

Rocket Performance

- The rocket equation
- Mass ratio and performance
- Structural and payload mass fractions
- Regression analysis
- Multistaging
- Optimal ΔV distribution between stages
- Trade-off ratios
- Parallel staging
- Modular staging



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Derivation of the Rocket Equation

- Momentum at time t :

$$M = mv$$

- Momentum at time $t + \Delta t$:

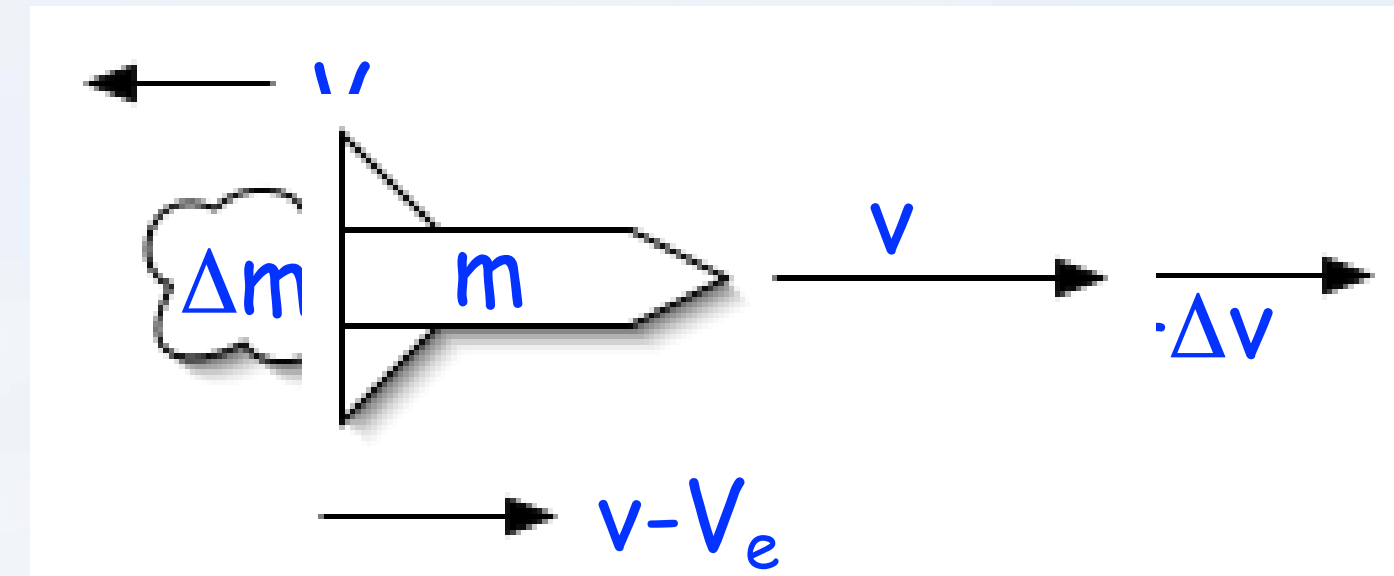
$$M = (m - \Delta m)(V + \Delta v) + \Delta m(v - V_e)$$

- Some algebraic manipulation gives:

$$m\Delta v = -\Delta mV_e$$

- Take to limits and integrate:

$$\int_{m_{initial}}^{m_{final}} \frac{dm}{m} = - \int_{V_{initial}}^{V_{final}} \frac{dv}{V_e}$$



The Rocket Equation

- Alternate forms

$$r \equiv \frac{m_{final}}{m_{initial}} = e^{-\frac{\Delta V}{V_e}}$$

$$\Delta v = -V_e \ln \left(\frac{m_{final}}{m_{initial}} \right) = -V_e \ln r$$

- Basic definitions / concepts

- Mass ratio

$$r \equiv \frac{m_{final}}{m_{initial}} \text{ or } \mathfrak{R} \equiv \frac{m_{initial}}{m_{final}}$$

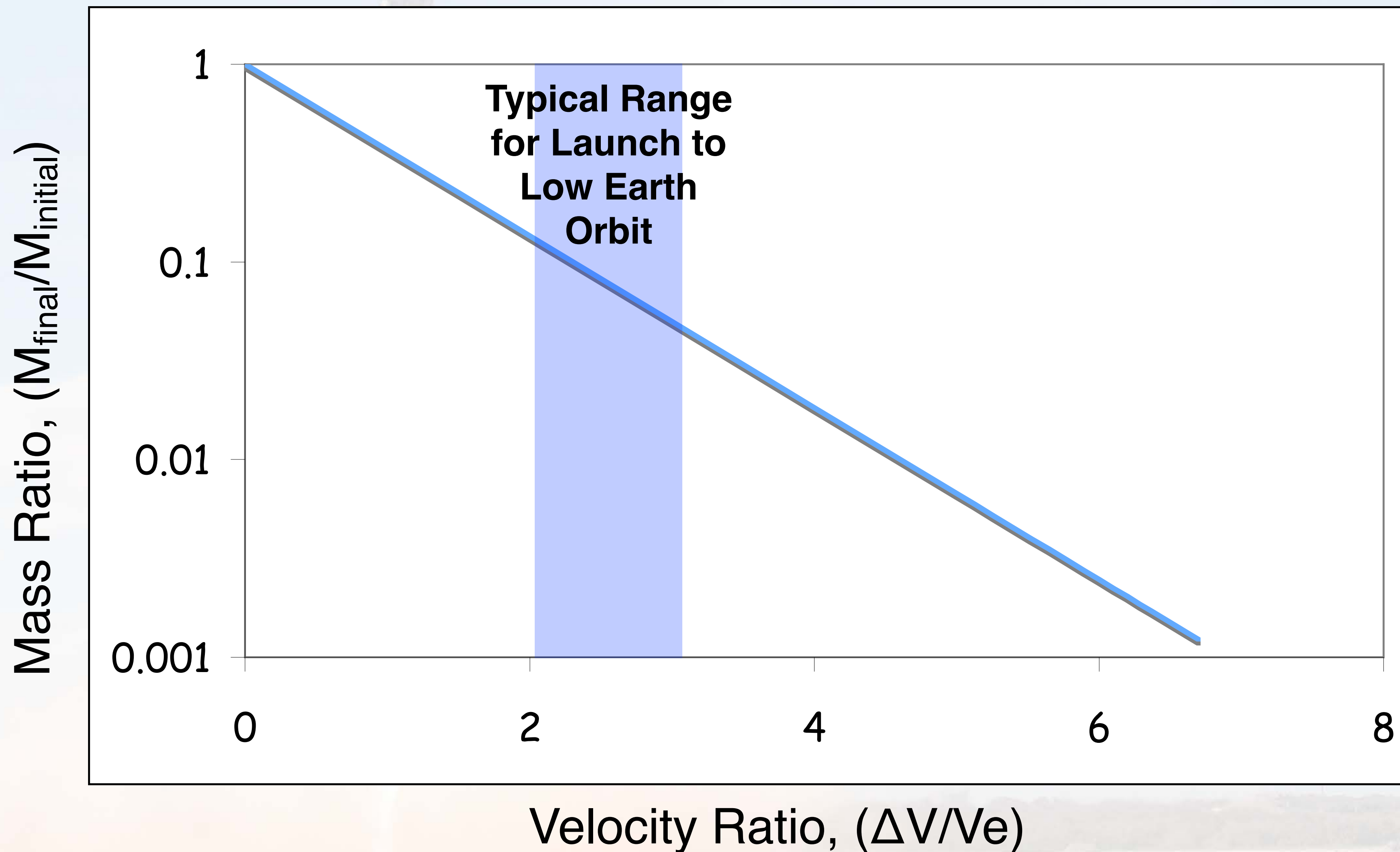
- Nondimensional velocity change

“Velocity ratio”

$$\nu \equiv \frac{\Delta V}{V_e}$$



Rocket Equation (First Look)



Sources and Categories of Vehicle Mass



Payload
Propellants
Structure
Propulsion
Avionics
Power
Mechanisms
Thermal
Etc.



Sources and Categories of Vehicle Mass

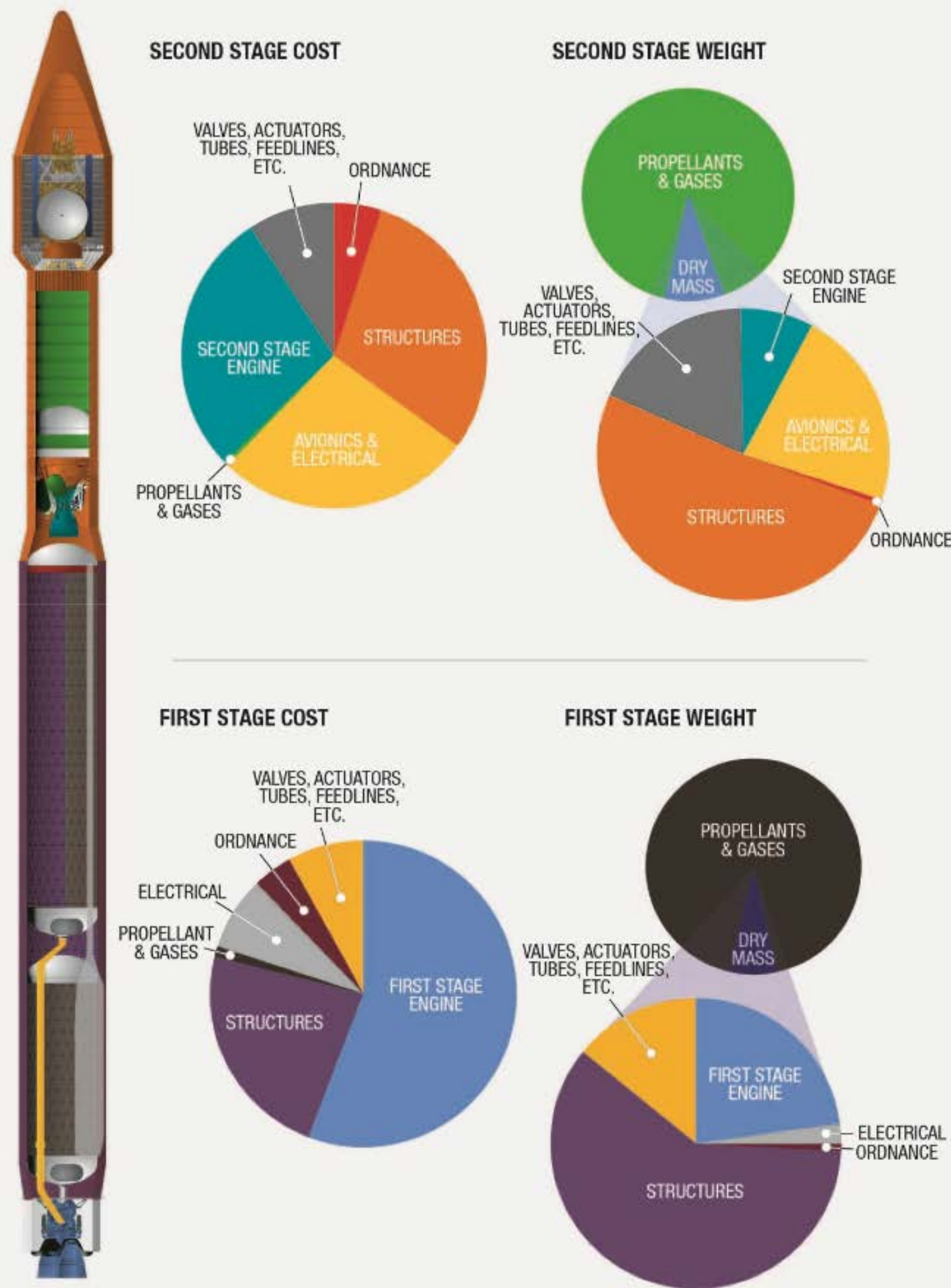


Payload
Propellants
Inert Mass
Structure
Propulsion
Avionics
Power
Mechanisms
Thermal
Etc.

