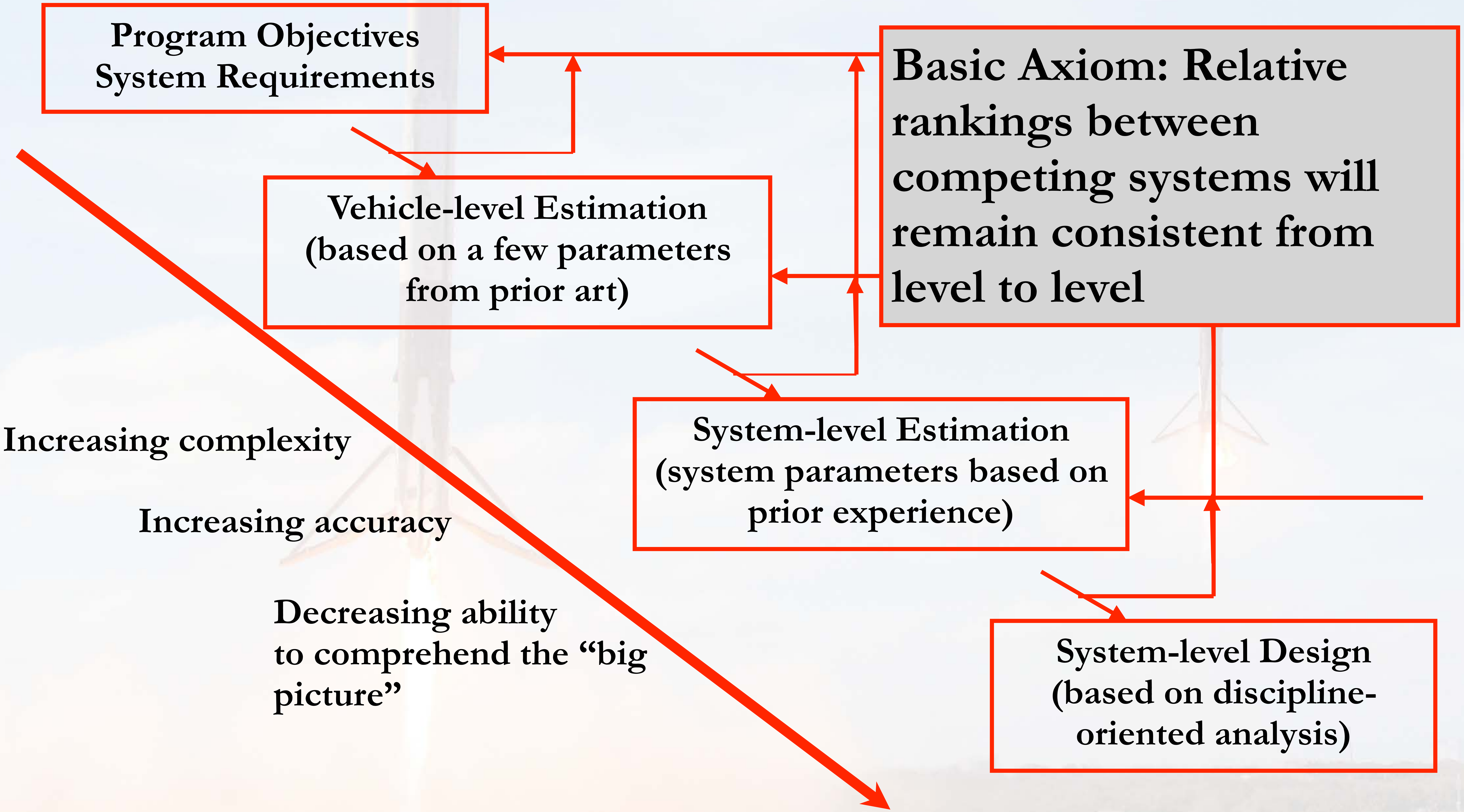
# Mass Estimating Relations

- Review of iterative design approach
- Mass Estimating Relations (MERs)
- Sample vehicle design analysis

# Overview of the Design Process



# Vehicle-Level Prelim Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- ΔV=9200 m/sec
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec (Ve=4214 m/sec)
  - δ=0.08

$$r = e^{-\frac{\Delta V}{Ve}} = 0.1127$$

$$\lambda = r - \delta = 0.0327$$

$$M_o = \frac{M_\ell}{\lambda} = 153,000\ kg$$

$$M_i = \delta M_o = 12,240\ kg$$

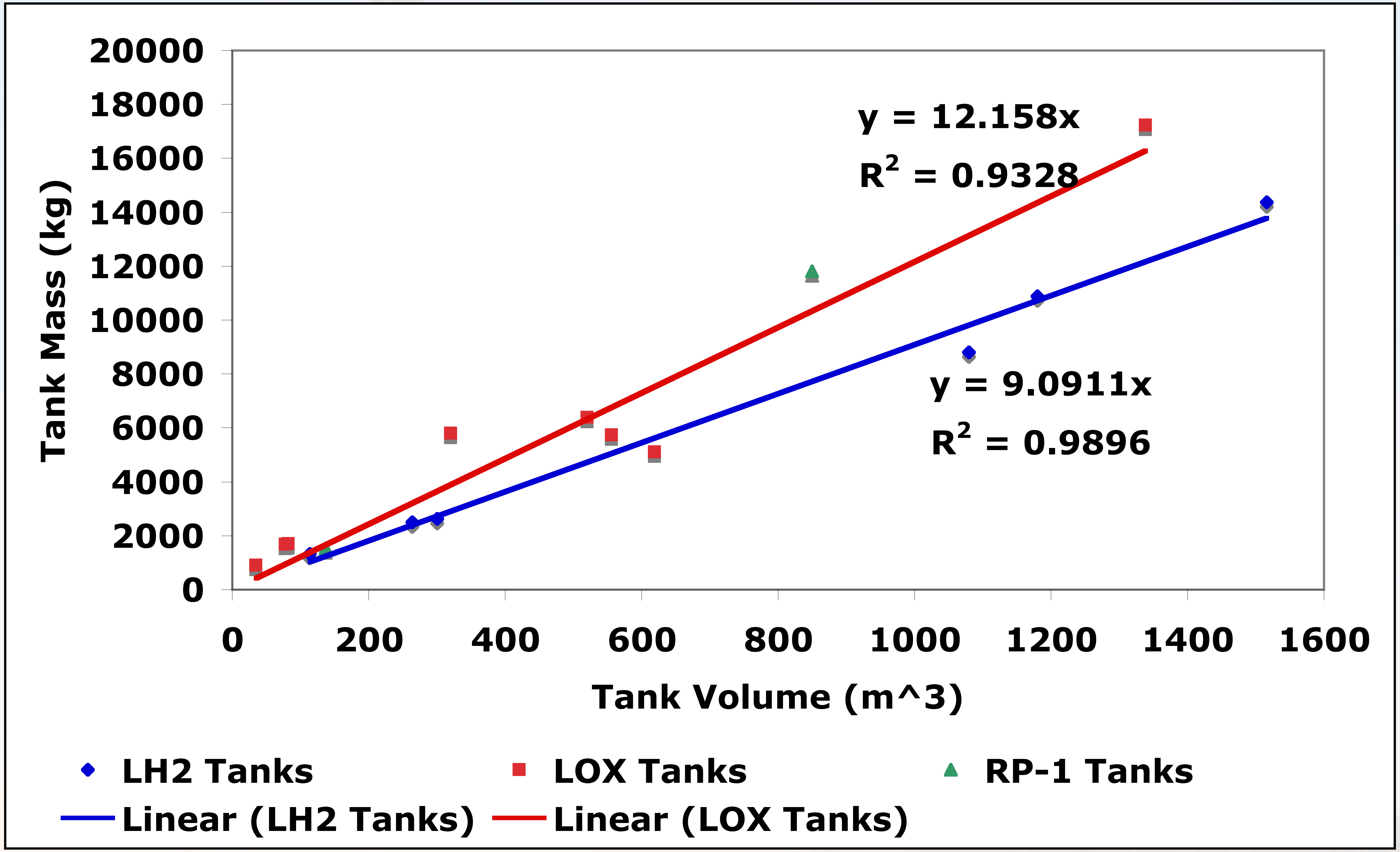
$$M_p = M_o(1 - r) = 135,800\ kg$$

# System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

$$\rho_{LOX} = 1140 \ \frac{kg}{m^3} \qquad \rho_{LH_2} = 71 \ \frac{kg}{m^3}$$

# Propellant Tank Regression Data



# Propellant Tank MERs (Volume)

- $LH_2$ tanks

$$M_{LH_2\ Tank}\langle kg\rangle = 9.09 V_{LH_2}\langle m^3\rangle$$

- All other tanks

$$M_{Tank}\langle kg\rangle = 12.16 V_{prop}\langle m^3\rangle$$

# Propellant Tank MERs (Mass)

- $LH_2$ tanks

$$\rho_{LH_2} = 71\ \frac{kg}{m^3} \Longrightarrow M_{LH_2\ Tank}\langle kg\rangle = 0.128 M_{LH_2}\langle kg\rangle$$

