## Ballistic Atmospheric Entry

- Planar state equations (from Orbital Mechanics lecture)
- Standard atmospheres
- Orbital decay due to drag
- Straight-line (no gravity) ballistic entry based on density and altitude
- Planetary entries (at least a start)
- Basic equations of planar motion


## Atmospheric Density with Altitude

Pressure=the integral of the atmospheric density in the column above the reference area

$$
\rho=f(h)
$$

$$
\begin{aligned}
& \qquad \begin{aligned}
P_{o}=\int_{o}^{\infty} \rho g d h=\rho_{o} g \int_{o}^{\infty} e^{-\frac{h}{h_{s}}} d h & =-\rho_{o} g h_{s}\left[e^{-\frac{h}{h_{s}}}\right]_{o}^{\infty} \\
& =-\rho_{o} g h_{s}[0-1]
\end{aligned} \\
& P_{o}=\rho_{o} g h_{s}
\end{aligned} \text { Earth: } \rho_{o}=1.226 \frac{\mathrm{~kg}}{\mathrm{~m}^{3}} ; h_{s}=7524 m ; \quad \begin{aligned}
& P_{o}(\text { calc })=90,400 P a ; P_{o}(a c t)=101,300 P a
\end{aligned}
$$

$$
\rho_{o}, P_{o}
$$

## Atmospheric Density with Altitude



Ref: V. L. Pisacane and R. C. Moore, Fundamentals of Space Systems Oxford University Press, 1994

## Energy Loss Due to Atmospheric Drag

$$
\begin{gathered}
\text { Drag } D \equiv \frac{1}{2} \rho v^{2} A c_{D} \\
\text { Drag acceleration } a_{d}=\frac{D}{m}=\frac{\rho v^{2}}{2} \frac{A c_{D}}{m} \\
\beta \equiv \frac{m}{c_{D} A}<==\text { Ballistic Coefficient } \\
a_{d}=\frac{\rho v^{2}}{2 \beta} \\
\text { orbital energy } \equiv E=-\frac{\mu}{2 a} \\
\frac{d E}{d t}=\frac{\mu}{2 a^{2}} \frac{d a}{d t}
\end{gathered}
$$

## Energy Loss Due to Atmospheric Drag

Since drag is highest at perigee, the first effect of atmospheric drag is to circularize the orbit (high perigee drag lowers apogee)

$$
\frac{d E_{d r a g}}{d t}=a_{d} v
$$

$$
v_{c i r c}^{2}=\frac{\mu}{a} \quad \frac{d E_{d r a g}}{d t}=-\frac{\rho v^{2}}{2 \beta} \sqrt{\frac{\mu}{a}}
$$

$$
\frac{d E_{d r a g}}{d t}=-\sqrt{\frac{\mu}{a}} \frac{\rho}{2 \beta} \frac{\mu}{a}=-\left(\frac{\mu}{a}\right)^{\frac{3}{2}} \frac{\rho}{2 \beta}
$$

## Derivation of Orbital Decay Due to Drag

Set orbital energy variation equal to energy lost by drag

$$
\begin{aligned}
& \frac{\mu}{2 a^{2}} \frac{d a}{d t}=-\frac{\rho}{2 \beta}\left(\frac{\mu}{a}\right)^{\frac{3}{2}} \\
& \frac{d a}{d t}=-\frac{\rho}{\beta} \sqrt{\mu a} \\
& \rho=\rho_{o} e^{-\frac{h}{h_{s}}} \quad a=h+r_{E} \Longrightarrow \frac{d a}{d t}=\frac{d h}{d t} \\
& \frac{d h}{d t}=-\frac{\sqrt{\mu\left(h+r_{E}\right)}}{\beta} \rho_{o} e^{-\frac{h}{h_{s}}}
\end{aligned}
$$

## Derivation of Orbital Decay (2)

This is a separable differential equation...