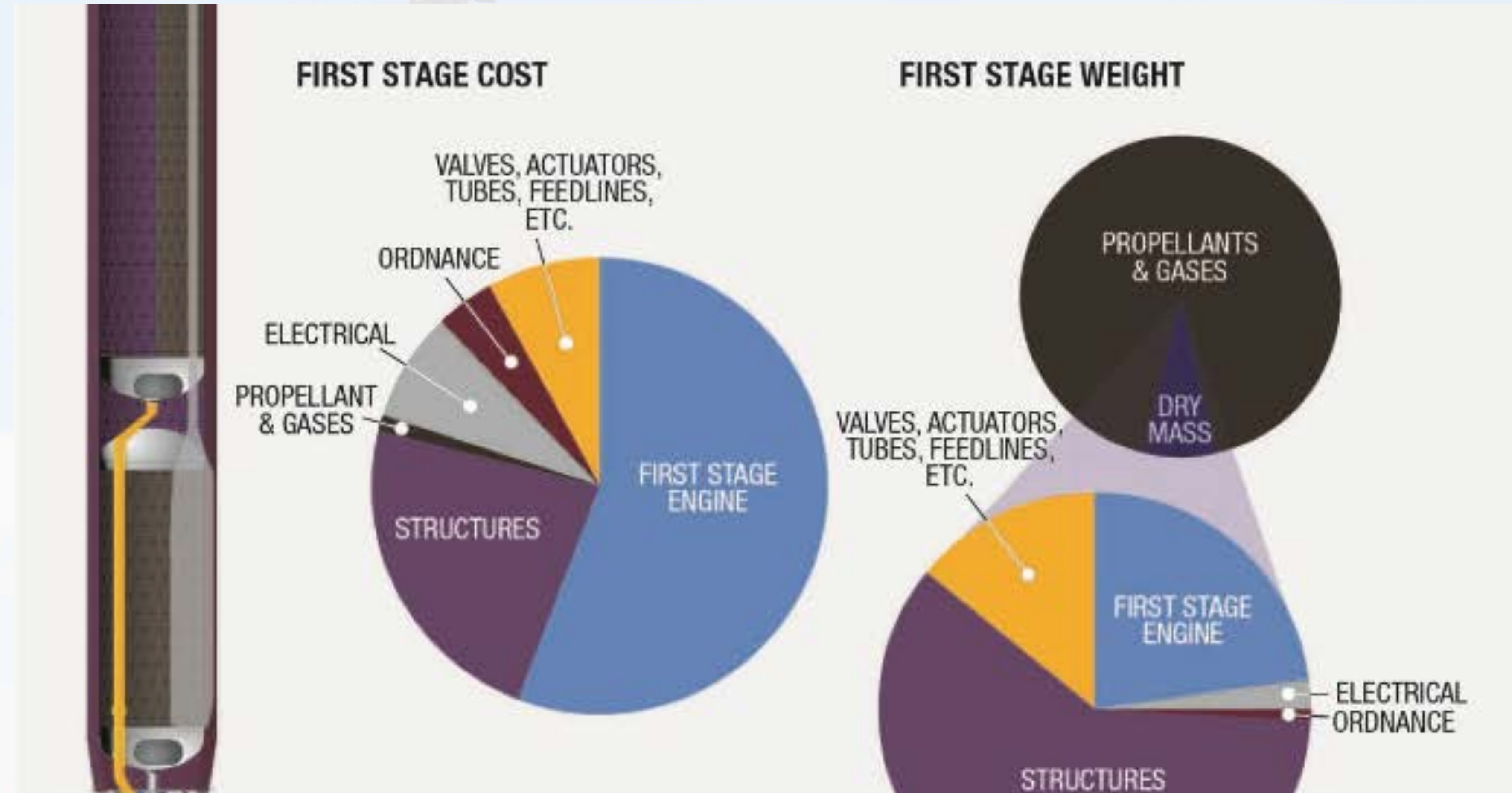


Cost and Weight Distribution - Atlas V

Launch Vehicle Cost and Weight by Major Element

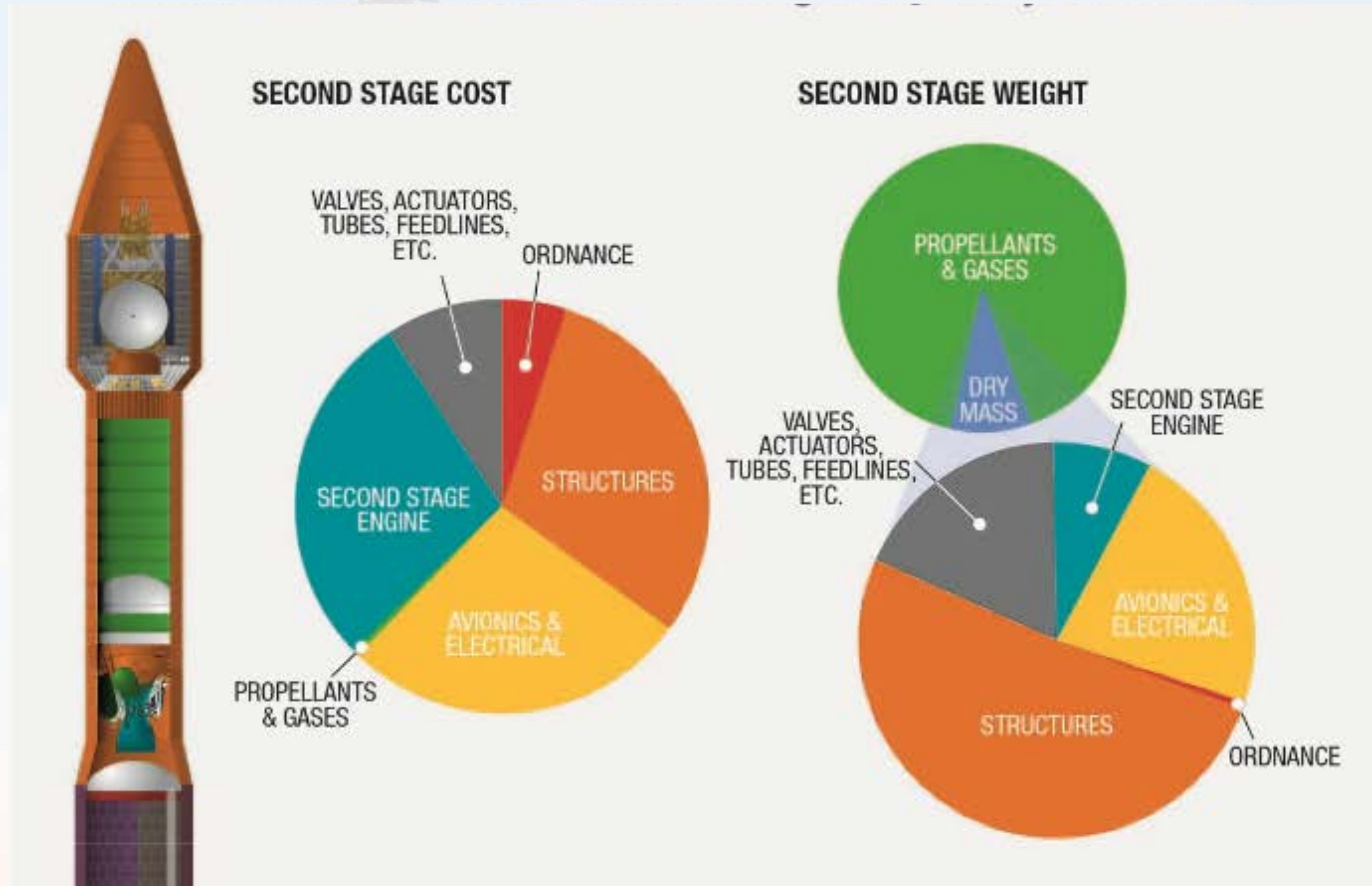


Cost and Weight - Atlas V First Stage



$$\text{Propellant mass fraction } PMF = \frac{\text{propellant mass}}{\text{total stage mass}}$$