- LOX tanks

$$\rho_{LOX} = 1140\frac{kg}{m^3} \Longrightarrow M_{LOX\ Tank}\langle kg\rangle = 0.0107 M_{LOX}\langle kg\rangle$$

- RP-1 tanks

$$\rho_{RP1} = 820\frac{kg}{m^3} \Longrightarrow M_{RP1\ Tank}\langle kg\rangle = 0.0148 M_{RP1}\langle kg\rangle$$

- $LCH_4$ tanks

$$\rho_{LCH4} = 420\ \frac{kg}{m^3} \Longrightarrow M_{LCH4\ Tank}\langle kg\rangle = 0.0290\ M_{LCH4}\langle kg\rangle$$

# Cryogenic Insulation MERs

$$M_{LH_2\ Insulation}\langle kg\rangle = 2.88 A_{tank}\langle\frac{kg}{m^2}\rangle$$

$$M_{LOX/LCH_4\ Insulation}\langle kg\rangle = 1.123 A_{\text{tank}}\left\langle\frac{kg}{m^2}\right\rangle$$

# LOX Tank Design

- Mass of LOX=116,400 kg

$$M_{LOX\ Tank} = 0.0107(116,400) = 1245\ kg$$

- Need area to find LOX tank insulation mass - assume a sphere

$$V_{LOX\ Tank} = \frac{M_{LOX}}{\rho_{LOX}} = 102.1\ m^3$$

$$r_{LOX\ Tank} = \left(\frac{V_{LOX}}{4\pi/3}\right)^{\frac{1}{3}} = 2.90\ m$$

$$A_{LOX\ Tank} = 4\pi r^2 = 105.6\ m^2$$

$$M_{LOX\ Insulation} = 1.123\langle\frac{kg}{m^2}\rangle(105.6\langle m^2\rangle) = 119\ kg$$

# $LH_2$ Tank Design

- Mass of $LH_2$=19,390 kg

$$M_{LH_2\ Tank}\langle kg\rangle = 0.128(19,390) = 2482\ kg$$

- Again, assume $LH_2$ tank is spherical

$$V_{LH_2\ Tank} = \frac{M_{LH_2}}{\rho_{LH_2}} = 273.1\ m^3$$

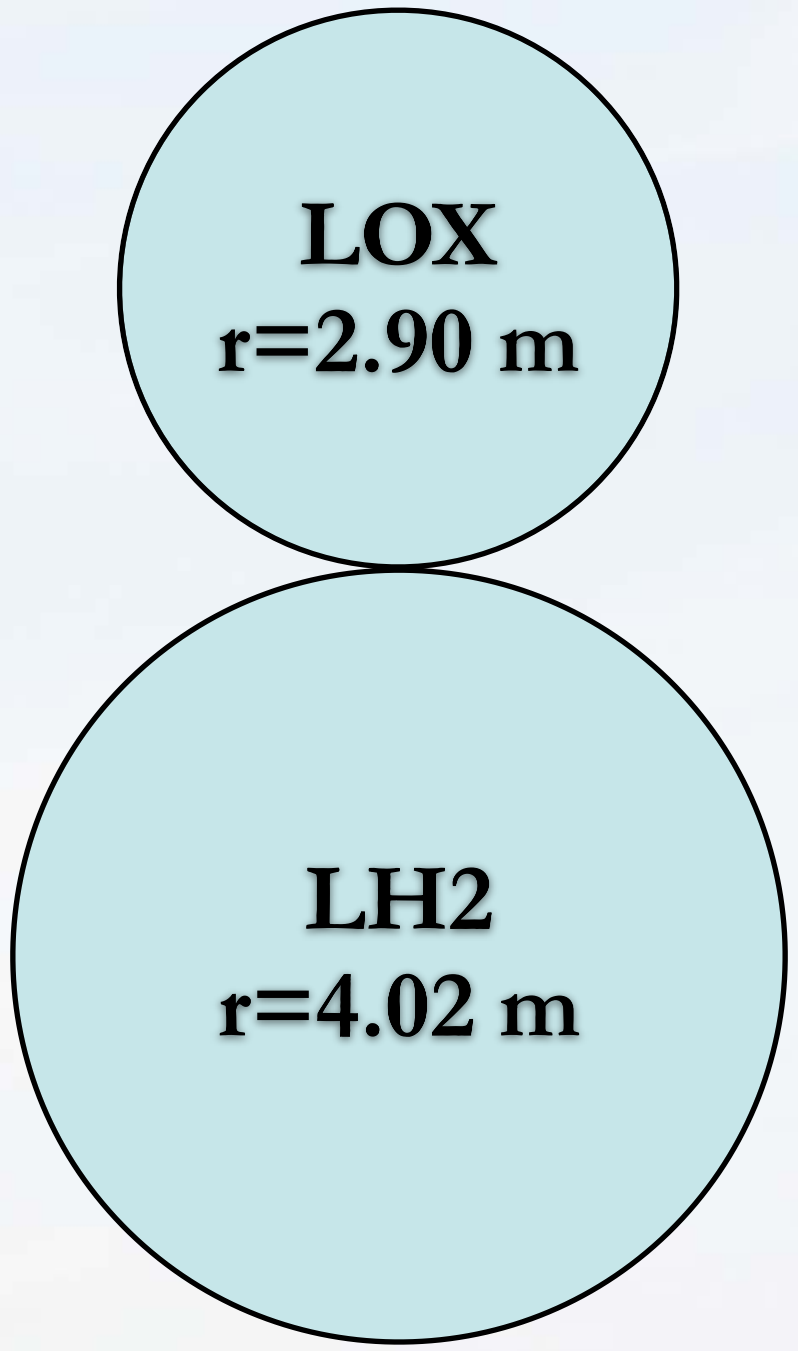
$$r_{LH_2\ Tank} = \left(\frac{V_{LH_2}}{4\pi/3}\right)^{\frac{1}{3}} = 4.02\ m$$

$$A_{LH_2\ Tank} = 4\pi r^2 = 203.6\ m^2$$

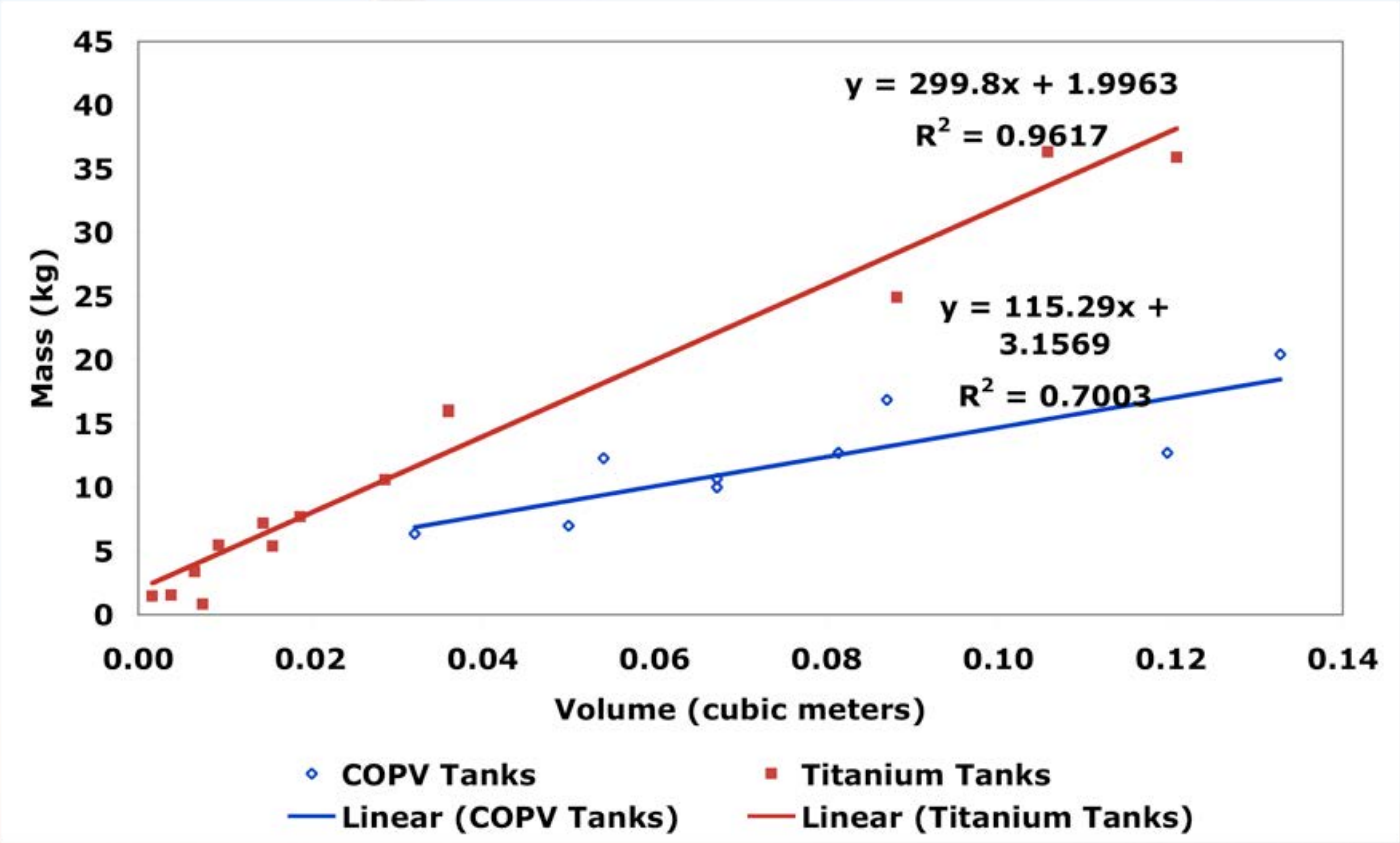
$$M_{LH_2\ Insulation} = 2.88\langle\frac{kg}{m^2}\rangle(203.6\langle m^2\rangle) = 586\ kg$$