$$
\begin{gathered}
\frac{1}{\sqrt{r_{E}+h}} e^{\frac{h}{h_{s}}} d h=-\frac{\sqrt{\mu}}{\beta} \rho_{o} d t \\
\int_{h_{o}}^{h} \frac{1}{\sqrt{r_{E}+h}} e^{\frac{h}{h_{s}}} d h=-\frac{\sqrt{\mu}}{\beta} \rho_{o} \int_{t_{o}}^{t} d t \\
\text { Assume } \sqrt{r_{E}+h} \sim \sqrt{r_{E}} \text { for } r_{E} \gg h \\
\frac{1}{\sqrt{r_{E}}} \int_{h_{o}}^{h} e^{\frac{h}{h_{s}}} d h=-\frac{\sqrt{\mu}}{\beta} \rho_{o}\left(t-t_{o}\right)
\end{gathered}
$$

## Derivation of Orbital Decay (3)

$$
\begin{gathered}
\frac{h_{s}}{\sqrt{r_{E}}}\left(e^{\frac{h}{h_{s}}}-e^{\frac{h_{o}}{h_{s}}}\right)=-\frac{\sqrt{\mu}}{\beta} \rho_{o}\left(t-t_{o}\right) \\
\left.e^{\frac{h}{h_{s}}}-e^{\frac{h_{o}}{h_{s}}}=-\frac{\sqrt{\mu r_{E}}}{h_{s} \beta} \rho_{o}\left(t-t_{o}\right)\right] \\
h(t)=h_{s} \ln \left[e^{\frac{h_{o}}{h_{s}}}-\frac{\sqrt{\mu r_{E}}}{h_{s} \beta} \rho_{o}\left(t-t_{o}\right)\right]
\end{gathered}
$$

Note that some variables typically use km, and others are in meters - you have to make sure unit conversions are done properly to make this work out correctly!

## Orbit Decay from Atmospheric Drag



## Time Until Orbital Decay

$$
e^{\frac{h}{h_{s}}}-e^{\frac{h_{o}}{h_{s}}}=-\frac{\sqrt{\mu r_{E}}}{h_{s} \beta} \rho_{o}\left(t-t_{o}\right)
$$

To find the time remaining ( $\mathrm{t}_{0}=0$ ) until the orbit reaches any given "critical" altitude, some algebra gives

$$
\begin{aligned}
t\left(h_{\text {crit }}\right)= & \frac{h_{s} \beta}{\sqrt{\mu r_{E}} \rho_{o}}\left(e^{\frac{h_{o}}{h_{s}}}-e^{\frac{h_{\text {crit }}}{h_{s}}}\right) \\
& t\left(h_{\text {crit }}\right) \propto \beta
\end{aligned}
$$

## Decay Time to $\mathrm{r}=120 \mathrm{~km}$



## Ballistic Entry (no lift)

$s=$ distance along the flight path

$$
\begin{aligned}
& \text { istance along the flight path } \\
& \frac{d v}{d t}=-g \sin \gamma-\frac{D}{m} \\
& \frac{d v}{d t}=\frac{d v}{d s} \frac{d s}{d t}=V \frac{d v}{d s}=\frac{1}{2} \frac{d\left(v^{2}\right)}{d s} \\
& \frac{1}{2} \frac{d\left(v^{2}\right)}{d s}=-g \sin \gamma-\frac{D}{m} \quad \operatorname{Drag} D \equiv \frac{1}{2} \rho v^{2} A c_{D} \\
& \frac{1}{2} \frac{d\left(v^{2}\right)}{d s}=-g \sin \gamma-\frac{\rho v^{2}}{2 m} A c_{D} \\
& \frac{\sin \gamma}{2} \frac{d\left(v^{2}\right)}{d h}=-g \sin \gamma-\frac{\rho v^{2}}{2 m} A c_{D}
\end{aligned}
$$