Cost and Weight - Atlas V Second Stage



Basic Vehicle Parameters

- Basic mass summary

$$m_o = m_{pl} + m_{pr} + m_{in}$$

m_o =initial mass

m_{pl} =payload mass

m_{pr} =propellant mass

m_{in} =inert mass

- Inert mass fraction

$$\delta \equiv \frac{m_{in}}{m_o} = \frac{m_{in}}{m_{pl} + m_{pr} + m_{in}}$$

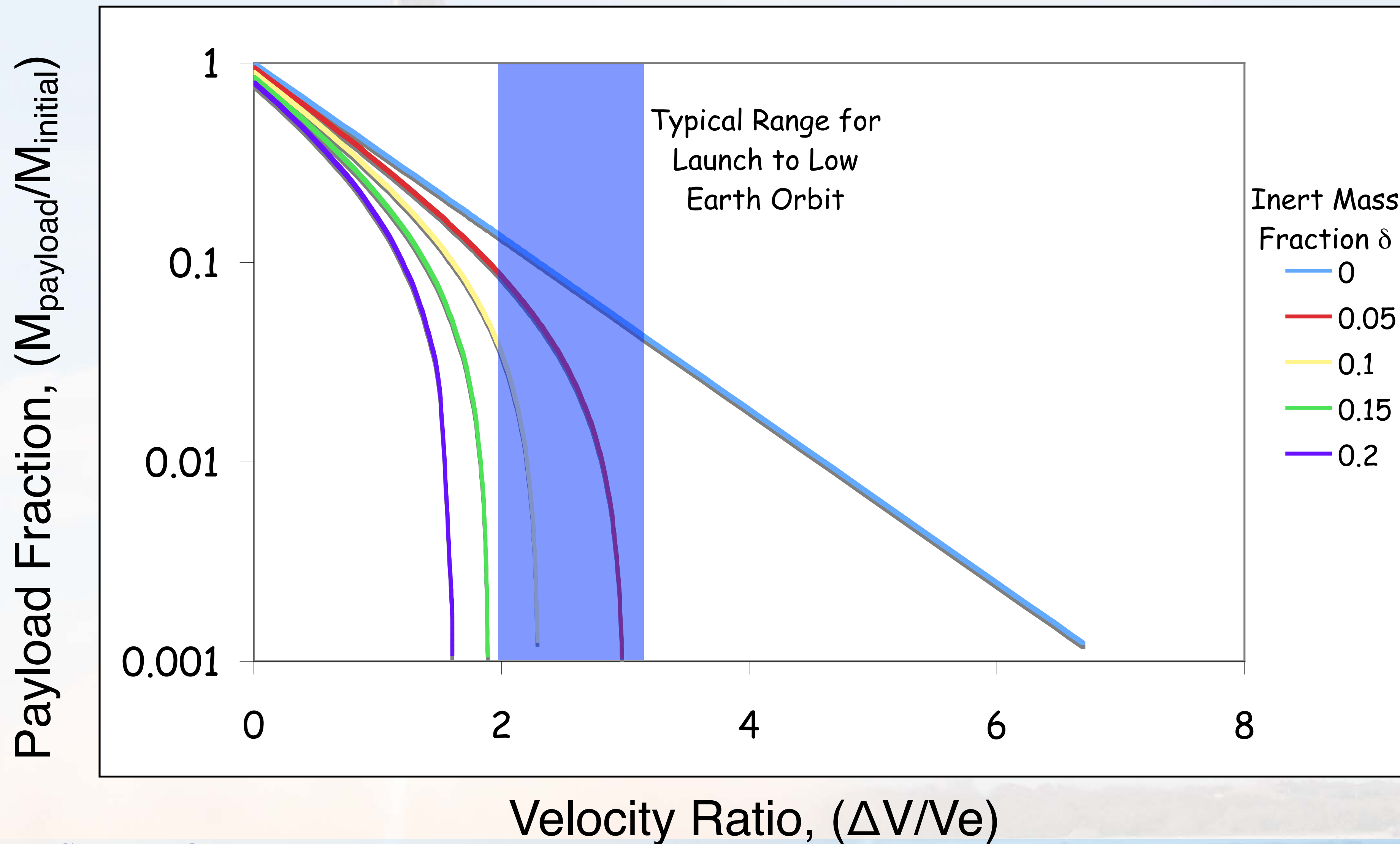
- Payload fraction

$$\lambda \equiv \frac{m_{pl}}{m_o} = \frac{m_{pl}}{m_{pl} + m_{pr} + m_{in}}$$

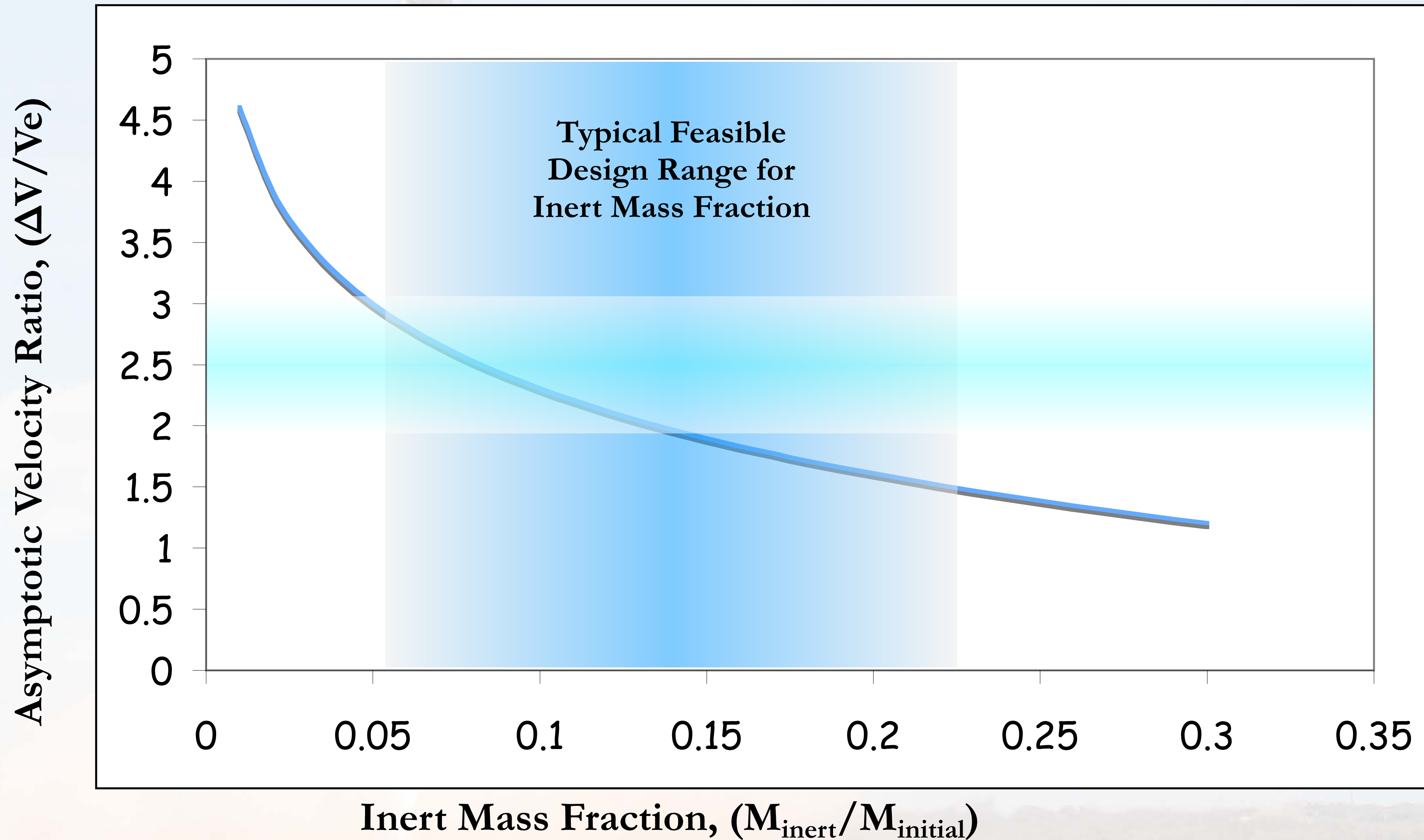
- Parametric mass ratio

$$r = \lambda + \delta$$

Rocket Equation (including Inert Mass)



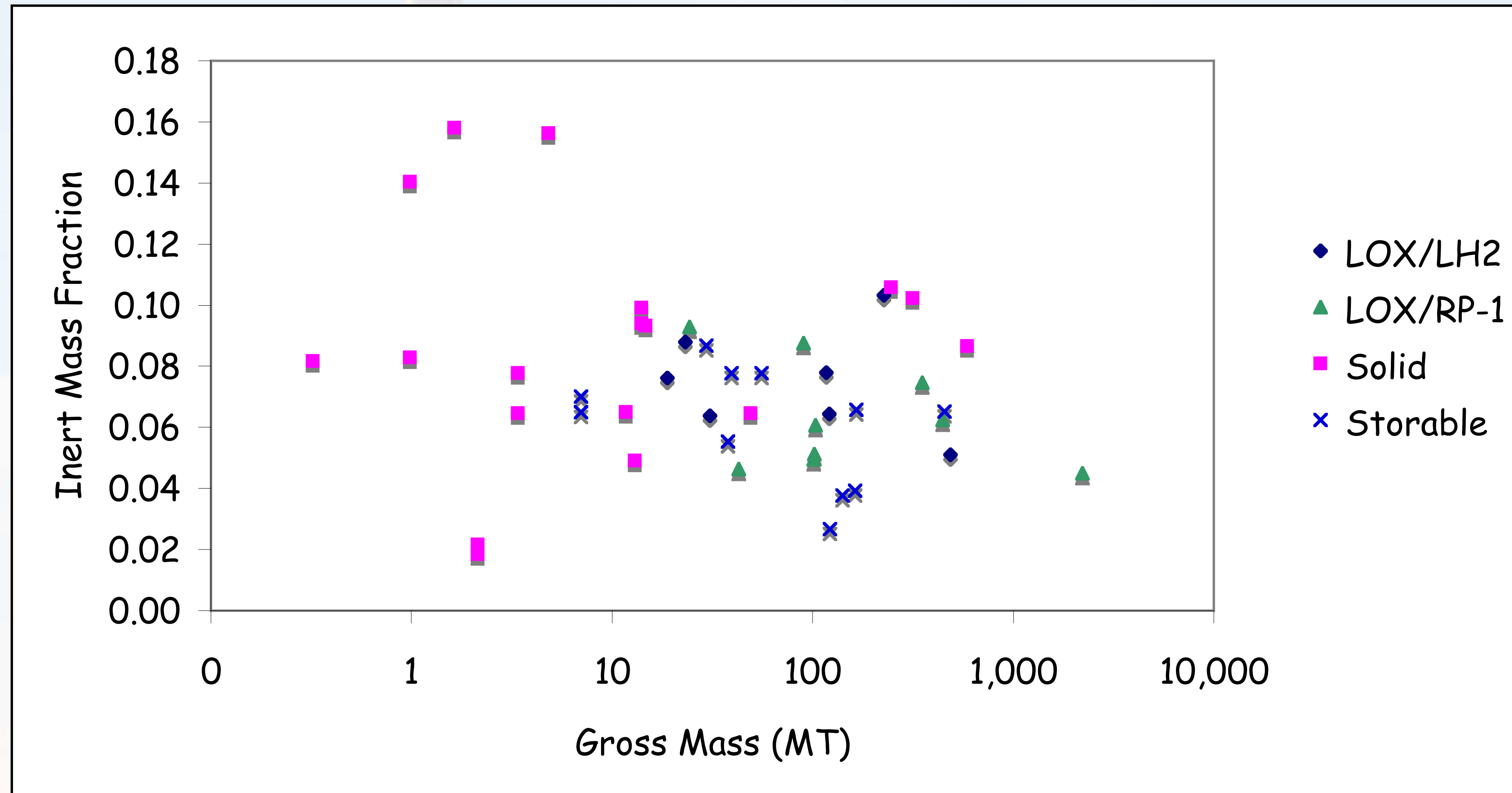
Limiting Performance Based on Inert Mass



Regression Analysis of Existing Vehicles

Veh/Stage	prop mass (lbs)	gross mass (lbs)	Type	Propellants	Isp vac (sec)	isp sl (sec)	sigma	eps	delta
Delta 6925 Stage 2	13,367	15,394	Storab	N2O4-A50	319.4		0.152	0.132	0.070
Delta 7925 Stage 2	13,367	15,394	Storab	N2O4-A50	319.4		0.152	0.132	0.065
Titan II Stage 2	59,000	65,000	Storab	N2O4-A50	316.0		0.102	0.092	0.087
Titan III Stage 2	77,200	83,600	Storab	N2O4-A50	316.0		0.083	0.077	0.055
Titan IV Stage 2	77,200	87,000	Storab	N2O4-A50	316.0		0.127	0.113	0.078
Proton Stage 3	110,000	123,000	Storab	N2O4-A50	315.0		0.118	0.106	0.078
Titan II Stage 1	260,000	269,000	Storab	N2O4-A50	296.0		0.035	0.033	0.027
Titan III Stage 1	294,000	310,000	Storab	N2O4-A50	302.0		0.054	0.052	0.038
Titan IV Stage 1	340,000	359,000	Storab	N2O4-A50	302.0		0.056	0.053	0.039
Proton Stage 2	330,000	365,000	Storab	N2O4-A50	316.0		0.106	0.096	0.066
Proton Stage 1	904,000	1,004,000	Storab	N2O4-A50	316.0	285.0	0.111	0.100	0.065
average					312.2	285.0	0.100	0.089	0.061
standard deviation					8.1		0.039	0.033	0.019

Inert Mass Fraction Data for Existing LVs



Regression Analysis

- Given a set of N data points (x_i, y_i)
- Linear curve fit: $y = Ax + B$
 - find A and B to minimize sum squared error

$$\text{error} = \sum_{i=1}^N (Ax_i + B - y_i)^2$$

- Analytical solutions exist, or use Solver in Excel
- Power law fit: $y = Bx^A$
 - Analytical solutions exist, or use Solver in Excel
- Polynomial, exponential, many other fits possible