# Current Design Sketch

- Masses
  - LOX Tank 1245 kg
  - LOX Tank Insulation 119 kg
  - $LH_2$ Tank 2482 kg
  - $LH_2$ Tank Insulation 586 kg



# High-Pressure Gas Tanks



# Pressurized Gas Tank MERs

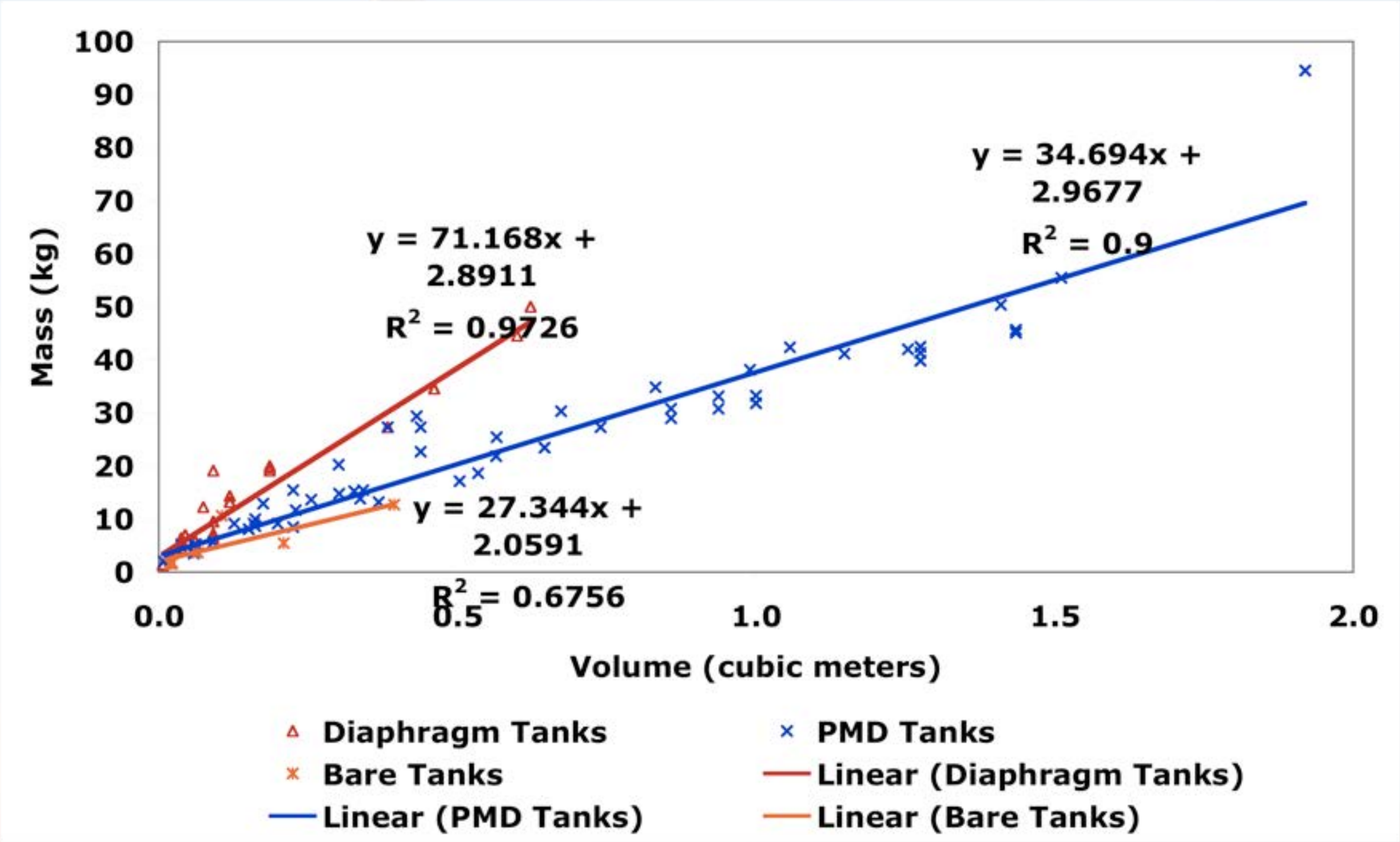
- COPV (Composite Overwrapped Pressure Vessel)

$$M_{COPV\ Tank}(kg) = 115.3\ V_{contents}(m^3) + 3$$

- Titanium tank

$$M_{COPV\ Tank}(kg) = 299.8\ V_{contents}(m^3) + 2$$

# Smaller Storable Liquids Tanks



# Small Liquid Tankage MERs

- Bare metal tanks

$$M_{Bare\ Tank}(kg) = 27.34\ V_{contents}(m^3) + 2$$

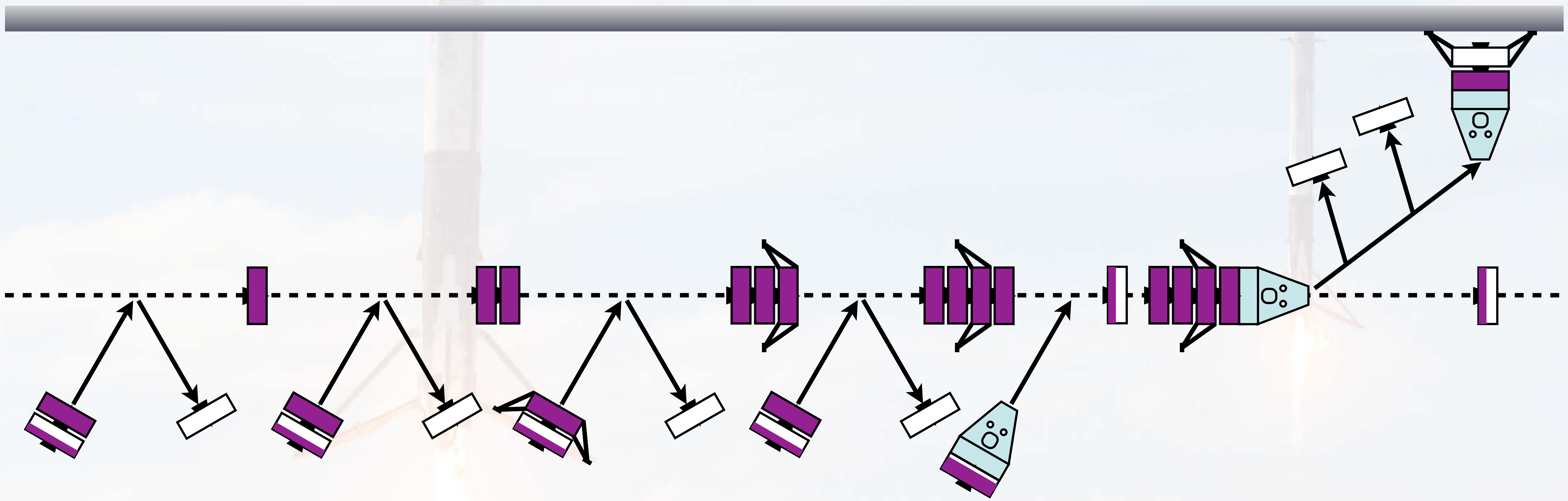
- Tanks with propellant management devices