## Ballistic Entry (2)

Exponential atmosphere $\Rightarrow \rho=\rho_{o} e^{-\frac{h}{h_{s}}}$

$$
\begin{gathered}
\frac{d \rho}{\rho_{o}}=e^{-\frac{h}{h_{s}}}\left(\frac{-d h}{h_{s}}\right)=\frac{\rho_{o} e^{-\frac{h}{h_{s}}}}{\rho_{o}}\left(\frac{-d h}{h_{s}}\right)=\frac{\rho}{\rho_{o}}\left(\frac{-d h}{h_{s}}\right) \\
d h=\frac{-h_{s}}{\rho} d \rho \\
\frac{\sin \gamma}{2} \frac{d\left(v^{2}\right)}{d h}=-g \sin \gamma-\frac{\rho v^{2}}{2 m} A c_{D} \\
\frac{\sin \gamma}{2} \frac{d\left(v^{2}\right)}{d \rho}\left(\frac{-\rho}{h_{s}}\right)=-g \sin \gamma-\frac{\rho v^{2}}{2} \frac{A c_{D}}{m} \\
\frac{d\left(v^{2}\right)}{d \rho}=\frac{2 g h_{s}}{\rho}+\frac{h_{s} v^{2}}{\sin \gamma} \frac{A c_{D}}{m}
\end{gathered}
$$

## Ballistic Entry (3)

Let $\beta \equiv \frac{m}{c_{D} A} \Rightarrow$ Ballistic Coefficient

$$
\frac{d\left(v^{2}\right)}{d \rho}-\frac{h_{s}}{\beta \sin \gamma} v^{2}=\frac{2 g h_{s}}{\rho}
$$

Assume $m g \ll D$ to get homogeneous ODE

$$
\frac{d\left(v^{2}\right)}{d \rho}-\frac{h_{s}}{\beta \sin \gamma} v^{2}=0
$$

$$
\frac{d\left(v^{2}\right)}{v^{2}}=\frac{h_{s}}{\beta \sin \gamma} d \rho
$$

Use $\left(v^{2}\right)$ as integration variable

$$
\int_{v_{e}}^{v} \frac{d\left(v^{2}\right)}{v^{2}}=\frac{h_{s}}{\beta \sin \gamma} \int_{0}^{\rho} d \rho \quad v_{e}=\text { velocity at entry }
$$

## Ballistic Entry (4)

Note that the effect of ignoring gravity is that there is no force perpendicular to velocity vector $\Rightarrow$ constant flight path angle $\gamma \Rightarrow$ straight line trajectories

$$
\begin{aligned}
\ln \frac{v^{2}}{v_{e}^{2}} & =2 \ln \frac{v}{v_{e}}=\frac{h_{s} \rho}{\beta \sin \gamma} \\
\frac{v}{v_{e}} & =\exp \left(\frac{h_{s} \rho}{2 \beta \sin \gamma}\right)
\end{aligned}
$$

$$
\frac{v}{v_{e}}=\exp \left(\frac{h_{s} \rho_{o}}{2 \beta \sin \gamma} \frac{\rho}{\rho_{o}}\right) \quad \text { Check units: }\left(\frac{m \frac{k g}{m^{3}}}{\frac{k g}{m^{2}}}\right)
$$

## Earth Entry, $\gamma=-60^{\circ}$



## What About Peak Deceleration?

$$
\begin{gathered}
n \equiv \frac{d v}{d t}=-\frac{\rho v^{2}}{2 \beta} \\
\text { To find } n_{\text {max }} \text {, set } \frac{d}{d t}\left(\frac{d v}{d t}\right)=\frac{d^{2} v}{d t^{2}}=0 \\
\frac{d^{2} v}{d t^{2}}=-\frac{1}{2 \beta}\left(2 \rho v \frac{d v}{d t}+v^{2} \frac{d \rho}{d t}\right)=0 \\
\frac{d^{2} v}{d t^{2}}=-\frac{1}{2 \beta}\left(-\frac{2 \rho^{2} v^{3}}{2 \beta}+v^{2} \frac{d \rho}{d t}\right)=0 \\
\frac{\rho^{2} v^{3}}{\beta}=v^{2} \frac{d \rho}{d t} \quad \rho^{2} v=\beta \frac{d \rho}{d t}
\end{gathered}
$$

## Peak Deceleration (2)

From exponential atmosphere,

$$
\frac{d \rho}{d t}=-\frac{\rho_{o}}{h_{s}} e^{-\frac{h}{h_{s}}} \frac{d h}{d t}=-\frac{\rho}{h_{s}} \frac{d h}{d t}
$$

From geometry, $\frac{d h}{d t}=v \sin \gamma$

$$
\begin{gathered}
\frac{d \rho}{d t}=-\frac{\rho v}{h_{s}} \sin \gamma \quad \rho^{2} v=\beta \frac{d \rho}{d t} \\
\rho^{2} v=\beta\left(-\frac{\rho v}{h_{s}} \sin \gamma\right)
\end{gathered}
$$