Solution of Least-Squares Linear Regression

$$\frac{\partial(\text{error})}{\partial A} = 2 \sum_{i=1}^N (Ax_i + B - y_i)x_i = 0$$

$$\frac{\partial(\text{error})}{\partial B} = 2 \sum_{i=1}^N (Ax_i + B - y_i) = 0$$

$$A \sum_{i=1}^N x_i^2 + B \sum_{i=1}^N x_i - \sum_{i=1}^N x_i y_i = 0$$

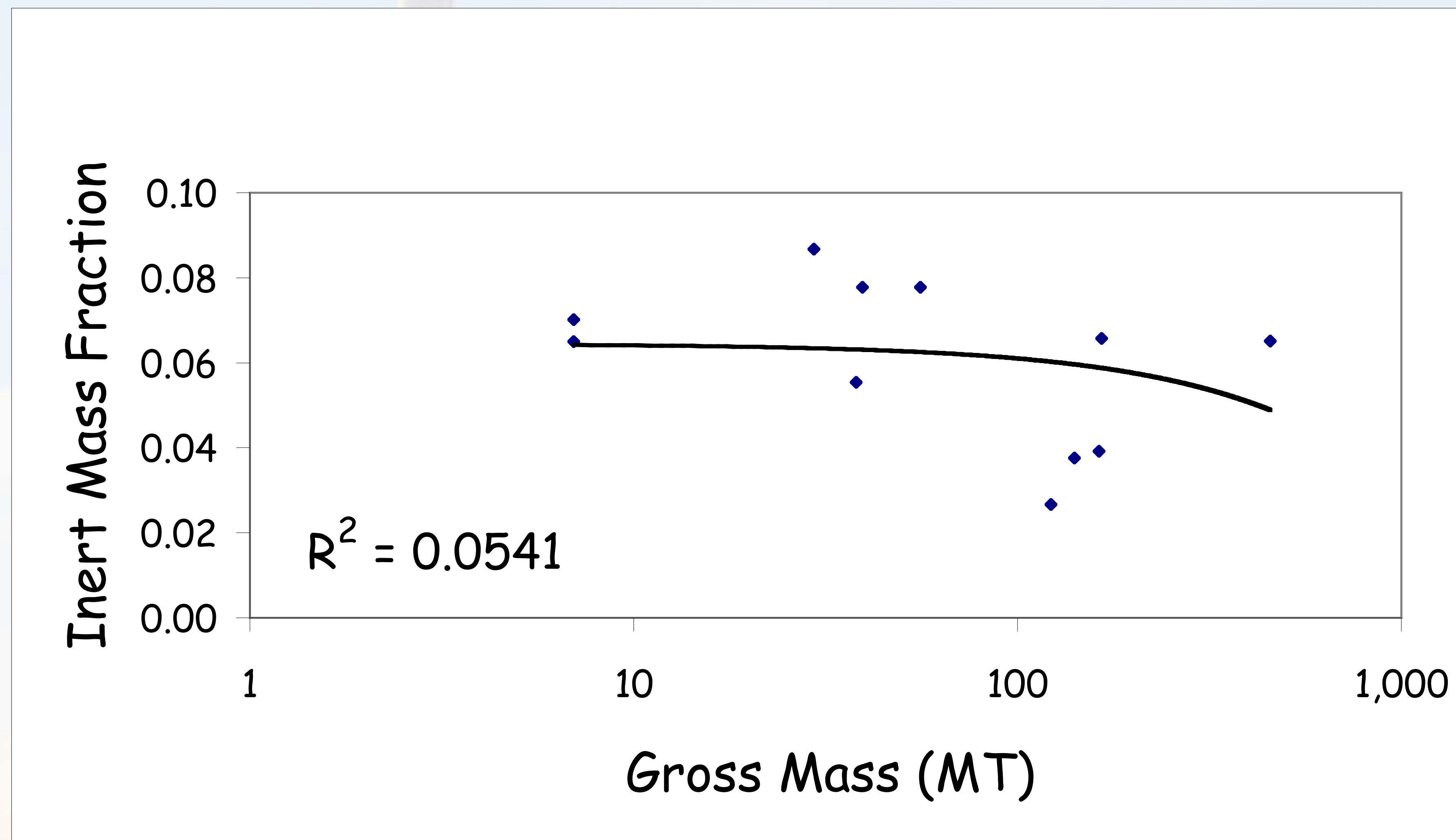
$$A \sum_{i=1}^N x_i + NB - \sum_{i=1}^N y_i = 0$$

$$A = \frac{N \sum x_i y_i - \sum x_i \sum y_i}{N \sum x_i^2 - (\sum x_i)^2}$$

$$B = \frac{\sum y_i \sum x_i^2 - \sum x_i \sum x_i y_i}{N \sum x_i^2 - (\sum x_i)^2}$$



Regression Analysis - Storables

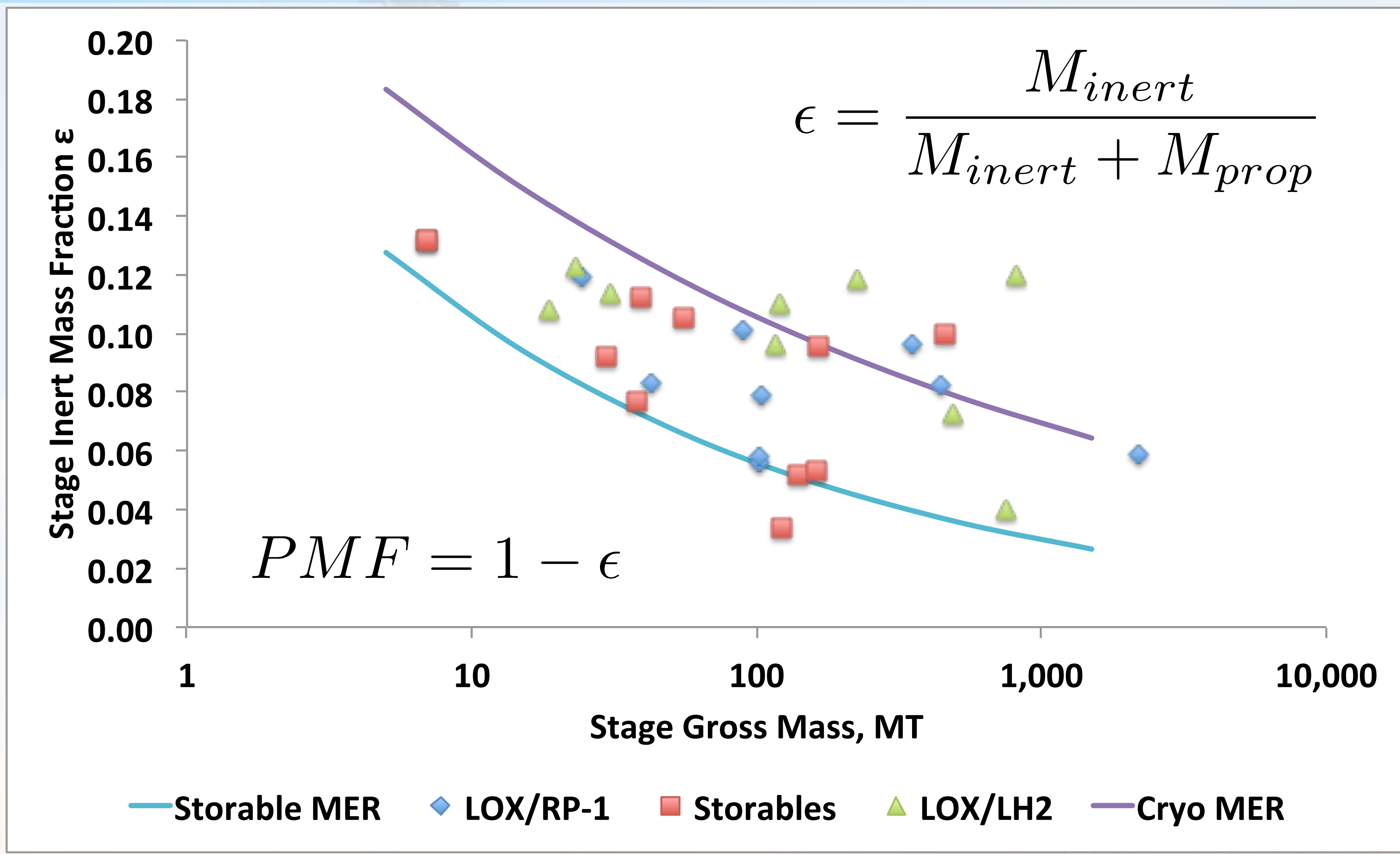


Regression Values for Design Parameters

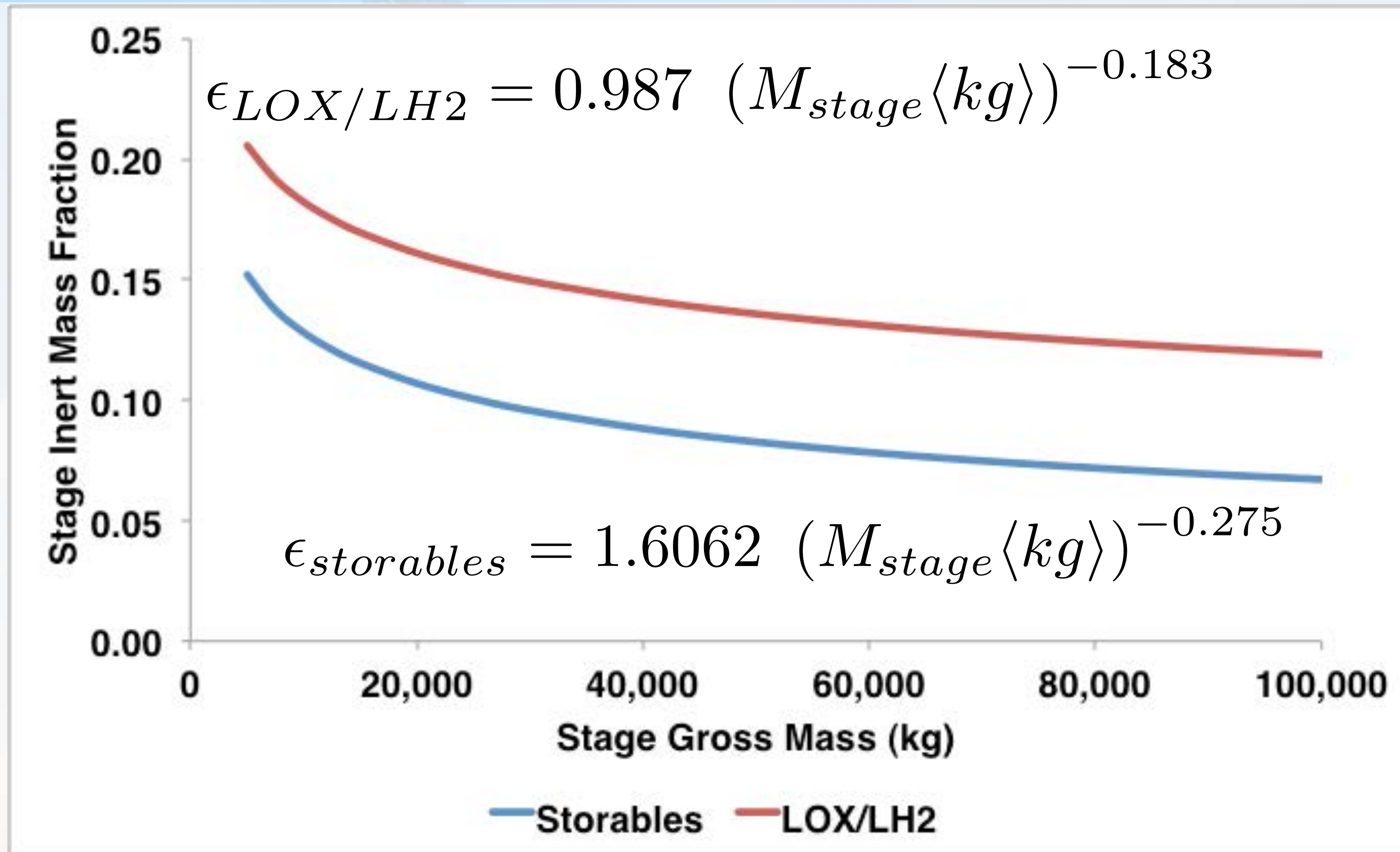
	Vacuum Ve (m/sec)	Inert Mass Fraction	Max ΔV (m/sec)
LOX/LH2	4273	0.075	11,070
LOX/RP-1	3136	0.063	8664
Storables	3058	0.061	8575
Solids	2773	0.087	6783



