
  - ✓+ for homework demonstrating extra effort (12 pts)
  - 0 for missing homework
- A detailed solution document is posted for each problem after the due date, which you should review to ensure you understand the techniques used

Term Project - Cislunar Space Transport



Term Project - Top Level Requirements

- Design a system to allow the construction and support of multiple habitats in cislunar space
  - Earth-Moon L1 for deep space staging
  - Low lunar orbit for lunar surface exploration
  - Lunar distant retrograde orbit for asteroid resource recovery
- Mission models
  - Human and cargo launch and human return from cislunar space
  - Details of mission models to follow

Term Project

- Work as individuals or two-person teams (your choice)
- Design an architecture to support cislunar operations in the most cost effective manner possible
- All vehicles will be conceptually designed from scratch (no “catalog engineering”!)
- Parametric design parameters will be provided for human spacecraft systems not ENAE791-relevant
- Design process should proceed throughout the term
- Formal design presentations at end of term

Orbital Mechanics: 500 years in 40 min.

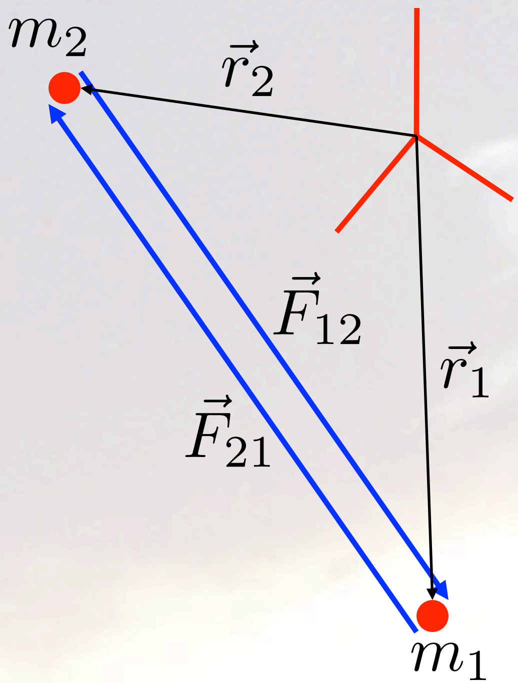
- Newton's Law of Universal Gravitation

$$F = \frac{G m_1 m_2}{r^2}$$

- Newton's First Law meets vector algebra

$$\overrightarrow{F} = m\overrightarrow{a}$$

Relative Motion Between Two Bodies



$$m_1 \frac{d^2\vec{r}_1}{dt^2} = G\frac{m_1 m_2}{|\vec{r}_{21}|^2}\frac{\vec{r}_{21}}{|\vec{r}_{21}|}$$

$$= G\frac{m_1 m_2}{|\vec{r}_{21}|^3}\vec{r}_{21} = G\frac{m_1 m_2}{|\vec{r}_{21}|^3}(\vec{r}_2 - \vec{r}_1)$$

$$m_2 \frac{d^2\vec{r}_2}{dt^2} = G\frac{m_1 m_2}{|\vec{r}_{12}|^3}(\vec{r}_1 - \vec{r}_2)$$

$\vec{F}_{12} =$ force due to body 1 on body 2

Gravitational Motion

$$\frac{d^2\vec{r}}{dt^2} = \frac{G}{r^3}\left[m_2\left(-\vec{r}\right) - m_1\left(\vec{r}\right)\right] = \frac{-G}{r^3}\left(m_1 + m_2\right)\vec{r}$$

$$\text{Let } r = |\vec{r}_{12}| = |\vec{r}_{21}| \qquad \text{Let } \vec{r} = \vec{r}_1 - \vec{r}_2$$

$$\text{Let } \mu = G(m_1 + m_2)$$

$$\boxed{\frac{d^2\vec{r}}{dt^2} + \mu\frac{\vec{r}}{r^3} = \vec{0}}$$

"Equation of Orbit" -
Orbital motion is simple harmonic motion

Orbital Angular Momentum

$$\vec{v} = \frac{d\vec{r}}{dt} \qquad \frac{d\vec{v}}{dt} + \mu\frac{\vec{r}}{r^3} = \vec{0}$$

$$\vec{r} \times \frac{d\vec{v}}{dt} + \frac{\mu}{r^3}\left(\vec{r} \times \vec{r}\right) = \vec{0} \qquad \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}$$

$$= \vec{v} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt} = \vec{r} \times \frac{d\vec{v}}{dt} = \vec{0}$$

$$\frac{d}{dt}\left(\vec{r} \times \vec{v}\right) = \vec{0} \qquad \vec{r} \times \vec{v} = \vec{constant} \qquad \vec{r} \times \vec{v} = \vec{h}$$

$\vec{h}$ is angular momentum vector (constant) $\Longrightarrow$
$\vec{r}$ and $\vec{v}$ are in a constant plane

