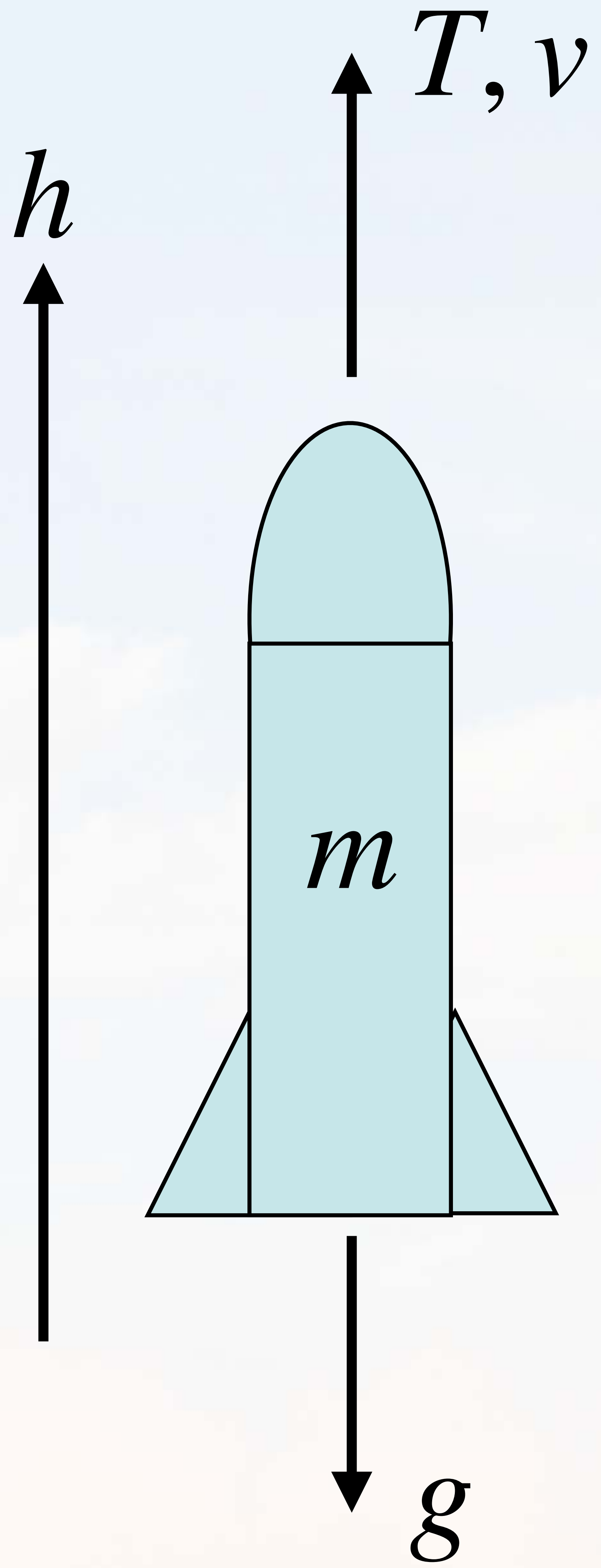
Sounding Rocket Trajectory Analysis

- Adding thrust to the equations of motion
- Vertical trajectory
- Effect of thrust duration

- Note: No in-class lectures Thursday 4/4 or Tuesday 4/9 – work on term projects

Sounding Rocket Analysis



$$\frac{dh}{dt} = v$$

$$m\frac{dv}{dt} = T - mg \qquad T = \dot{m}v_e, \quad \dot{m} = -\frac{dm}{dt}$$

$$m\frac{dv}{dt} = -\frac{dm}{dt}v_e - mg$$

$$dv = -v_e\frac{dm}{m} - g\ dt$$

Integrating Accelerations for Velocity

$$\int_0^v dv = -g\int_0^t dt - v_e\int_{m_0}^{m}\frac{dm}{m}$$

Assume that thrust is constant $\rightarrow \dot{m}$ is constant

$$m = m_0 - \dot{m}t \implies t = \frac{m_0 - m}{\dot{m}}$$

$$v = -gt - v_e \ln\frac{m}{m_0} = \frac{dh}{dt}$$

Gravity loss

Integrating Velocity for Altitude

$$\frac{dh}{dt} = -gt - v_e \ln \frac{m_0 - \dot{m}t}{m_0}$$

$$\int_0^h dh = -g\int_0^t t dt - v_e \int_0^t \ln \frac{m_0 - \dot{m}t}{m_0} dt$$

$$\text{Let } u = \frac{m_0 - \dot{m}t}{m_0} \quad \Longrightarrow \quad du = -\frac{\dot{m}}{m_0} dt$$

$$\text{Limits: } t = 0 \Rightarrow u = 1;\; t = t_f \Rightarrow u_f = \frac{m_f}{m_0}$$

Solving for Altitude at Burnout

$$\int_0^{h_{bo}} dh = -g\int_0^{t_{bo}} t\,dt + \frac{v_e m_0}{\dot{m}}\int_1^{u_f} \ln u\,du$$

$$h_{bo} = -\frac{1}{2}g t_{bo}^2 + \frac{v_e m_0}{\dot{m}}\left[u\ln u - u\right]_1^{u_f}$$

$$h_{bo} = -\frac{1}{2}g t_{bo}^2 + \frac{v_e}{\dot{m}}\left[m_f \ln\frac{m_f}{m_0} - m_f + m_0\right]$$

$$\dot{m} = \frac{m_0 - m_f}{t_{bo}}$$

Maximizing Burnout Altitude

$$v_{bo} = -gt_{bo} - v_e \ln \frac{m_f}{m_0}$$

$$h_{bo} = -\frac{1}{2} g t_{bo}^2 + \frac{v_e t_{bo}}{m_0 - m_f} \left[m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right]$$

$$\frac{\partial h_{bo}}{\partial t_{bo}} = -gt_{bo} + \frac{v_e}{m_0 - m_f} \left[m_f \ln \frac{m_f}{m_0} - m_f + m_0 \right] = 0$$