Economy of Scale for Stage Size



Stage Inert Mass Fraction Estimation



Calculating M_0 from M_{pl}

- Given Δv and I_{sp}

$$r = e^{-\frac{\Delta v}{gI_{sp}}}$$

Given δ

$$\lambda = r - \delta \implies m_o = \frac{m_{pl}}{\lambda}$$

Given ϵ

$$m_{stage} = m_{in} + m_{pr}; \quad m_{in} = \epsilon m_{stage}; \quad m_o = m_{pl} + m_{stage}$$

$$r = \frac{m_{pl} + m_{in}}{m_{pl} + m_{in} + m_{pr}} = \frac{m_{pl} + \epsilon m_{stage}}{m_{pl} + m_{stage}}$$

$$m_{stage} = \left(\frac{1-r}{r-\epsilon} \right) m_{pl} \implies m_o = \left(\frac{1-\epsilon}{r-\epsilon} \right) m_{pl}$$

The Rocket Equation for Multiple Stages

- Assume two stages

$$\Delta V_1 = -V_{e1} \ln \left(\frac{m_{final1}}{m_{initial1}} \right) = -V_{e1} \ln(r_1)$$

$$\Delta V_2 = -V_{e2} \ln \left(\frac{m_{final2}}{m_{initial2}} \right) = -V_{e2} \ln(r_2)$$

- Assume $V_{e1} = V_{e2} = V_e$

$$\Delta V_1 + \Delta V_2 = -V_e \ln(r_1) - V_e \ln(r_2) = -V_e \ln(r_1 r_2)$$

Multistage Inert Mass Fraction

- Total inert mass fraction

$$\delta_0 = \frac{m_{in,1} + m_{in,2} + m_{in,3}}{m_0} = \frac{m_{in,1}}{m_0} + \frac{m_{in,2}}{m_0} + \frac{m_{in,3}}{m_0}$$

$$\delta_0 = \frac{m_{in,1}}{m_0} + \frac{m_{in,2}}{m_{0,2}} \frac{m_{0,2}}{m_0} + \frac{m_{in,3}}{m_{0,3}} \frac{m_{0,3}}{m_{0,2}} \frac{m_{0,2}}{m_0}$$

- Convert to dimensionless parameters

$$\delta_0 = \delta_1 + \delta_2 \lambda_1 + \delta_3 \lambda_2 \lambda_1$$

- General form of the equation

$$\delta_0 = \sum_{j=1}^{n \text{ stages}} \left[\delta_j \prod_{\ell=1}^{j-1} \lambda_\ell \right]$$

Multistage Payload Fraction

- Total payload fraction (3 stage example)

$$\lambda_0 = \frac{m_{pl}}{m_0} = \frac{m_{pl}}{m_{0,3}} \frac{m_{0,3}}{m_{0,2}} \frac{m_{0,2}}{m_0}$$

- Convert to dimensionless parameters

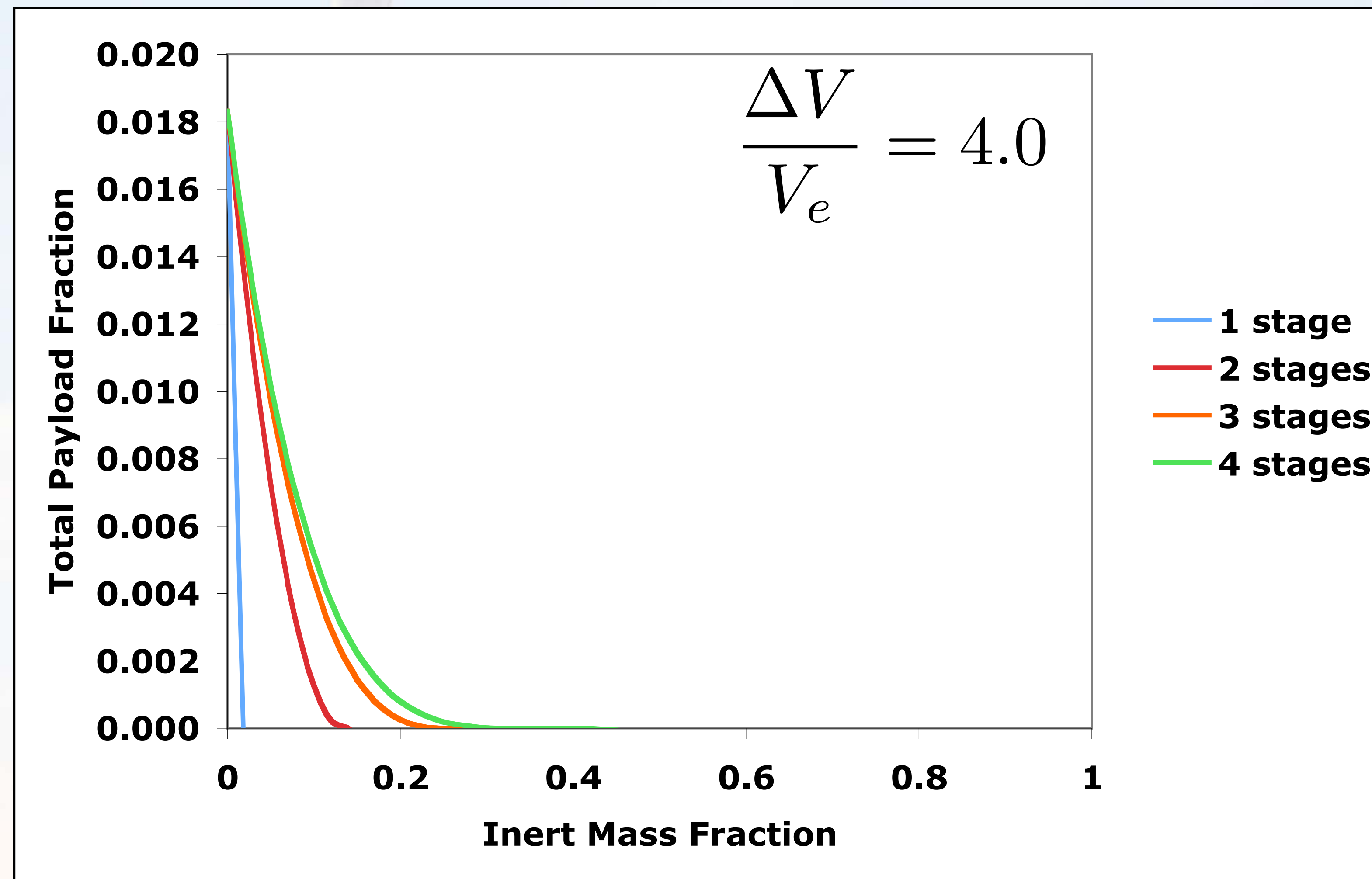
$$\lambda_0 = \lambda_3 \lambda_2 \lambda_1$$

- Generic form of the equation

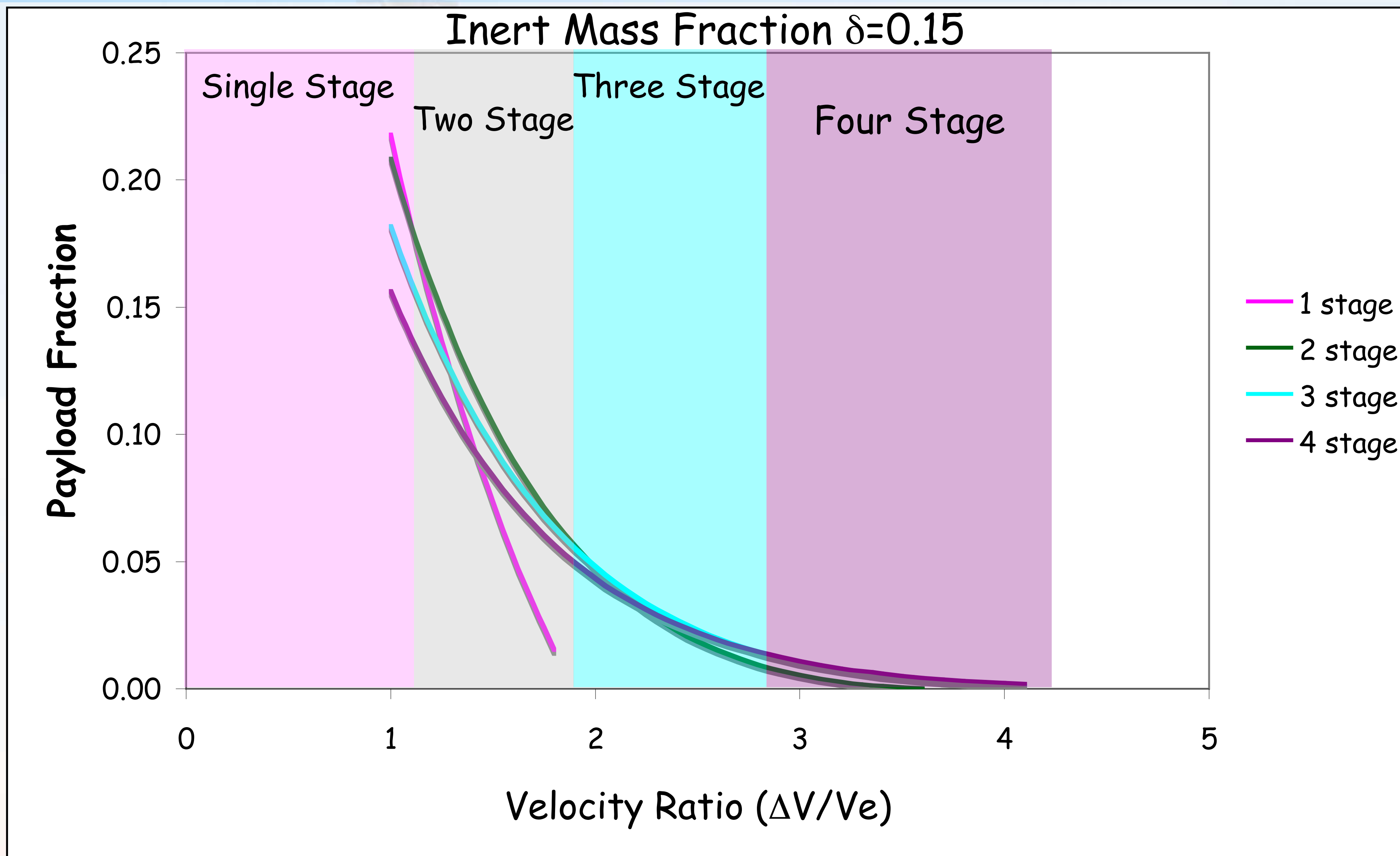
$$\lambda_0 = \prod_{j=1}^{n \text{ stages}} \lambda_j$$