Remember that this refers to the conditions at max deceleration

$$
\rho_{n_{\max }}=-\frac{\beta}{h_{s}} \sin \gamma
$$

## Critical $\beta$ for Deceleration Before Impact

At surface, $\rho=\rho_{o}$

$$
\beta_{\text {crit }}=-\frac{\rho_{o} h_{s}}{\sin \gamma} \quad \Leftarrow \text { Vround at point of maximum deceleration }
$$

How large is maximum deceleration?

$$
\begin{gathered}
\frac{d v}{d t}=\frac{\rho v^{2}}{2 \beta} \Rightarrow\left|\frac{d v}{d t}\right|_{\max }=\frac{\rho_{n_{\max }} v^{2}}{2 \beta} \\
\left|\frac{d v}{d t}\right|_{\max }=\frac{v^{2}}{2 \beta}\left(-\frac{\beta}{h_{s}} \sin \gamma\right)=-\frac{1}{2} \frac{v^{2}}{h_{s}} \sin \gamma
\end{gathered}
$$

Note that this value of $v$ is actually $v_{n_{\max }}$

## Peak Deceleration (3)

From page 16,

$$
\begin{gathered}
\frac{v}{v_{e}}=\exp \left(\frac{h_{s} \rho}{2 \beta \sin \gamma}\right) \\
\frac{v_{n_{\max }}}{v_{e}}=\exp \left[\frac{h_{s}}{2 \beta \sin \gamma}\left(-\frac{\beta}{h_{s}} \sin \gamma\right)\right]=e^{-\frac{1}{2}} \\
\left|\frac{d v}{d t}\right|_{\max }=-\frac{1}{2} \frac{\left(v_{e} e^{-\frac{1}{2}}\right)^{2}}{h_{s}} \sin \gamma=-\frac{v_{e}^{2} \sin \gamma}{2 h_{s} e}
\end{gathered}
$$

Note that the velocity at which maximum deceleration occurs is always a fixed fraction of the entry velocity - it doesn't depend on ballistic coefficient, flight path angle, or anything else! Also, the magnitude of the maximum deceleration is not a function of ballistic coefficient - it is dependent on the entry trajectory ( $\mathrm{v}_{\mathrm{e}}$ and $\gamma$ ) but not spacecraft parameters (i.e., ballistic coefficient).

## Terminal Velocity

Full form of ODE -

$$
\frac{d\left(v^{2}\right)}{d \rho}-\frac{h_{s}}{\beta \sin \gamma} v^{2}=\frac{2 g h_{s}}{\rho}
$$

At terminal velocity, $v=$ constant $\equiv v_{T}$

$$
\begin{aligned}
& -\frac{h_{s}}{\beta \sin \gamma} v_{T}^{2}=\frac{2 g h_{s}}{\rho} \\
& v_{T}^{2}=\sqrt{-\frac{2 g \beta \sin \gamma}{\rho}}
\end{aligned}
$$

## "Cannon Ball" $\gamma=-90^{\circ}$ Ballistic Entry

6.75" diameter sphere, $\mathrm{c}_{\mathrm{D}}=0.2, \mathrm{~V}_{\mathrm{E}}=6000 \mathrm{~m} / \mathrm{sec}$

|  | Iron | Aluminum | Balsa Wood |
| :--- | :---: | :---: | :---: |
| Weight | 40 lb | 15.6 lb | 14.5 oz |
| $\beta\left(\mathrm{kg} / \mathrm{m}^{2}\right)$ | 3938 | 1532 | 89 |
| $\rho_{\mathrm{md}}\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ | 0.555 | 0.216 | 0.0125 |
| $\mathrm{~h}_{\text {md }}(\mathrm{m})$ | 5600 | 12,300 | 32,500 |
| $V_{\text {impact }}(\mathrm{m} / \mathrm{s})$ | 1998 | 355 | $0^{\star}$ |
| $V_{\text {term }}(\mathrm{m} / \mathrm{sec})$ | 251 | 156 | 38 |