$$M_{PMD\ Tank}(kg) = 34.69\ V_{contents}(m^3) + 3$$

- Titanium tanks with positive expulsion bladders

$$M_{Diaphragm\ Tank}(kg) = 71.17\ V_{contents}(m^3) + 3$$

# Minimum Cost Lunar Architecture



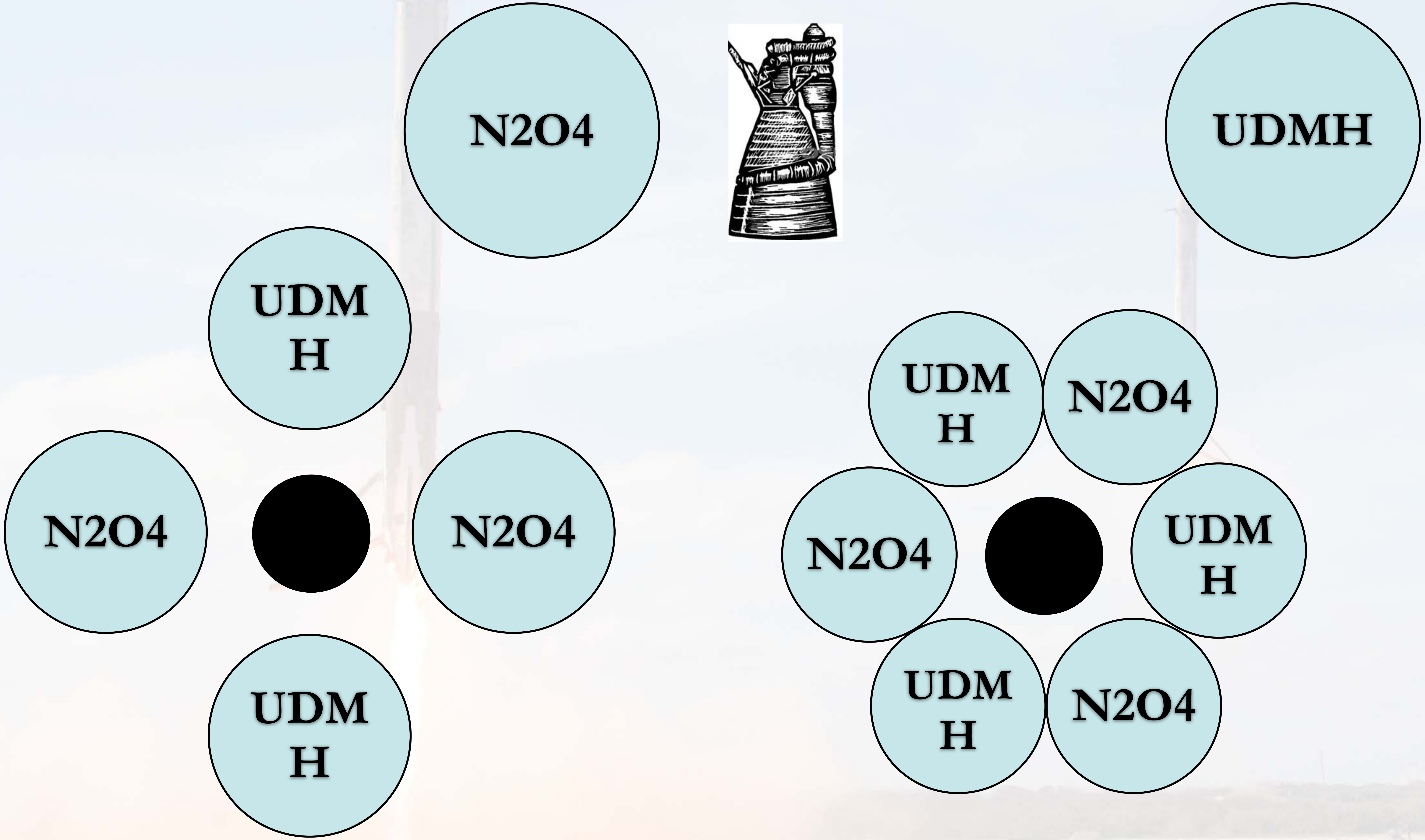
# Orbital Maneuvering Stage (OMS)

- Gross mass 6950 kg
  - Inert mass 695 kg
  - Propellant mass 6255 kg
  - Mixture ratio $N_2O_4$/UDMH = 2.0 (by mass)
- $N_2O_4$ tank
  - Mass = 4170 kg
  - Density = 1450 kg/m$^3$
  - Volume = 2.876 m$^3$
- UDMH tank
  - Mass = 2085 kg
  - Density = 793 kg/m$^3$
  - Volume = 2.629 m$^3$

# $N_2O_4$ Tank Sizing

- Need total $N_2O_4$ volume = 2.876 $m^3$
- Single PMD tank
  - Radius = 0.882 m
  - Mass = 102.8 kg
- Dual PMD tanks
  - Radius = 0.700 m
  - Mass = 52.9 kg (x2 = 105.8 kg)
- Triple PMD tanks
  - Radius = 0.612 m

# Tank Configuration Issues



# Other Structural MERs

- Fairings and shrouds

$$M_{fairing}\langle kg\rangle = 4.95\left(A_{fairing}\langle m^2\rangle\right)^{1.15}$$

- Avionics

$$M_{avionics}\langle kg\rangle = 10\left(M_o\langle kg\rangle\right)^{0.361}$$

- Wiring

$$M_{wiring}\langle kg\rangle = 1.058\sqrt{M_o\langle kg\rangle}\,\ell^{0.25}$$

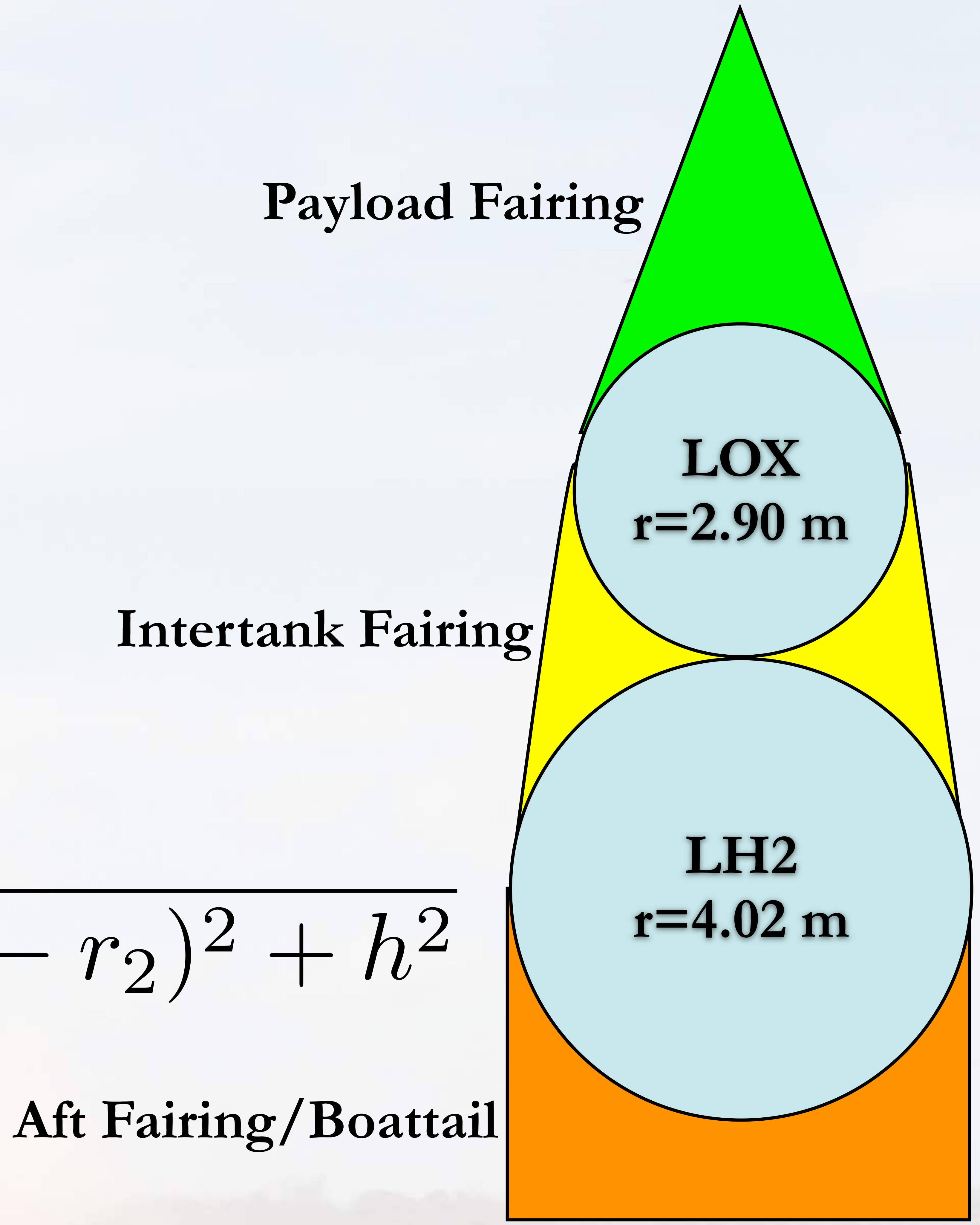
# External Fairings - First Cut

- Masses
  - LOX Tank 1245 kg
  - LOX Tank Insulation 119 kg
  - $LH_2$ Tank 2482 kg
  - $LH_2$ Tank Insulation 586 kg