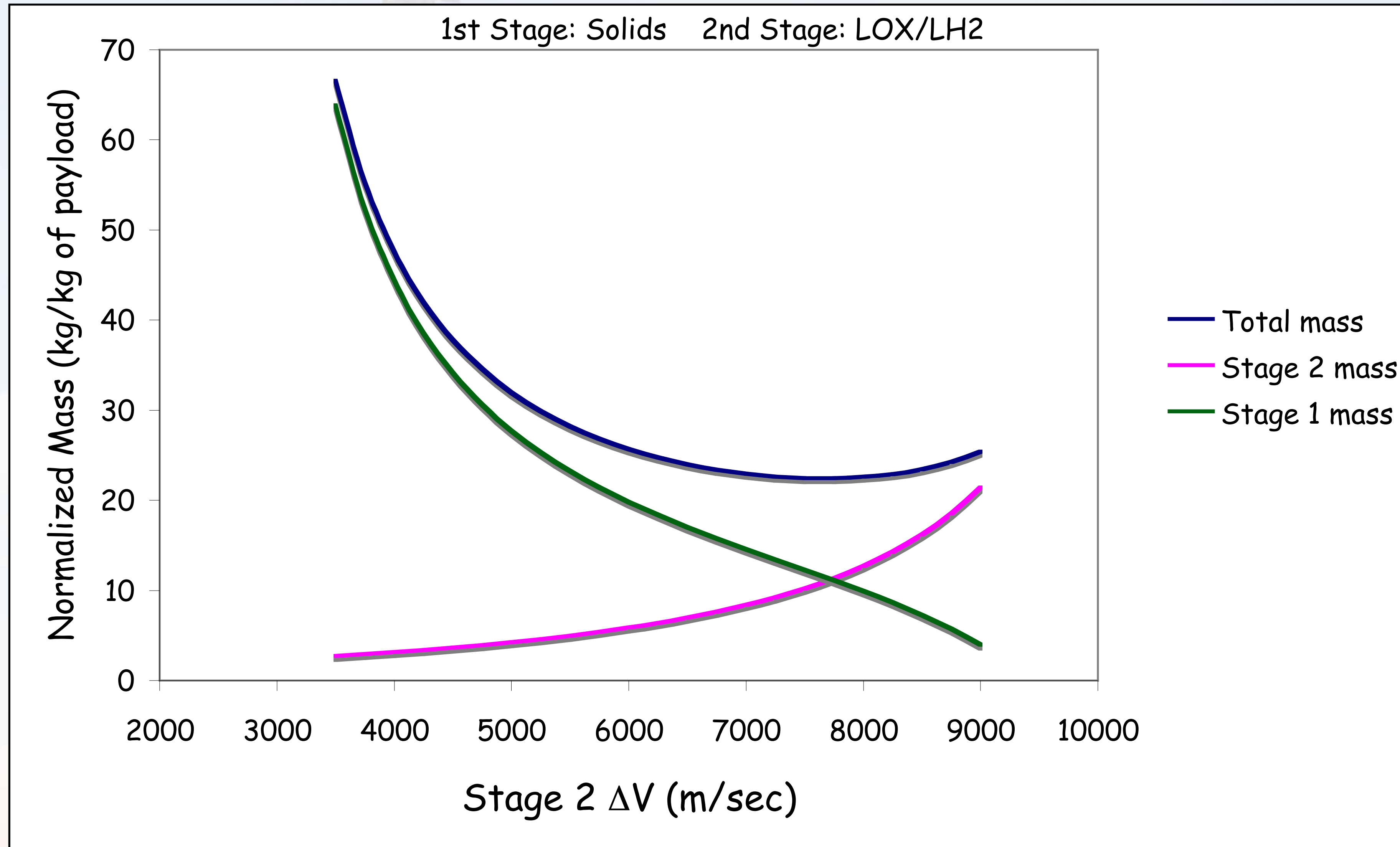
Effect of δ and $\Delta V/V_e$ on Payload



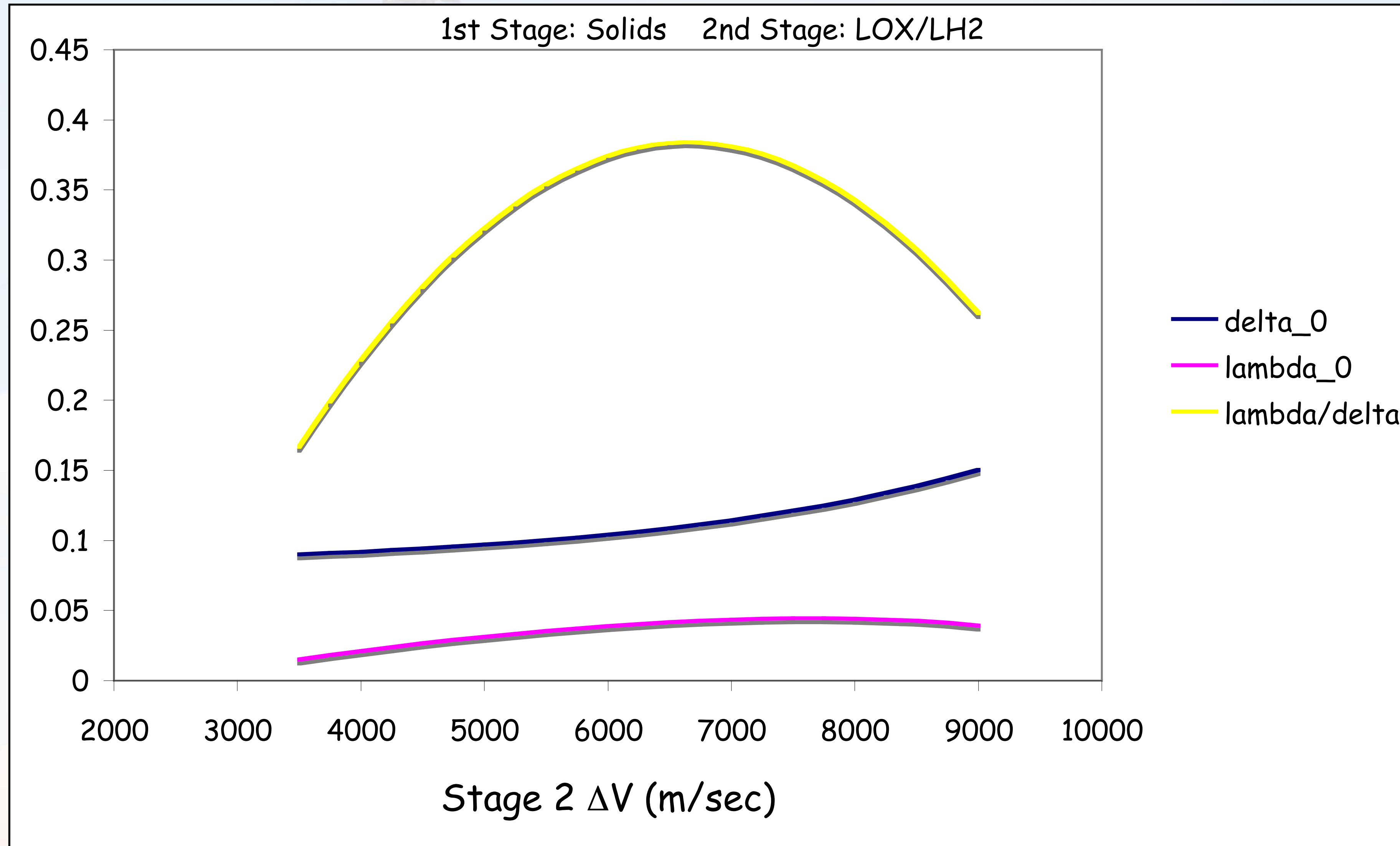
Effect of Staging



Effect of ΔV Distribution



ΔV Distribution and Design Parameters



Lagrange Multipliers

- Given an objective function

$$y = f(x)$$

subject to constraint function

$$z = g(x)$$

- Create a new objective function

$$y = f(x) + \lambda[g(x) - z]$$

- Solve simultaneous equations

$$\frac{\partial y}{\partial x} = 0 \qquad \frac{\partial y}{\partial \lambda} = 0$$



Optimum ΔV Distribution Between Stages

- Maximize payload fraction (2 stage case)

$$\lambda_0 = \lambda_1 \lambda_2 = (r_1 - \delta_1)(r_2 - \delta_2)$$

subject to constraint function

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

- Create a new objective function

$$\lambda_o = \left(e^{\frac{-\Delta V_1}{V_{e,1}}} - \delta_1 \right) \left(e^{\frac{-\Delta V_2}{V_{e,2}}} - \delta_2 \right) + K [\Delta V_1 + \Delta V_2 - \Delta V_{total}]$$

➔ Very messy for partial derivatives!

Optimum ΔV Distribution (continued)

- Use substitute objective function

$$\max (\lambda_o) \iff \max [\ln (\lambda_o)]$$

- Create a new constrained objective function

$$\ln (\lambda_o) = \ln (r_1 - \delta_1) + \ln (r_2 - \delta_2) + K [\Delta V_1 + \Delta V_2 - \Delta V_{total}]$$

- Take partials and set equal to zero

$$\frac{\partial [\ln (\lambda_o)]}{\partial r_1} = 0 \quad \frac{\partial [\ln (\lambda_o)]}{\partial r_2} = 0 \quad \frac{\partial [\ln (\lambda_o)]}{\partial K} = 0$$

Optimum ΔV Special Cases

- “Generic” partial of objective function

$$\frac{\partial [\ln(\lambda_o)]}{\partial r_i} = \frac{1}{r_i - \delta_i} + K \frac{V_{e,i}}{r_i} = 0$$

- Special case: $\delta_1 = \delta_2$ $V_{e,1} = V_{e,2}$

$$r_1 = r_2 \implies \Delta V_1 = \Delta V_2 = \frac{\Delta V_{total}}{2}$$

- Same principle holds for n stages

$$r_1 = r_2 = \dots = r_n \implies$$

$$\Delta V_1 = \Delta V_2 = \dots = \Delta V_n = \frac{\Delta V_{total}}{n}$$

Sensitivity to Inert Mass

ΔV for multistaged rocket

$$\Delta V_{tot} = \sum_{k=1}^{n \text{ stages}} \Delta V_k = \sum_{k=1}^n V_{e,k} \ln \left(\frac{m_{o,k}}{m_{f,k}} \right)$$

where

$$m_{o,k} = m_{pl} + m_{pr,k} + m_{in,k} + \sum_{j=k+1}^n (m_{pr,j} + m_{in,j})$$

$$m_{f,k} = m_{pl} + m_{in,k} + \sum_{j=k+1}^n (m_{pr,j} + m_{in,j})$$

Finding Payload Sensitivity to Inert Mass

- Given the equation linking mass to ΔV , take

$$\frac{\partial(\Delta V_{tot})}{\partial m_{pl}} dm_{pl} + \frac{\partial(\Delta V_{tot})}{\partial m_{in,j}} dm_{in,j} = 0$$

and solve to find

$$\left. \frac{dm_{pl}}{dm_{in,k}} \right|_{\partial(\Delta V_{tot})=0} = \frac{-\sum_{j=1}^k V_{e,j} \left(\frac{1}{m_{o,j}} - \frac{1}{m_{f,j}} \right)}{\sum_{l=1}^N V_{e,l} \left(\frac{1}{m_{o,l}} - \frac{1}{m_{f,l}} \right)}$$

- This equation shows the “trade-off ratio” - Δ payload resulting from a change in inert mass for stage k (for a vehicle with N total stages)

Trade-off Ratio Example: Gemini-Titan II



	Stage 1	Stage 2
m_o (kg)	150,500	32,630
m_f (kg)	39,370	6099
v_e (m/sec)	2900	3097
$\frac{\partial m_{pl}}{\partial m_{in,k}}$	-0.1164	-1

Payload Sensitivity to Propellant Mass

- In a similar manner, solve to find

$$\left. \frac{dm_{pl}}{dm_{pr,k}} \right|_{\partial(\Delta V_{tot})=0} = \frac{-\sum_{j=1}^k V_{e,j} \left(\frac{1}{m_{o,j}} \right)}{\sum_{l=1}^N V_{e,l} \left(\frac{1}{m_{o,l}} - \frac{1}{m_{f,l}} \right)}$$

- This equation gives the change in payload mass as a function of additional propellant mass (all other parameters held constant)

Trade-off Ratio Example: Gemini-Titan II

	Stage 1	Stage 2
Initial Mass (kg)	150,500	32,630
Final Mass (kg)	39,370	6099
V_e (m/sec)	2900	3097
$dm_{pl} / dm_{in,k}$	-0.1164	-1
$dm_{pl} / dm_{pr,k}$	0.04124	0.2443



Payload Sensitivity to Exhaust Velocity

- Use the same technique to find the change in payload resulting from additional exhaust velocity for stage k

$$\left. \frac{dm_{pl}}{dV_{e,k}} \right|_{\partial(\Delta V_{tot})=0} = \frac{\sum_{j=1}^k \ln \left(\frac{m_{o,j}}{m_{f,j}} \right)}{\sum_{l=1}^N V_{e,l} \left(\frac{1}{m_{o,l}} - \frac{1}{m_{f,l}} \right)}$$

- This trade-off ratio (unlike the ones for inert and propellant masses) has units - kg / (m / sec)