Fun and Games with Algebra

$$\frac{d\vec{v}}{dt} + \mu\frac{\vec{r}}{r^3} = \vec{0} \qquad \frac{d\vec{v}}{dt}\times\vec{h} + \frac{\mu}{r^3}\left(\vec{r}\times\vec{h}\right) = \vec{0}$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = \frac{d\vec{v}}{dt}\times\vec{h} + \cancelto{0}{\vec{v}\times\frac{d\vec{h}}{dt}}$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r}\times\vec{h}\right) = -\frac{\mu}{r^3}\left(\vec{r}\times\vec{r}\times\vec{v}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\frac{\mu}{r^3}\left[\left(\vec{r}\cdot\vec{v}\right)\vec{r} - \left(\vec{r}\cdot\vec{r}\right)\vec{v}\right]$$

$$\vec{r}\cdot\vec{v} = rv\cos\gamma = r\frac{dr}{dt}$$

More Algebra, More Fun

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\frac{\mu}{r^3}\left[r\frac{dr}{dt}\vec{r} - r^2\frac{d\vec{r}}{dt}\right]$$

$$\frac{d}{dt}\left(\frac{\vec{r}}{r}\right) = \frac{\left(r\frac{d\vec{r}}{dt} - \vec{r}\frac{dr}{dt}\right)}{r^2} = \left(\frac{1}{r}\frac{d\vec{r}}{dt} - \frac{\vec{r}}{r^2}\frac{dr}{dt}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h}\right) = -\mu\left(\frac{1}{r^2}\frac{dr}{dt}\vec{r} - \frac{1}{r}\frac{d\vec{r}}{dt}\right) = \mu\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$$

$$\frac{d}{dt}\left(\vec{v}\times\vec{h} - \mu\frac{\vec{r}}{r}\right) = \vec{0}$$

Orientation of the Orbit

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \overrightarrow{\text{constant}}$$

$$\vec{v} \times \vec{h} - \mu \frac{\vec{r}}{r} = \mu \vec{e}$$

$\vec{e} \equiv$ eccentricity vector, in orbital plane

$\vec{e}$ points in the direction of periapsis

$$\vec{r} \cdot \vec{v} \times \vec{h} - \vec{r} \cdot \mu \frac{\vec{r}}{r} = \mu \left(\vec{r} \cdot \vec{e} \right)$$

$$\vec{r} \times \vec{v} \cdot \vec{h} - \mu \frac{\vec{r} \cdot \vec{r}}{r} = \mu r e \cos \theta$$

$$\vec{h} \cdot \vec{h} - \mu \frac{r^2}{r} = \mu r e \cos \theta$$

Position in Orbit

$$h^2 - \mu r = \mu r e \cos\theta$$

$$\boxed{r = \frac{h^2/\mu}{1 + e\cos\theta}}$$

θ = true anomaly: angular travel from perigee passage

$$\text{at } \theta = \pm\frac{\pi}{2};\ \cos\theta = 0;\ r = p \equiv h^2/\mu$$

Relating Velocity and Orbital Elements

$$\mu\vec{e} = \vec{v} \times \vec{h} - \mu\frac{\vec{r}}{r}$$

$$\mu\vec{e} \cdot \mu\vec{e} = \vec{v} \times \vec{h} \cdot \vec{v} \times \vec{h} - 2\mu\left(\vec{v} \times \vec{h}\right) \cdot \frac{\vec{r}}{r} + \mu^2\left(\frac{\vec{r}}{r} \cdot \frac{\vec{r}}{r}\right)$$

$$\mu^2 e^2 = v^2 h^2 - 2\mu\frac{h^2}{r} + \mu^2$$

$$e^2 = \frac{v^2}{\mu}p - 2\frac{p}{r} + 1$$

Vis-Viva Equation

$$p \equiv a(1-e^2) = \frac{1-e^2}{\frac{2}{r}-\frac{v^2}{\mu}}$$

$$a = \left(\frac{2}{r}-\frac{v^2}{\mu}\right)^{-1}$$

$$\boxed{v^2 = \mu\left(\frac{2}{r}-\frac{1}{a}\right)}$$

<--Vis-Viva Equation

$$\frac{v^2}{2}-\frac{\mu}{r} = -\frac{\mu}{2a}$$

Energy in Orbit

- Kinetic Energy

$$K.E. = \frac{1}{2}mv^2 \Rightarrow \frac{K.E.}{m} = \frac{v^2}{2}$$

- Potential Energy

$$P.E. = -\frac{m\mu}{r} \Rightarrow \frac{P.E.}{m} = -\frac{\mu}{r}$$

- Total Energy

$$Const. = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

<--Vis-Viva Equation

Suborbital Tourism - Spaceship Two

How Close are we to Space Tourism?

- Energy for 100 km vertical climb

$$-\frac{\mu}{r_E + 100\ km} + \frac{\mu}{r_E} = 0.965\ \frac{km^2}{sec^2} = 0.965\ \frac{MJ}{kg}$$

- Energy for 200 km circular orbit

$$-\frac{\mu}{2(r_E + 200\ km)} + \frac{\mu}{r_E} = 32.2\ \frac{km^2}{sec^2} = 32.2\ \frac{MJ}{kg}$$

- Energy difference is a factor of 33!

Implications of Vis-Viva

- Circular orbit (r=a)

$$v_{circular} = \sqrt{\frac{\mu}{r}}$$

- Parabolic escape orbit (a tends to infinity)

$$v_{escape} = \sqrt{\frac{2\mu}{r}}$$

- Relationship between circular and parabolic orbits

$$v_{escape} = \sqrt{2} v_{circular}$$

Some Useful Constants

- Gravitation constant $\mu = GM$
  - Earth: 398,604 km^3/sec^2
  - Moon: 4667.9 km^3/sec^2
  - Mars: 42,970 km^3/sec^2
  - Sun: 1.327×10^{11} km^3/sec^2
- Planetary radii
  - $r_{Earth} = 6378$ km
  - $r_{Moon} = 1738$ km
  - $r_{Mars} = 3393$ km