ENAE 791 PROBLEM SET 1 – SPRING, 2014

DUE 2/20/14

(1) A critical parameter for atmospheric entry is the flight path angle γ at the point at which aerodynamic forces become significant, which is termed entry interface r_e.
   (a) Starting out in an initial circular orbit of radius r_1, find an equation for the required Δv for the deorbit burn so that you encounter entry interface r_e at a flight path angle γ_e.
   (b) What is the elapsed time between deorbit and entry interface?

(2) We are going to "reverse engineer" the SpaceX Falcon 9 v1.1 launch vehicle family. Some critical parameters: (note that throughout this course, "MT" refers to metric tons, or kg$\times 10^3$).

   First stage:
   m_{prop} : 385 MT
   m_{inert} : 19 MT
   I_{sp} : 300 sec

   Second stage:
   m_{prop} : 93 MT
   m_{inert} : 6 MT
   I_{sp} : 340 sec
   Payload to LEO: 13.15 MT

   (a) What is the vehicle gross mass?
   (b) Calculate the Δv's for the first and second stages, and the total Δv for this launch vehicle
   (c) Calculate the inert mass fraction δ and stage inert mass fraction ϵ for each stage
   (d) Use the mass estimating relations from the lecture notes to calculate predicted ϵ for each stage. How is SpaceX doing compared to the predicted values?
   (e) Using the mass fractions obtained, find the Δv distribution which would theoretically maximize the payload mass fraction
   (f) Calculate the trade-off ratios for each stage
      (i) Effect of inert mass change on payload
      (ii) Effect of marginal propellant mass on payload
      (iii) Effect of additional exhaust velocity on payload

(3) SpaceX would like to recover and reuse the first stage by reserving propellent to decelerate and propulsively land the empty stage back at the launch site. Assume this will require a Δv of 300 m/sec.
   (a) How much propellant would be required for this return and landing maneuver?
   (b) Since that amount of propellant is no longer available for launching the payload, how much payload can the vehicle carry to orbit (the same Δv as you calculated in 2b) on a mission which recovers the first stage?

(4) The Falcon Heavy (which is scheduled to fly later this year) uses two additional Falcon 9 first stages as "strap-on boosters" for the core vehicle, which is otherwise a standard Falcon 9. Assume the two first stages used as boosters have an additional 3MT of inert mass due to the need for aerodynamic fairings and other modifications. Assume that the Falcon Heavy must achieve the same Δv as you calculated in 2b.
  (a) If all three first stages modules burn together, what is the payload to orbit?
  (b) For initial flights, the center core will operate at 70% thrust while the boosters are burning, then burn to completion and be jettisoned. What payload does this approach provide?
  (c) For high-performance missions, the two outer boosters will be "cross-strapped": their propellant tanks will also supply the engines in the center core until booster depletion. At that point, the boosters will be jettisoned and the fully fueled center module will continue to burn. What payload to orbit would this provide?
  (d) SpaceX is quoting a 53 MT payload to LEO. Based on your calculations, is this optimistic, pessimistic, or just about right?

(5) Write a computer routine (program, MatLab script, or Excel spreadsheet, whatever works for you) to numerically integrate the planar orbital motion state equations derived in class. Starting with the conditions in a circular Earth orbit at an altitude of 500 km, propagate the orbit forward through one orbital period. What are the position and velocity errors in your numerical prediction as compared to the calculated orbital state? (Note: we're going to be adding on to this program throughout the term to incorporate atmospheric drag, lift, and launch thrust, as well as out-of-plane motions. It's in your enlightened self-interest to write the code cleanly enough you can continue to modify and reuse it throughout the term.)