Trade-off Ratio Example: Gemini-Titan II

	Stage 1	Stage 2
Initial Mass (kg)	150,500	32,630
Final Mass (kg)	39,370	6099
V_e (m / sec)	2900	3097
$dm_{pl} / dm_{in,k}$	-0.1164	-1
$dm_{pl} / dm_{pr,k}$	0.04124	0.2443
$dm_{pl} / dV_{e,k}$ (kg / m / sec)	2.87	6.459



Parallel Staging



- Multiple dissimilar engines burning simultaneously
- Frequently a result of upgrades to operational systems
- General case requires “brute force” numerical performance analysis

Parallel-Staging Rocket Equation

- Momentum at time t :

$$M = mv$$

- Momentum at time $t + \Delta t$:
(subscript “b” = boosters; “c” = core vehicle)

$$M = (m - \Delta m_b - \Delta m_c)(v + \Delta v) + \Delta m_b(v - V_{e,b}) + \Delta m_c(v - V_{e,c})$$

- Assume thrust (and mass flow rates) constant

Parallel-Staging Rocket Equation

- Rocket equation during booster burn

$$\Delta V = -\bar{V}_e \ln \left(\frac{m_{final}}{m_{initial}} \right) = -\bar{V}_e \ln \left(\frac{m_{in,b} + m_{in,c} + \chi m_{pr,c} + m_{0,2}}{m_{in,b} + m_{pr,b} + m_{in,c} + m_{pr,c} + m_{0,2}} \right)$$

where χ = fraction of core propellant remaining after booster burnout, and where

$$\bar{V}_e = \frac{V_{e,b} \dot{m}_b + V_{e,c} \dot{m}_c}{\dot{m}_b + \dot{m}_c} = \frac{V_{e,b} m_{pr,b} + V_{e,c} (1 - \chi) m_{pr,c}}{m_{pr,b} + (1 - \chi) m_{pr,c}}$$

Analyzing Parallel-Staging Performance

Parallel stages break down into pseudo-serial stages:

- Stage “0” (boosters and core)

$$\Delta V_0 = -\bar{V}_e \ln \left(\frac{m_{in,b} + m_{in,c} + \chi m_{pr,c} + m_{0,2}}{m_{in,b} + m_{pr,b} + m_{in,c} + m_{pr,c} + m_{0,2}} \right)$$

- Stage “1” (core alone)

$$\Delta V_1 = -V_{e,c} \ln \left(\frac{m_{in,c} + m_{0,2}}{m_{in,c} + \chi m_{pr,c} + m_{0,2}} \right)$$

- Subsequent stages are as before

Parallel Staging Example: Space Shuttle

- 2 x solid rocket boosters (data below for single SRB)
 - Gross mass 589,670 kg
 - Empty mass 86,183 kg
 - V_e 2636 m/sec
 - Burn time 124 sec
- External tank (space shuttle main engines)
 - Gross mass 750,975 kg
 - Empty mass 29,930 kg
 - V_e 4459 m/sec
 - Burn time 480 sec
- “Payload” (orbiter + P/L) 125,000 kg



Shuttle Parallel Staging Example

$$V_{e,b} = 2636 \frac{m}{sec} \qquad V_{e,c} = 4459 \frac{m}{sec}$$

$$\chi = \frac{480 - 124}{480} = 0.7417$$

$$\bar{V}_e = \frac{2636(1,007,000) + 4459(721,000)(1 - .7417)}{1,007,000 + 721,000(1 - .7417)} = 2921 \frac{m}{sec}$$

$$\Delta V_0 = -2921 \ln \frac{862,000}{3,062,000} = 3702 \frac{m}{sec}$$

$$\Delta V_1 = -4459 \ln \frac{154,900}{689,700} = 6659 \frac{m}{sec}$$

$$\Delta V_{tot} = 10,360 \frac{m}{sec}$$



Atlas V – 5-m Payload Fairing

Vehicle Naming Convention: Atlas V xyz

1st Digit = x = PLF Diameter (meters, approx 4 or 5)
 2nd Digit = y = No. of SRBs (0 to 5)
 3rd Digit = z = No. of Centaur Engines (1 or 2)

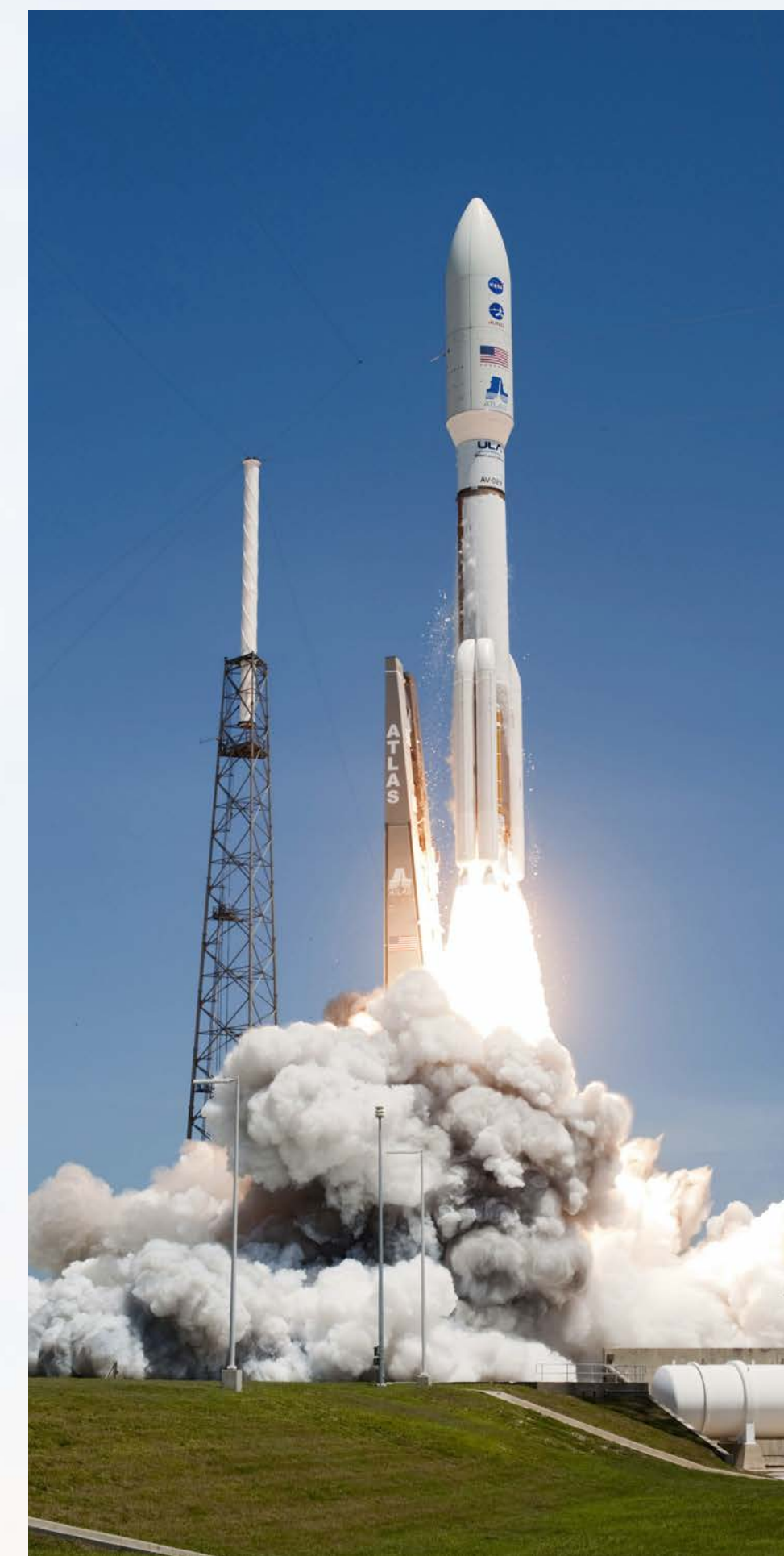
	SRBs	400 Series	500 Series
0			
1			
2			
3			
4			
5			

x	PLF Diameter	4 meter	5 meter
y	Number of SRBs	0 thru 3	0 thru 5
z	Number of Centaur Engines	1 or 2	1 or 2

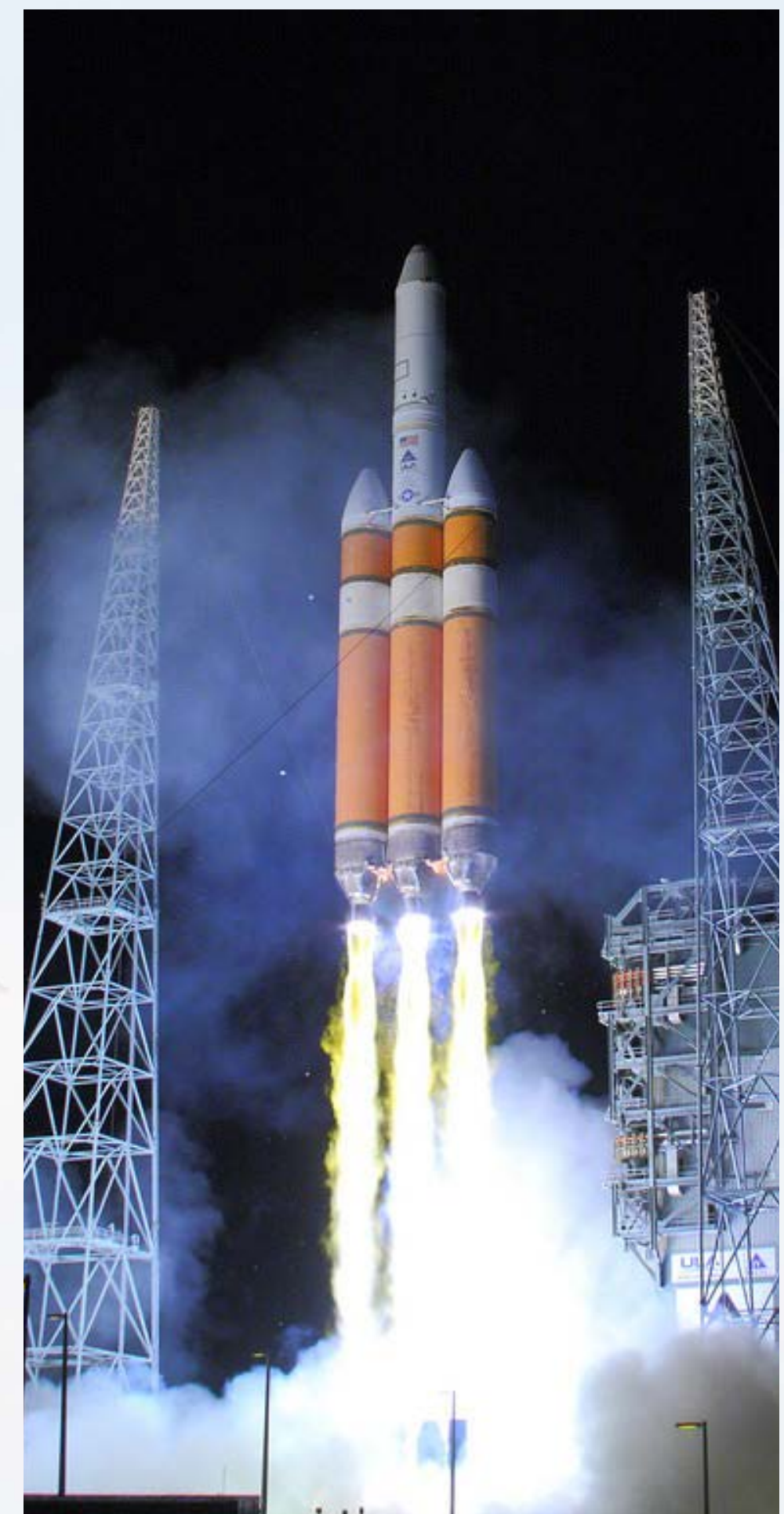
AVUG11_F010401_02c



Atlas V 501
3775 kg (GTO)