$$A_{cone} = \pi r \sqrt{r^2 + h^2}$$

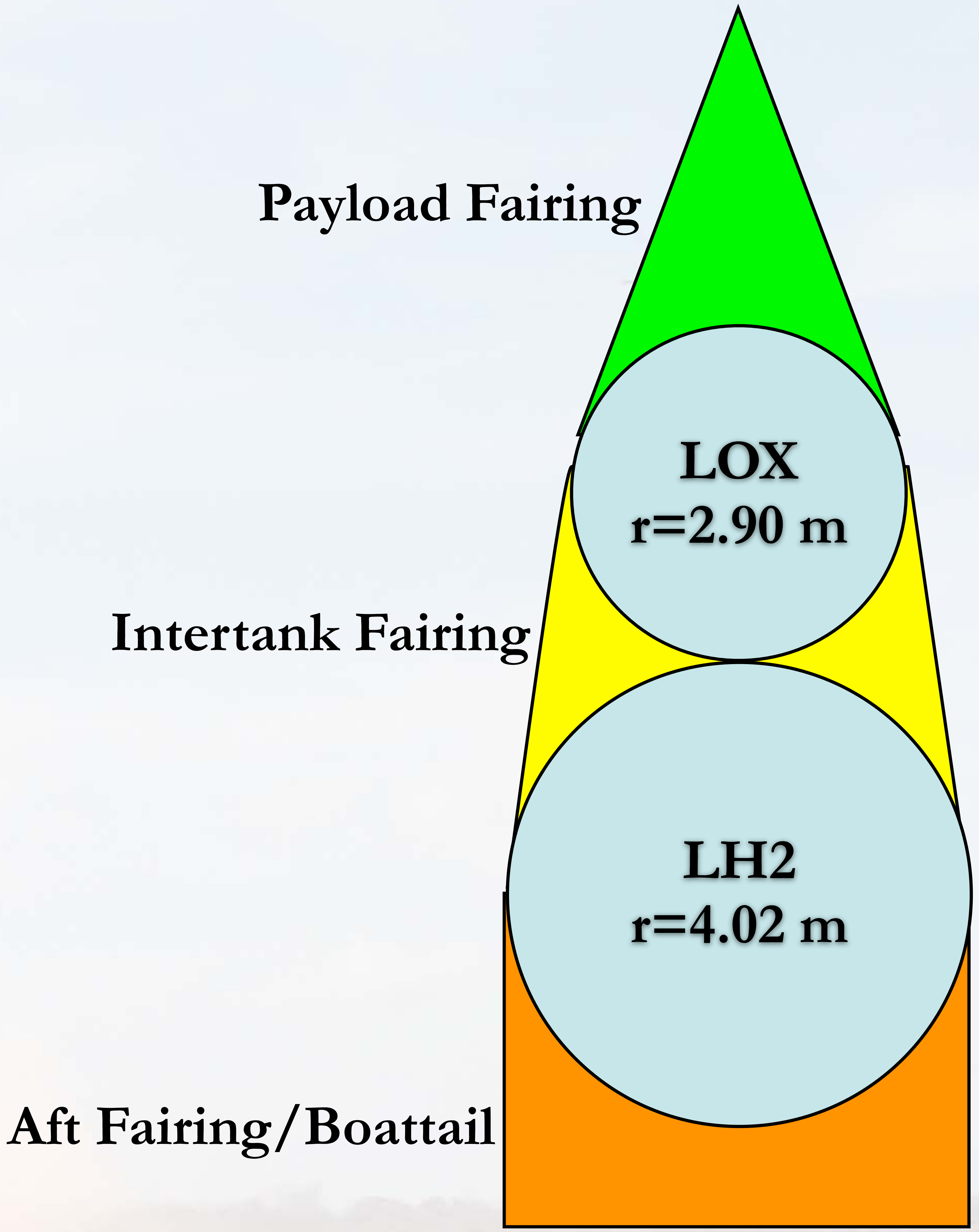
$$A_{frustrum} = \pi (r_1 + r_2) \sqrt{(r_1 - r_2)^2 + h^2}$$

$$A_{cylinder} = 2\pi r h$$



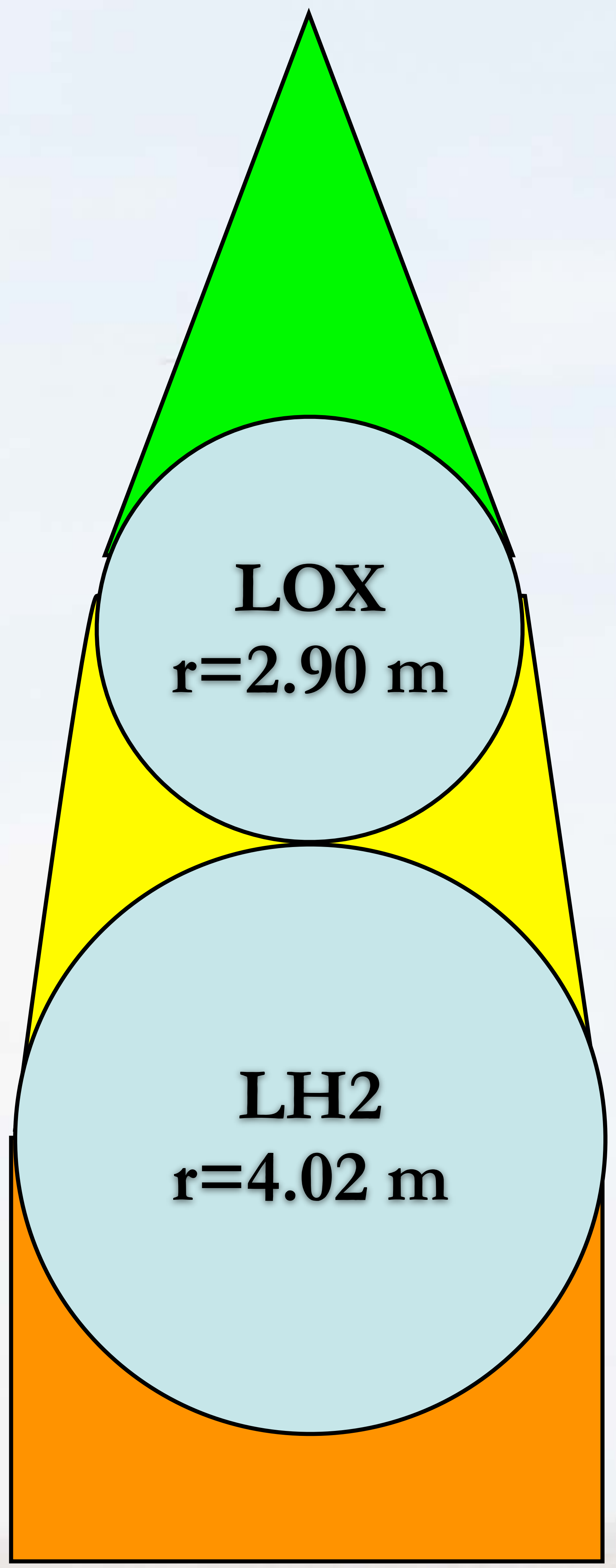
# External Fairings - First Cut

- Assumptions
  - P/L fairing $h$ — 7 m
  - P/L fairing $r$ — 2.9 m
  - I/T fairing $h$ — 7 m
  - I/T fairing $r_1$ — 4.02 m
  - I/T fairing $r_2$ — 2.9 m
  - Aft fairing $h$ — 7 m
  - Aft fairing $r$ — 4.02 m



# Fairing Analysis

- Payload Fairing
  - Area 69.03 $m^2$
  - Mass 645 kg
- Intertank Fairing
  - Area 154.1 $m^2$
  - Mass 1624 kg
- Aft Fairing
  - Area 176.8 $m^2$



# Avionics and Wiring Masses

- Avionics

$$M_{avionics}\langle kg\rangle = 10\left(153,000\right)^{0.361} = 744\ kg$$

- Wiring

$$M_{wiring}\langle kg\rangle = 1.058\sqrt{153,000}(21\ m)^{0.25} = 886\ kg$$

# Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass

$$M_{Rocket\ Engine}(kg) = 7.81\times 10^{-4}T(N) + 3.37\times 10^{-5}T(N)\sqrt{\frac{A_e}{A_t}} + 59$$

- Solid Rocket Motor

$$M_{Motor\ Casing} = 0.135 M_{propellants}$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55\times 10^{-4}T(N)$$

# Propulsion MERs (continued)

- Gimbal Mass

$$M_{Gimbals}(kg) = 237.8\left[\frac{T(N)}{P_0(Pa)}\right]^{.9375}$$

- Gimbal Torque

$$\tau_{Gimbals}(N \cdot m) = 990{,}000\left[\frac{T(N)}{P_0(Pa)}\right]^{1.25}$$

# Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out

$$\frac{5}{6}(1.3) = 1.083 > 1$$

- Chamber pressure = 1000 psi = 6897 kPa
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio $A_e/A_t$=30