*Artifact of assumption that $D \gg m g$

## Nondimensional Ballistic Coefficient

$$
\frac{v}{v_{e}}=\exp \left(\frac{h_{s} \rho_{o}}{2 \beta \sin \gamma} \frac{\rho}{\rho_{o}}\right)=\exp \left(\frac{P_{o}}{2 \beta g \sin \gamma} \frac{\rho}{\rho_{o}}\right)
$$

Let $\widehat{\beta} \equiv \frac{\beta}{\rho_{o} h_{s}}=\frac{\beta g}{P_{o}}$ (Nondimensional form of ballistic coefficient) Note that we are using the estimated value of $P_{o}=\rho_{o} g h_{s}$, not the actual surface pressure.

$$
\frac{v}{v_{e}}=\exp \left(\frac{1}{2 \widehat{\beta} \sin \gamma} \frac{\rho}{\rho_{o}}\right)
$$

$$
\beta_{c r i t}=-\frac{\rho_{o} h_{s}}{\sin \gamma} \quad \widehat{\beta}_{c r i t}=-\frac{1}{\sin \gamma}
$$

## Entry Velocity Trends, $\gamma=-90^{\circ}$



## Ballistic Entry, Again (this time using altitude)

$s=$ distance along the flight path

$$
\frac{d v}{d t}=-g \sin \gamma-\frac{D}{m}
$$

Again assuming $D \gg g$,


$$
\begin{aligned}
& \frac{d v}{d t}=-\frac{D}{m} \quad \text { Drag } D \equiv \frac{1}{2} \rho v^{2} A c_{D} \\
& \frac{d v}{d t}=-\frac{\rho c_{D} A}{2 m} v^{2}
\end{aligned}
$$

Separating the variables,

$$
\frac{d v}{v^{2}}=-\frac{\rho}{2 \beta} d t
$$

## Calculating the Entry Velocity Profile

$$
\begin{aligned}
& \frac{d h}{d t}=v \sin \gamma \Rightarrow d t=\frac{d h}{v \sin \gamma} \\
& \frac{d v}{v^{2}}=-\frac{\rho}{2 \beta v \sin \gamma} d h \Rightarrow \frac{d v}{v}=-\frac{\rho}{2 \beta \sin \gamma} d h \\
& \frac{d v}{v}=-\frac{\rho_{o}}{2 \beta \sin \gamma} e^{-\frac{h}{h s}} d h \\
& \int_{v_{e}}^{v} \frac{d v}{v}=-\frac{\rho_{o}}{2 \beta \sin \gamma} \int_{h_{e}}^{h} e^{-\frac{h}{h s}} d h \\
& \ln \frac{v}{v_{e}}=\frac{\rho_{o} h_{s}}{2 \beta \sin \gamma}\left[e^{-\frac{h}{h s}}\right]_{h_{e}}^{h}=\frac{1}{2 \widehat{\beta} \sin \gamma}\left[e^{-\frac{h}{h s}}-e^{-\frac{h_{e}}{h s}}\right]
\end{aligned}
$$

## Deriving the Entry Velocity Function

Remember that $e^{-\frac{h_{e}}{h s}}=\frac{\rho_{e}}{\rho_{o}} \approx 0$

$$
\frac{v}{v_{e}}=\exp \left[\frac{1}{2 \widehat{\beta} \sin \gamma} e^{-\frac{h}{h s}}\right]
$$

We have a parametric entry equation in terms of nondimensional velocity ratios, ballistic coefficient, and altitude. To bound the nondimensional altitude variable between 0 and 1, rewrite as

$$
\frac{v}{v_{e}}=\exp \left[\frac{1}{2 \widehat{\beta} \sin \gamma} e^{-\frac{h}{h e} \frac{h_{e}}{h_{s}}}\right]
$$

$\frac{h_{e}}{h_{s}}$ and $\widehat{\beta}$ are the only variables that relate to a specific planet