Atlas V 551
8900 kg (GTO)



Atlas V Heavy
13,000 kg (GTO)

Modular Staging

- Use identical modules to form multiple stages
- Have to cluster modules on lower stages to make up for nonideal ΔV distributions
- Advantageous from production and development cost standpoints



Module Analysis

- All modules have the same inert mass and propellant mass
- Because δ varies with payload mass, not all modules have the same δ !
- Use module-oriented parameters

$$\varepsilon \equiv \frac{m_{in}}{m_{in} + m_{pr}} \quad \sigma \equiv \frac{m_{in}}{m_{pr}}$$

- Conversions

$$\varepsilon = \frac{\delta}{1 - \lambda} \quad \sigma = \frac{\delta}{1 - \delta - \lambda}$$

Rocket Equation for Modular Boosters

- Assuming n modules in stage 1,

$$r_1 = \frac{n(m_{in}) + m_{o2}}{n(m_{in} + m_{pr}) + m_{o2}} = \frac{n\varepsilon + \frac{m_{o2}}{m_{mod}}}{n + \frac{m_{o2}}{m_{mod}}}$$

- If all 3 stages use same modules, n_j for stage j ,

$$r_1 = \frac{n_1\varepsilon + n_2 + n_3 + \rho_{pl}}{n_1 + n_2 + n_3 + \rho_{pl}}$$

where

$$\rho_{pl} \equiv \frac{m_{pl}}{m_{mod}}; m_{mod} = m_{in} + m_{pr}$$

Example: Conestoga 1620 (EER)



- Small launch vehicle (1 flight, 1 failure)
- Payload 900 kg
- Module gross mass 11,400 kg
- Module empty mass 1,400 kg
- Exhaust velocity 2754 m/sec
- Staging pattern
 - 1st stage - 4 modules
 - 2nd stage - 2 modules
 - 3rd stage - 1 module
 - 4th stage - Star 48V (gross mass 2200 kg, empty mass 140 kg, V_e 2842 m/sec)



Conestoga 1620 Performance

- 4th stage ΔV

$$\Delta V_4 = -V_{e4} \ln \frac{m_{f4}}{m_{o4}} = -2842 \ln \frac{900 + 140}{900 + 2200} = 3104 \frac{\text{m}}{\text{sec}}$$

- Treat like three-stage modular vehicle; $M_{pl}=3100$ kg

$$\epsilon = \frac{m_{in}}{m_{mod}} = \frac{1400}{11400} = 0.1228$$

$$\rho_{pl} = \frac{m_{pl}}{m_{mod}} = \frac{3100}{11400} = 0.2719$$

$$n_1 = 4; n_2 = 2; n_3 = 1$$

Constellation 1620 Performance (cont.)

$$r_1 = \frac{n_1 \epsilon + n_2 + n_3 + \rho_{pl}}{n_1 + n_2 + n_3 + \rho_{pl}} = \frac{4 \times 0.1228 + 2 + 1 + 0.2719}{4 + 2 + 1 + 0.2719} = 0.5175$$

$$r_2 = \frac{n_2 \epsilon + n_3 + \rho_{pl}}{n_2 + n_3 + \rho_{pl}} = \frac{2 \times 0.1228 + 1 + 0.2719}{2 + 1 + 0.2719} = 0.4638$$

$$r_3 = \frac{n_3 \epsilon + \rho_{pl}}{n_3 + \rho_{pl}} = \frac{1 \times 0.1228 + 0.2719}{1 + 0.2719} = 0.3103$$

$$V_1 = 1814 \frac{\text{m}}{\text{sec}}; \quad V_2 = 2116 \frac{\text{m}}{\text{sec}}$$

$$V_3 = 3223 \frac{\text{m}}{\text{sec}}; \quad V_4 = 3104 \frac{\text{m}}{\text{sec}}$$

$$V_{total} = 10,257 \frac{\text{m}}{\text{sec}}$$

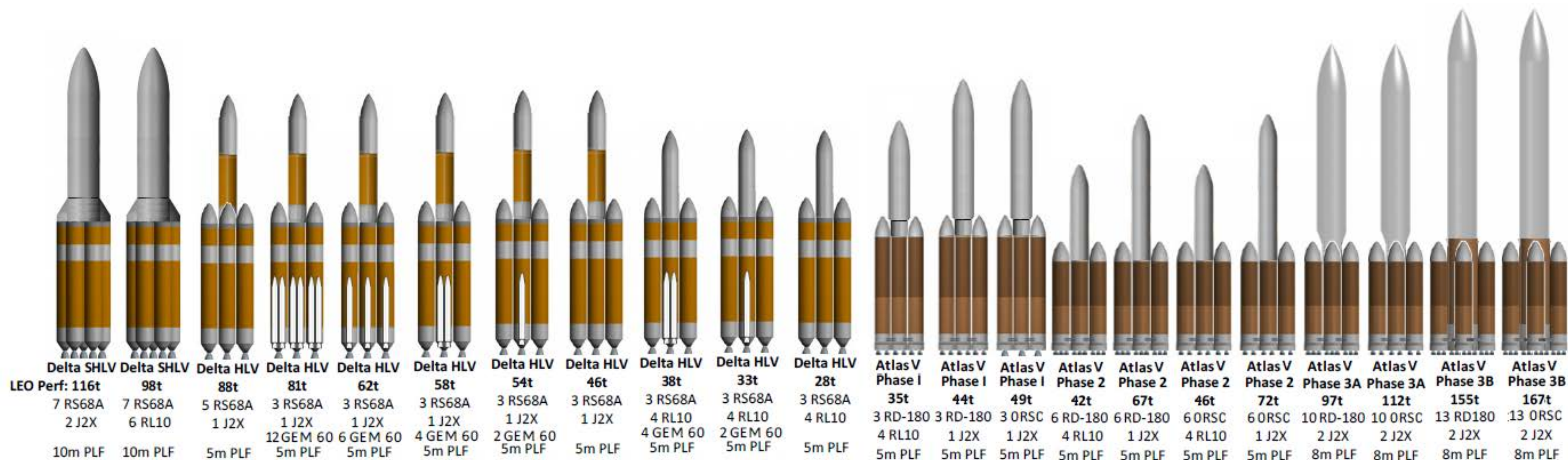
Discussion about Modular Vehicles

- Modularity has several advantages
 - Saves money (smaller modules cost less to develop)
 - Saves money (larger production run = lower cost/module)
 - Allows resizing launch vehicles to match payloads
- Trick is to optimize number of stages, number of modules / stage to minimize total number of modules
- Generally close to optimum by doubling number of modules at each lower stage
- Have to worry about packing factors, complexity

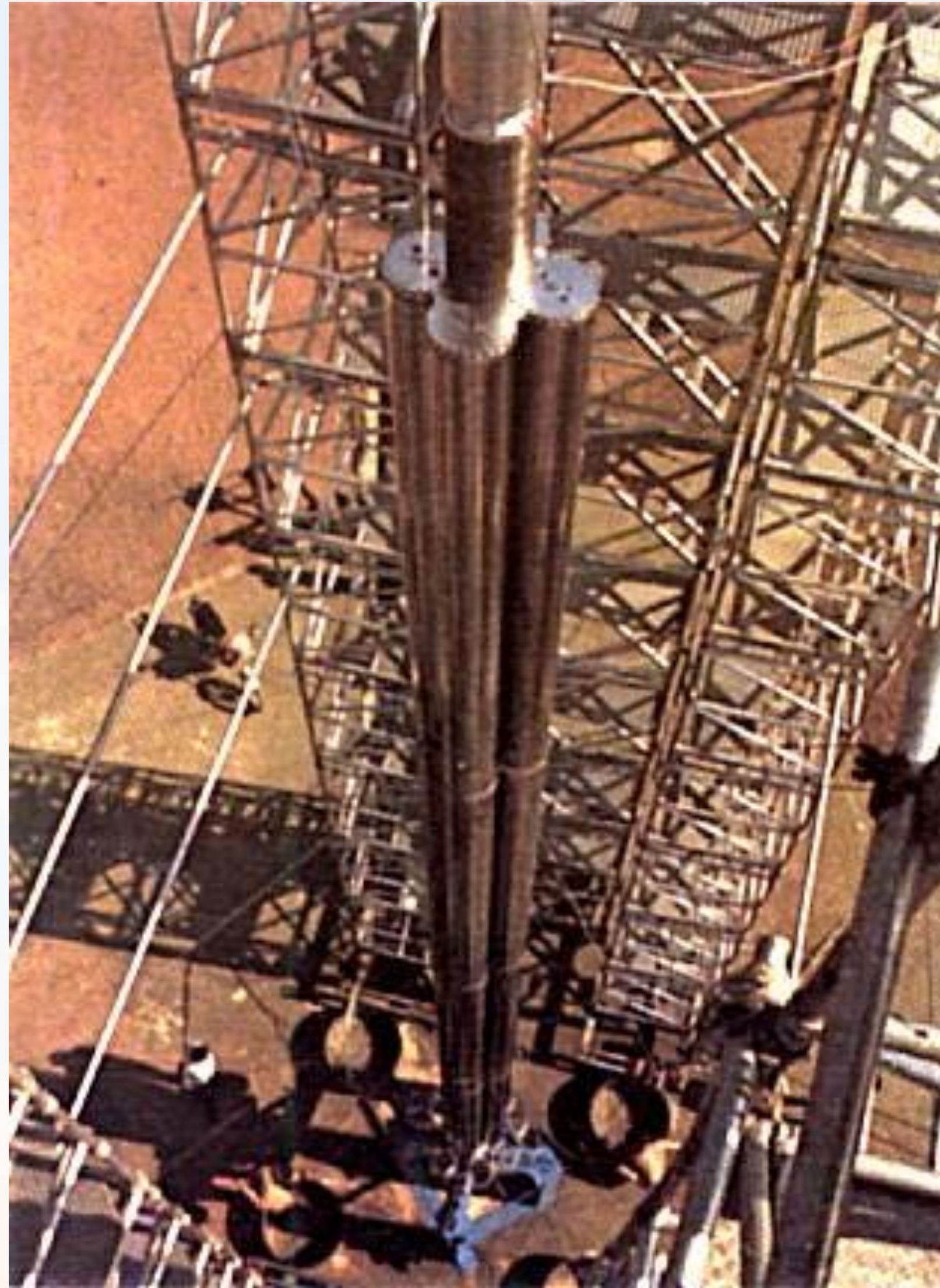
Delta and Atlas Growth Options



Comparison of Heavy Lift Options



OTRAG - 1977-1983



Modular Example

- Let's build a launch vehicle out of seven Space Shuttle Solid Rocket Boosters
 - $M_{in}=86,180$ kg
 - $M_{pr}=503,500$ kg
 - $T=14,685,000$ N ($T/W=2.98$)

$$\varepsilon \equiv \frac{m_{in}}{m_{in} + m_{pr}} = 0.1461 \quad \sigma \equiv \frac{m_{in}}{m_{pr}} = 0.1711$$

- Look at possible approaches to sequential firing

Modular Sequencing - SRB Example

- Assume no payload
- All seven firing at once - $\Delta V_{\text{tot}}=5138$ m/sec
- 3-3-1 sequence - $\Delta V_{\text{tot}}=9087$ m/sec
- 4-2-1 sequence - $\Delta V_{\text{tot}}=9175$ m/sec
- 2-2-2-1 sequence - $\Delta V_{\text{tot}}=9250$ m/sec
- 2-1-1-1-1-1 sequence - $\Delta V_{\text{tot}}=9408$ m/sec
- 1-1-1-1-1-1-1 sequence - $\Delta V_{\text{tot}}=9418$ m/sec
- Sequence limited by need to balance thrust laterally