# Propulsion Mass Estimates

- Rocket Engine Thrust (each)

$$T(N) = \frac{m_0 g (T/W)_0}{n_{engines}} = 324{,}900\ N$$

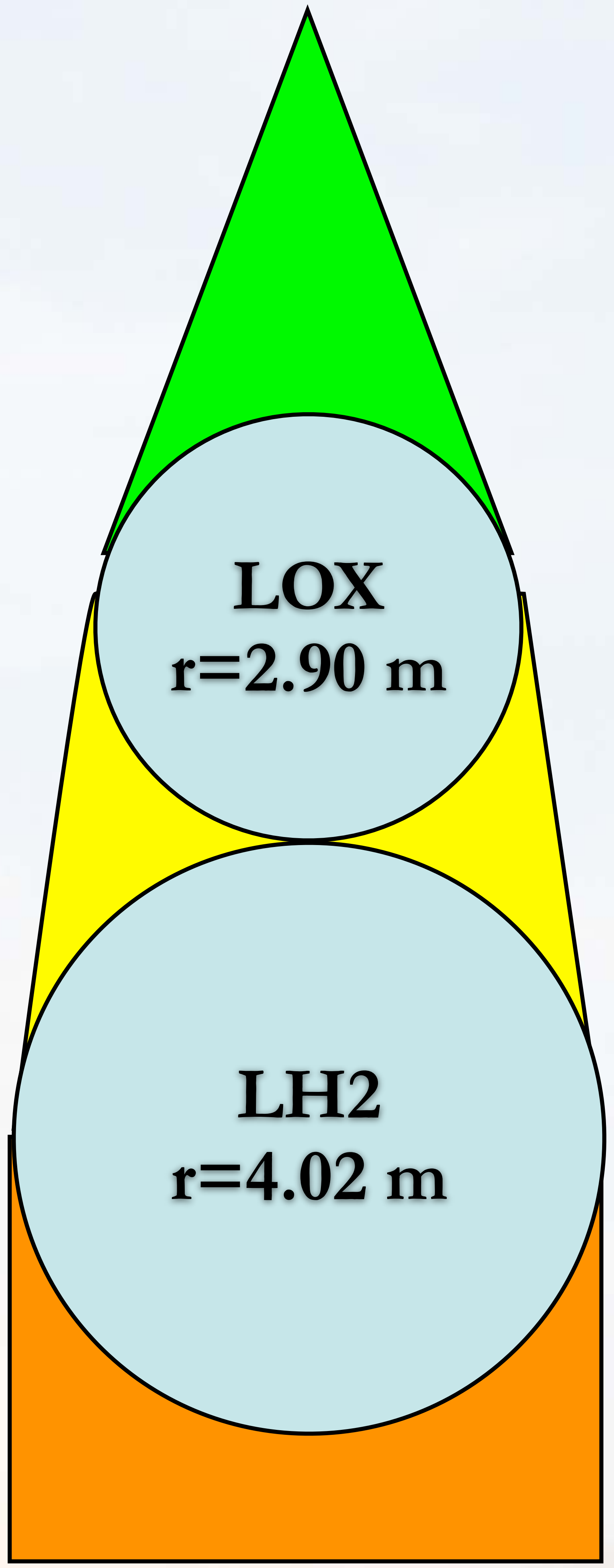
- Rocket Engine Mass (each)

$$M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4}(324{,}900) + 3.37 \times 10^{-5}(324{,}900)\sqrt{30} + 59 = 373\ kg$$

- Thrust Structure Mass

$$M_{Thrust\ \ Structure}(kg) = 2.55 \times 10^{-4}(324{,}900) = 82.8\ kg$$

# First Pass Vehicle Configuration

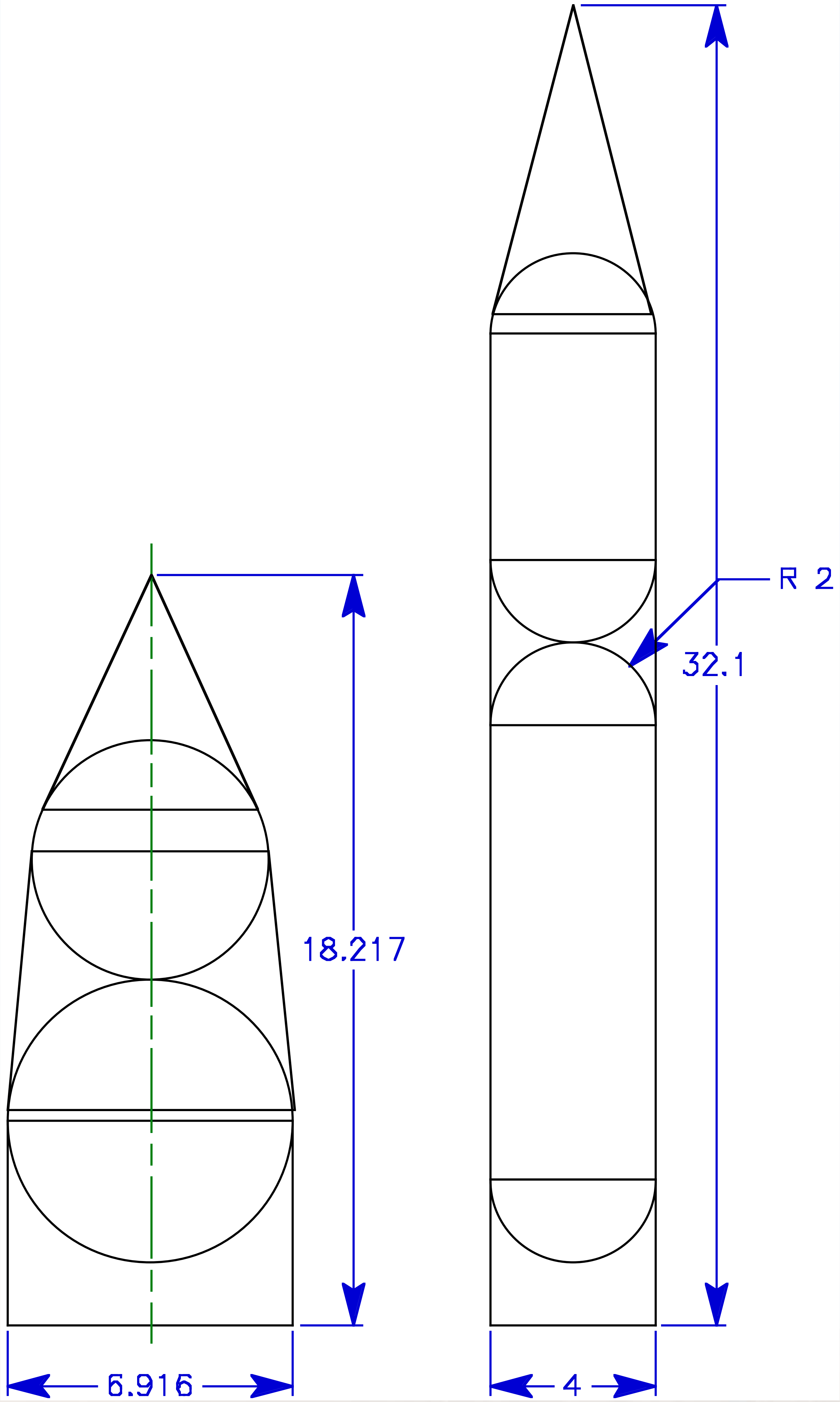


# Mass Summary - First Pass

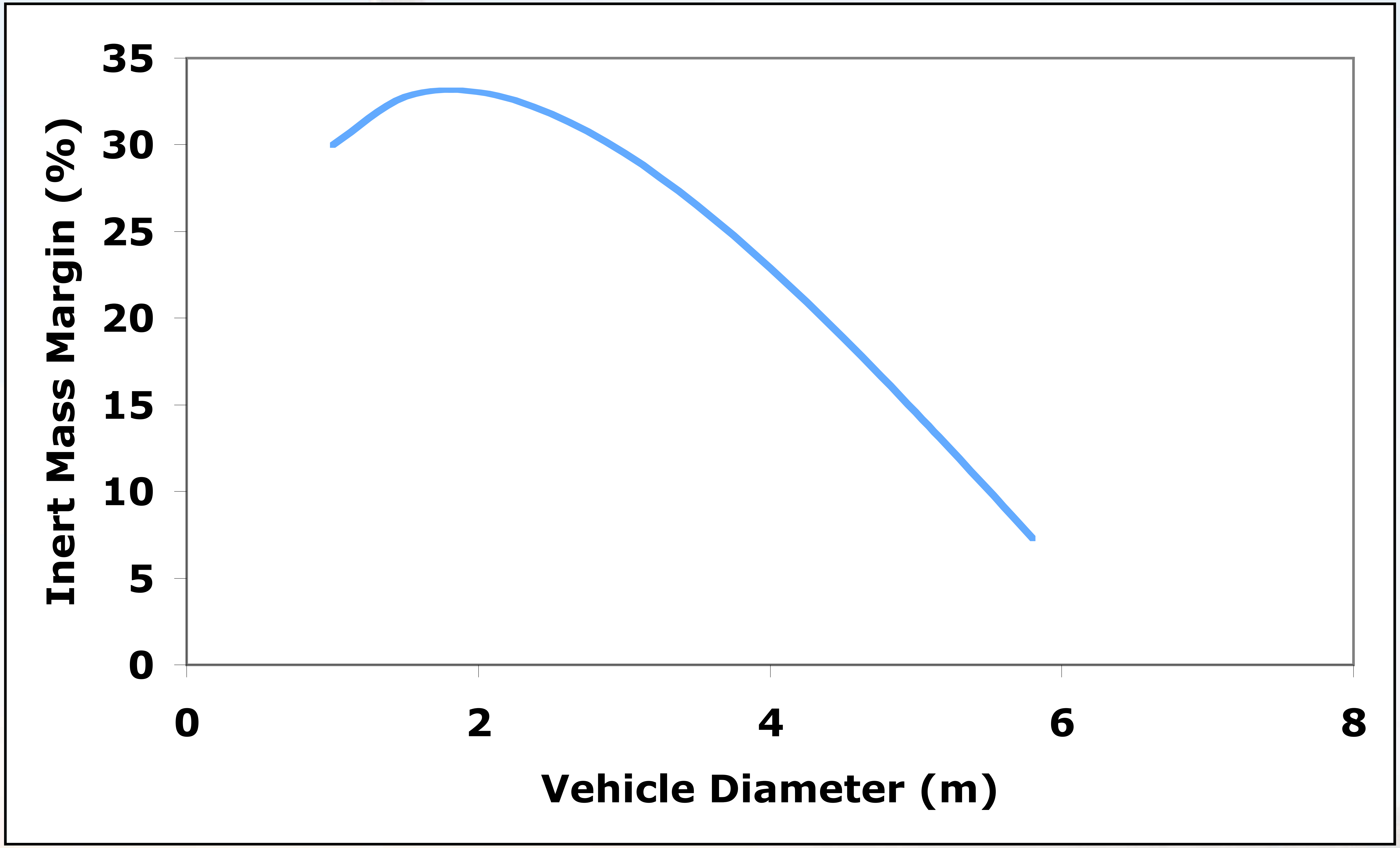
| | |
|---|---|
| Initial Inert Mass Estimate | **12,240 kg** |
| LOX Tank | 1245 kg |
| LH2 Tank | 2482 kg |
| LOX Insulation | 119 kg |
| LH2 Insulation | 586 kg |
| Payload Fairing | 645 kg |
| Intertank Fairing | 1626 kg |
| Aft Fairing | 1905 kg |
| Engines | 2236 kg |
| Thrust Structure | 497 kg |
| Gimbals | 81 kg |
| Avionics | 744 kg |
| Wiring | 886 kg |
| Reserve | - |
| Total Inert Mass | **13,052 kg** |
| Design Margin | **-6.22 %** |

# Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis



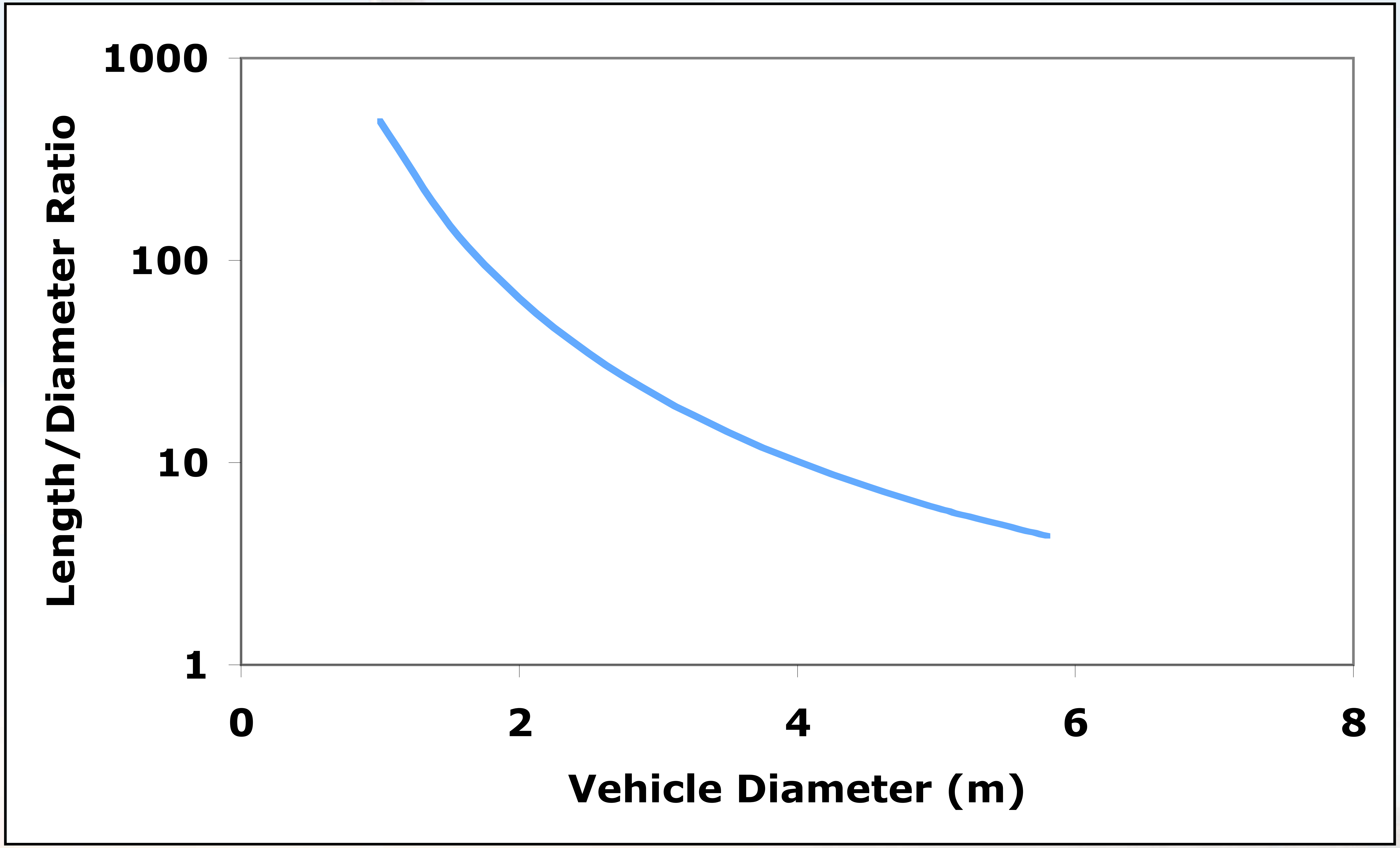
# Effect of Vehicle Diameter on Mass Margin



# Effect of Mass-Optimal Diameter Choice

- Mass-optimal vehicle has diameter=1.814 m
- Mass margin goes from -6.22% to +33.1%
- Vehicle length=155 m
- Length/diameter ratio=86 – approximately equivalent to piece of spaghetti
- No volume for six rocket engines in aft fairing
- Infeasible configuration

# Effect of Diameter on Vehicle L/D



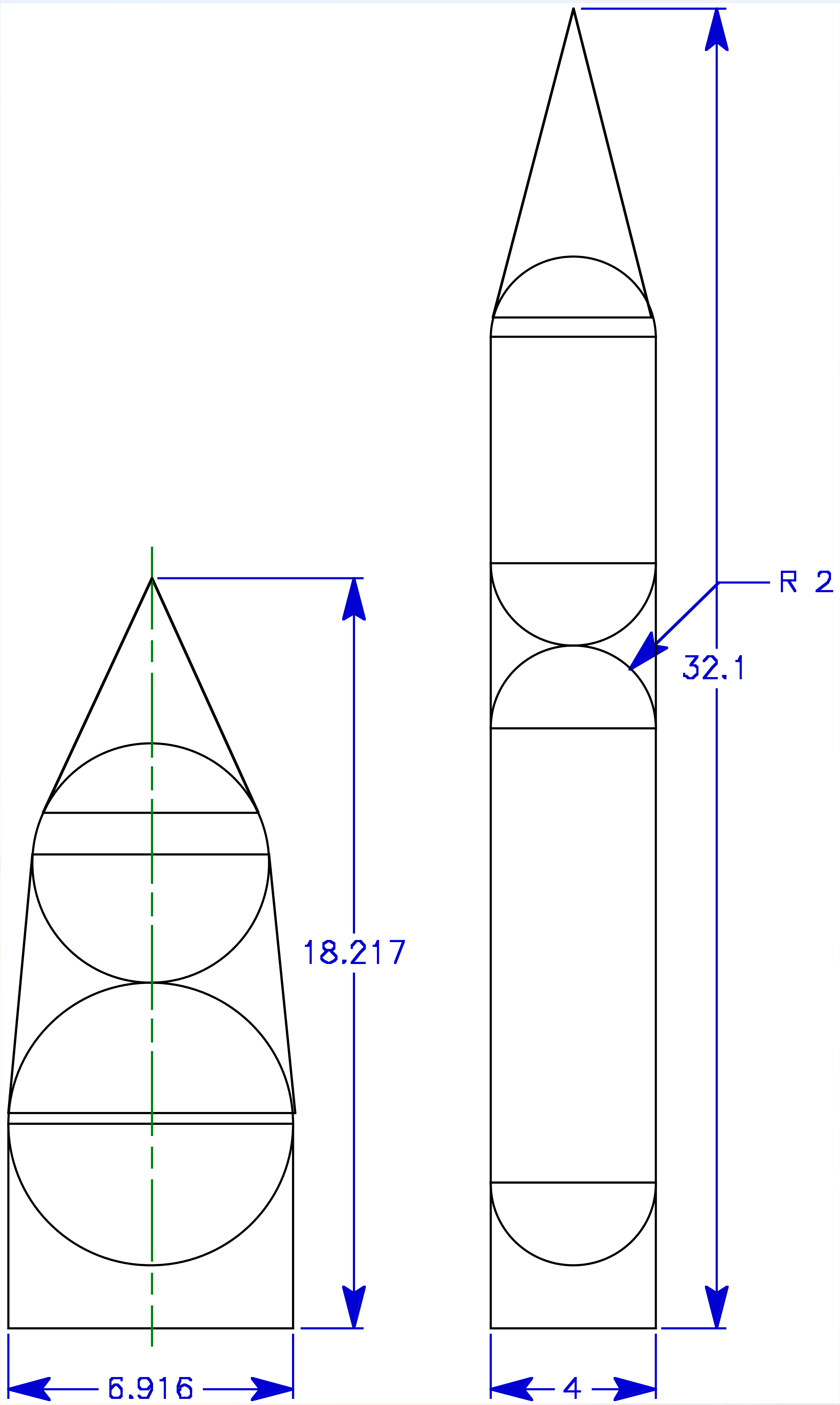
# S-IC Barge Delivery (10m diameter)