## Earth Entry, $\gamma=-90^{\circ}$



## Deceleration as a Function of Altitude

Start with

$$
\begin{aligned}
& \frac{v}{v_{e}}=\exp \left[\frac{1}{2 \widehat{\beta} \sin \gamma} e^{-\frac{h}{h e} \frac{h_{e}}{h_{s}}}\right] \quad \text { Let } B \equiv \frac{1}{2 \widehat{\beta} \sin \gamma} \\
& \frac{v}{v_{e}}=\exp \left(B e^{-\frac{h}{h_{s}}}\right) \\
& \frac{d}{d t}\left(\frac{v}{v_{e}}\right)=\exp \left(B e^{-\frac{h}{h_{s}}}\right) \frac{d}{d t}\left(B e^{-\frac{h}{h_{s}}}\right) \\
& \frac{d v}{d t}=v_{e} \exp \left(B e^{-\frac{h}{h_{s}}}\right) \frac{-B}{h_{s}}\left(e^{-\frac{h}{h_{s}}}\right) \frac{d h}{d t} \\
& \frac{d h}{d t}=v \sin \gamma=v_{e} \sin \gamma \exp B e^{-\frac{h}{h_{s}}}
\end{aligned}
$$

## Parametric Deceleration

$$
\begin{aligned}
& \frac{d v}{d t}=v_{e} \exp \left(B e^{-\frac{h}{h_{s}}}\right) \frac{-B}{h_{s}}\left(e^{-\frac{h}{h_{s}}}\right) v_{e} \sin \gamma \exp \left(B e^{-\frac{h}{h_{s}}}\right) \\
& \frac{d v}{d t}=\frac{-B v_{e}^{2}}{h_{s}} \sin \gamma\left(e^{-\frac{h}{h_{s}}}\right) \exp \left(2 B e^{-\frac{h}{h_{s}}}\right) \\
& \frac{d v}{d t}=\frac{-v_{e}^{2}}{2 h_{s} \widehat{\beta}}\left(e^{-\frac{h}{h_{s}}}\right) \exp \left(\frac{1}{\widehat{\beta} \sin \gamma} e^{-\frac{h}{h_{s}}}\right) \\
& \operatorname{Let} n_{\text {ref }} \equiv \frac{v_{e}^{2}}{h_{s}}, \nu \equiv \frac{d v / d t}{n_{\text {ref }}}, \varphi \equiv \frac{h_{e}}{h_{s}} \\
& \nu=\frac{-1}{2 \widehat{\beta}}\left(e^{-\varphi \frac{h}{h_{e}}}\right) \exp \left(\frac{1}{\widehat{\beta} \sin \gamma} e^{-\varphi \frac{h}{h_{e}}}\right)
\end{aligned}
$$

## Nondimensional Deceleration, $\gamma=-90^{\circ}$



## Deceleration Equations

Nondimensional Form

$$
\nu=\frac{-1}{2 \widehat{\beta}}\left(e^{-\varphi \frac{h}{h_{e}}}\right) \exp \left(\frac{1}{\widehat{\beta} \sin \gamma} e^{-\varphi \frac{h}{h_{e}}}\right)
$$

Dimensional Form

$$
n=\frac{-\rho_{o} V_{e}^{2}}{2 \beta}\left(e^{-\frac{h}{h_{s}}}\right) \exp \left(\frac{\rho_{o} h_{s}}{\beta \sin \gamma} e^{-\frac{h}{h_{s}}}\right)
$$

Note that these equations result in values $<0$ (reflecting deceleration) - graphs are absolute values of deceleration for clarity.

## Dimensional Deceleration, $\gamma=-90^{\circ}$



## Altitude of Maximum Deceleration

Returning to shorthand notation for deceleration

$$
\begin{aligned}
& \nu=-B \sin \gamma\left(e^{-\frac{h}{h_{s}}}\right) \exp \left(2 B e^{-\frac{h}{h_{s}}}\right) \\
& \nu=-B \sin \gamma\left(e^{-\eta}\right) \exp \left(2 B e^{-\eta}\right) \quad \text { Let } \eta \equiv \frac{h}{h_{s}} \\
& \frac{d \nu}{d \eta}=-B \sin \gamma\left[\frac{d}{d \eta}\left(e^{-\eta}\right) \exp \left(2 B e^{-\eta}\right)+\left(e^{-\eta}\right) \frac{d}{d \eta} \exp \left(2 B e^{-\eta}\right)\right] \\
& \frac{d \nu}{d \eta}=-B \sin \gamma\left[-\left(e^{-\eta}\right) \exp \left(2 B e^{-\eta}\right)+\left(e^{-\eta}\right)\left(-2 B e^{-\eta}\right) \exp \left(2 B e^{-\eta}\right)\right] \\
& \frac{d \nu}{d \eta}=B \sin \gamma e^{-\eta} \exp \left(2 B e^{-\eta}\right)\left[1+\left(2 B e^{-\eta}\right)\right]=0
\end{aligned}
$$