# S-IVB Air Transport (7m diameter)

# Atlas/Delta Delivery System (4-5m diam)

# SpaceX Falcon 9 Delivery (3.7m diam)

# Second Pass Vehicle Configuration

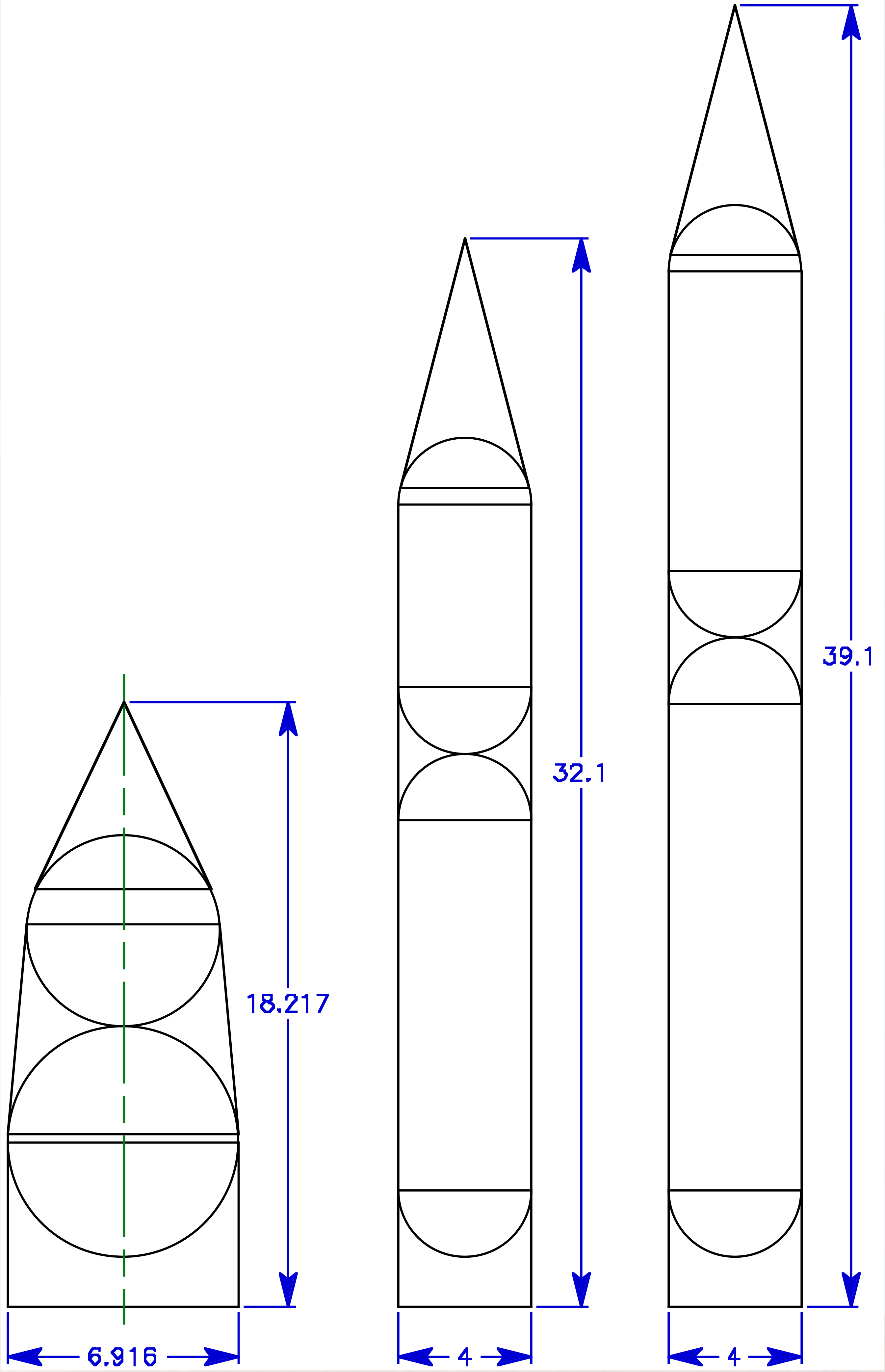


# Mass Summary - Second Pass

| | | |
|---|---|---|
| Initial Inert Mass Estimate | **12,240 kg** | **12,240 kg** |
| LOX Tank | 1245 kg | 1245 kg |
| LH2 Tank | 2482 kg | 2482 kg |
| LOX Insulation | 119 kg | 56 kg |
| LH2 Insulation | 586 kg | 145 kg |
| Payload Fairing | 645 kg | 402 kg |
| Intertank Fairing | 1626 kg | 448 kg |
| Aft Fairing | 1905 kg | 579 kg |
| Engines | 2236 kg | 2236 kg |
| Thrust Structure | 497 kg | 497 kg |
| Gimbals | 81 kg | 81 kg |
| Avionics | 744 kg | 744 kg |
| Wiring | 886 kg | 1044 kg |
| Reserve | - | - |
| Total Inert Mass | **13,052 kg** | **9960 kg** |
| Design Margin | **-6.22** % | **+22.9** % |

# Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in $m_0$ and $m_i$, to reach 30% mass margin
- Modify diameter to keep L/D≤10 and iterate again for optimal initial mass estimate



# Vehicle-Level Prelim Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- ΔV=9200 m/sec
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec (Ve=4214 m/sec)
  - δ=0.08323
- Diameter=4.2 m
- L/D=9.7

$$r = e^{-\frac{\Delta V}{V e}} = 0.1127$$

$$\lambda = r - \delta = 0.0294$$

$$M_o = \frac{M_\ell}{\lambda} = 169,800\ kg$$

$$M_i = \delta M_o = 14,130\ kg$$

$$M_p = M_o(1-r) = 150,700\ kg$$

# Mass Summary - Third Pass

| | | | |
|---|---|---|---|
| Initial Inert Mass Estimate | 12,240 kg | 12,240 kg | 14,130 kg |
| LOX Tank | 1245 kg | 1245 kg | 1382 kg |
| LH2 Tank | 2482 kg | 2482 kg | 2755 kg |
| LOX Insulation | 119 kg | 56 kg | 62 kg |
| LH2 Insulation | 586 kg | 145 kg | 160 kg |
| Payload Fairing | 645 kg | 402 kg | 427 kg |
| Intertank Fairing | 1626 kg | 448 kg | 501 kg |
| Aft Fairing | 1905 kg | 579 kg | 626 kg |
| Engines | 2236 kg | 2236 kg | 2443 kg |
| Thrust Structure | 497 kg | 497 kg | 552 kg |
| Gimbals | 81 kg | 81 kg | 90 kg |
| Avionics | 744 kg | 744 kg | 773 kg |
| Wiring | 886 kg | 1044 kg | 1101 kg |
| Reserve | - | - | – |
| Total Inert Mass | 13,052 kg | 9960 kg | 10,870 kg |
| Design Margin | -6.22 % | +22.9 % | +30.0 % |

# Mass Budgeting

| | Estimates | Budgeted | Margins |
|---|---|---|---|
| Initial Inert Mass Estimate | 14,131 kg | 14,131 kg | |
| LOX Tank | 1382 kg | 1589 kg | 207 kg |
| LH2 Tank | 2755 kg | 3168 kg | 413 kg |
| LOX Insulation | 62 kg | 72 kg | 9 kg |
| LH2 Insulation | 160 kg | 184 kg | 24 kg |
| Payload Fairing | 427 kg | 491 kg | 64 kg |
| Intertank Fairing | 501 kg | 576 kg | 75 kg |
| Aft Fairing | 626 kg | 720 kg | 94 kg |
| Engines | 2443 kg | 2809 kg | 366 kg |
| Thrust Structure | 552 kg | 634 kg | 83 kg |
| Gimbals | 90 kg | 103 kg | 13 kg |
| Avionics | 773 kg | 889 kg | 116 kg |
| Wiring | 1101 kg | 1267 kg | 165 kg |
| Reserve | – | 1630 kg | |
| Total Inert Mass | 10,870 kg | 12,500 kg | |

# References


- C. R. Glatt, WAATS - *A Computer Program for Weights Analysis of Advanced Transportation Systems* NASA CR-2420, September 1974.
- I. O. MacConochie and P. J. Klich, *Techniques for the Determination of Mass Properties of Earth-to-Orbit Transportation Systems* NASA TM-78661, June 1978.
- Willie Heineman, Jr., *Fundamental Techniques of Weight Estimating and Forecasting for Advanced Manned Spacecraft and Space Stations* NASA TN-D-6349, May 1971
- Willie Heineman, Jr., *Mass Estimation and Forecasting for Aerospace Vehicles Based on Historical Data* NASA JSC-26098, November 1994