## Altitude of Maximum Deceleration

$$
\begin{aligned}
& 1+\left(2 B e^{-\eta}\right)=0 \Rightarrow e^{\eta}=-2 B \\
& \eta_{n_{\max }}=\ln (-2 B) \\
& \eta_{n_{\max }}=\ln \left(\frac{-1}{\widehat{\beta} \sin \gamma}\right)
\end{aligned}
$$

Converting from parametric to dimensional form gives

$$
h_{n_{\max }}=h_{s} \ln \left(\frac{-\rho_{o} h_{s}}{\beta \sin \gamma}\right)
$$

Altitude of maximum deceleration is independent of entry velocity!

## Altitude of Maximum Deceleration



## Magnitude of Maximum Deceleration

Start with the equation for acceleration -

$$
\nu=\frac{-1}{2 \widehat{\beta}} e^{-\eta} \exp \left(\frac{1}{\widehat{\beta} \sin \gamma} e^{-\eta}\right)
$$

and insert the value of $\eta$ at the point of maximum deceleration

$$
\begin{aligned}
\eta_{n_{\max }} & =\ln \left(\frac{-1}{\widehat{\beta} \sin \gamma}\right) \Rightarrow e^{-\eta}=-\widehat{\beta} \sin \gamma \\
\nu_{n_{\max }} & =\frac{-1}{2 \widehat{\beta}}(-\widehat{\beta} \sin \gamma) \exp \left(\frac{-\widehat{\beta} \sin \gamma}{\widehat{\beta} \sin \gamma}\right) \Rightarrow \nu_{n_{\max }}=\frac{\sin \gamma}{2 e} \\
n_{\max } & =\frac{v_{e}^{2}}{h_{s}} \frac{\sin \gamma}{2 e}
\end{aligned}
$$

Maximum deceleration is not a function of ballistic coefficient!

## Peak Ballistic Deceleration for Earth Entry



## Velocity at Maximum Deceleration

Start with the equation for velocity

$$
\frac{v}{v_{e}}=\exp \left[\frac{1}{2 \widehat{\beta} \sin \gamma} e^{-\eta}\right]
$$

and insert the value of $\eta$ at the point of maximum deceleration

$$
\begin{aligned}
& \eta_{n_{\max }}=\ln \left(\frac{-1}{\widehat{\beta} \sin \gamma}\right) \Rightarrow e^{-\eta}=-\widehat{\beta} \sin \gamma \\
& \frac{v}{v_{e}}=\exp \left[\frac{-\widehat{\beta} \sin \gamma}{2 \widehat{\beta} \sin \gamma}\right] \Rightarrow v_{n_{\max }}=\frac{v_{e}}{\sqrt{e}} \cong 0.606 v_{e}
\end{aligned}
$$

Velocity at maximum deceleration is independent of everything except $v_{e}$

## Planetary Entry - Physical Data

|  | Radius <br> $(\mathrm{km})$ | $\mu$ <br> $\left(\mathrm{km}^{3} / \mathrm{sec}^{2}\right)$ | $\rho_{O}$ <br> $\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$ | $\mathrm{h}_{\mathrm{s}}$ <br> $(\mathrm{km})$ | $\mathrm{v}_{\text {esc }}$ <br> $(\mathrm{km} / \mathrm{sec})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Earth | 6378 | 398,604 | 1.225 | 7.524 | 11.18 |
| Mars | 3393 | 42,840 | 0.0993 | 27.7 | 5.025 |
| Venus | 6052 | 325,600 | 16.02 | 6.227 | 10.37 |

Ballistic Entry ENAE 791 - Launch and Entry Vehicle Design

## Comparison of Planetary Atmospheres



## Planetary Entry Profiles



## Planetary Entry Deceleration Comparison



## Check on Approximation Formulas



Ballistic Entry
MARYLAND