For maximum burnout altitude, $t_{bo} \longrightarrow 0$

Solving for Height Attained After Burnout

$$\text{Energy } E = \frac{1}{2}mv^2 + mgh = \text{constant}$$

$$\frac{1}{2}mv_{bo}^2 + mgh_{bo} = mgh_{max}$$

$$\frac{1}{2g}\left(-gt_{bo} - v_e \ln\frac{m_f}{m_0}\right)^2 - \frac{1}{2}gt_{bo}^2 + \frac{v_e t_{bo}}{m_0 - m_f}\left(m_f \ln\frac{m_f}{m_0} - m_f + m_0\right) = h_{max}$$

Maximum Altitude

Simplifying assumption: as $t_{bo} \rightarrow 0, h_{bo} \rightarrow 0$

$$\frac{1}{2g} v_{bo}^2 \approx h_{max}$$

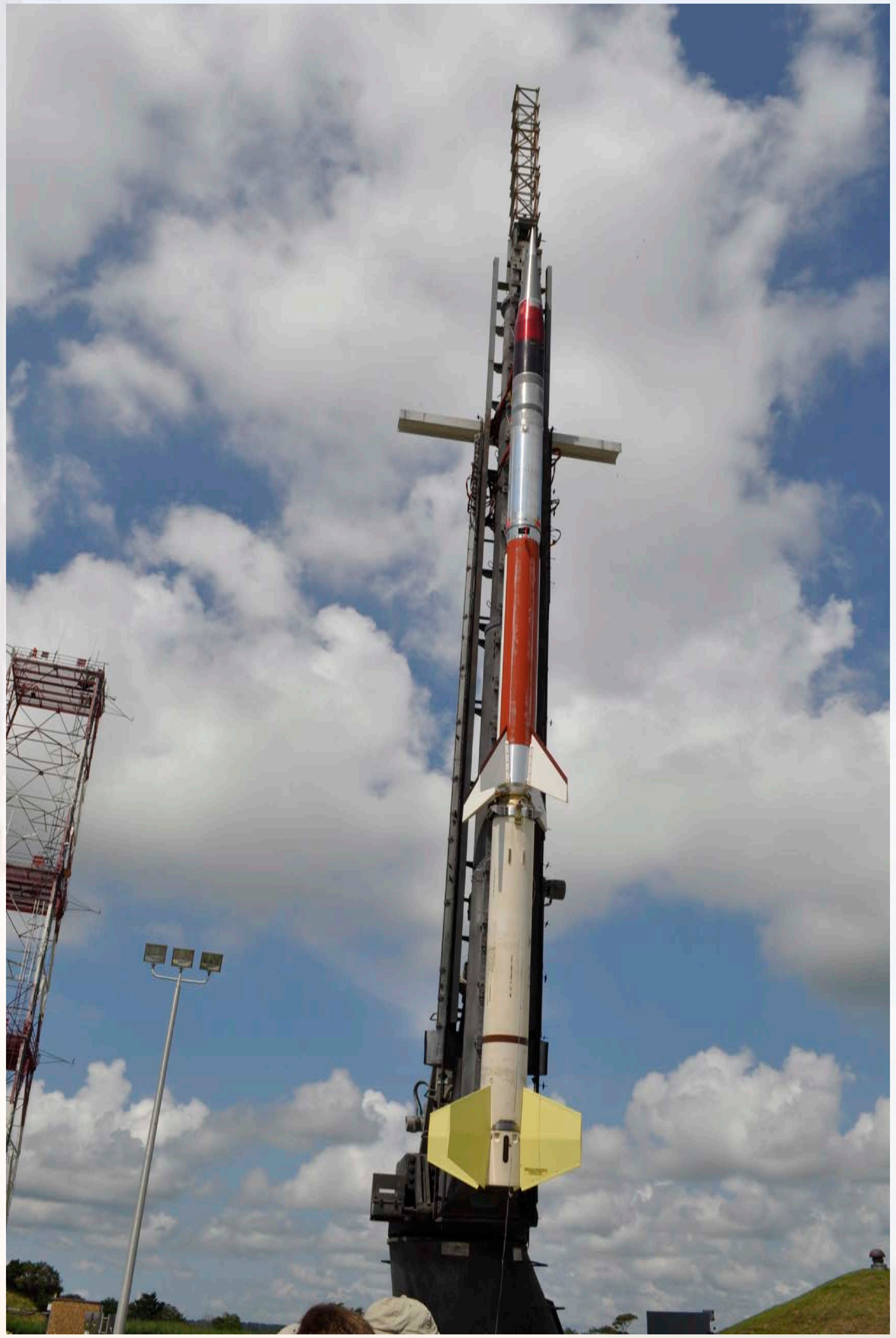
$$\frac{v_e^2}{2g} \ln^2\left(\frac{m_f}{m_0}\right) \approx h_{max}$$

Example: $\frac{m_f}{m_0} = 0.2, \quad v_e = 3000 \ \frac{m}{sec} \Longrightarrow h_{max} = 1189 \ km$

Variation of Altitude with Burn Time

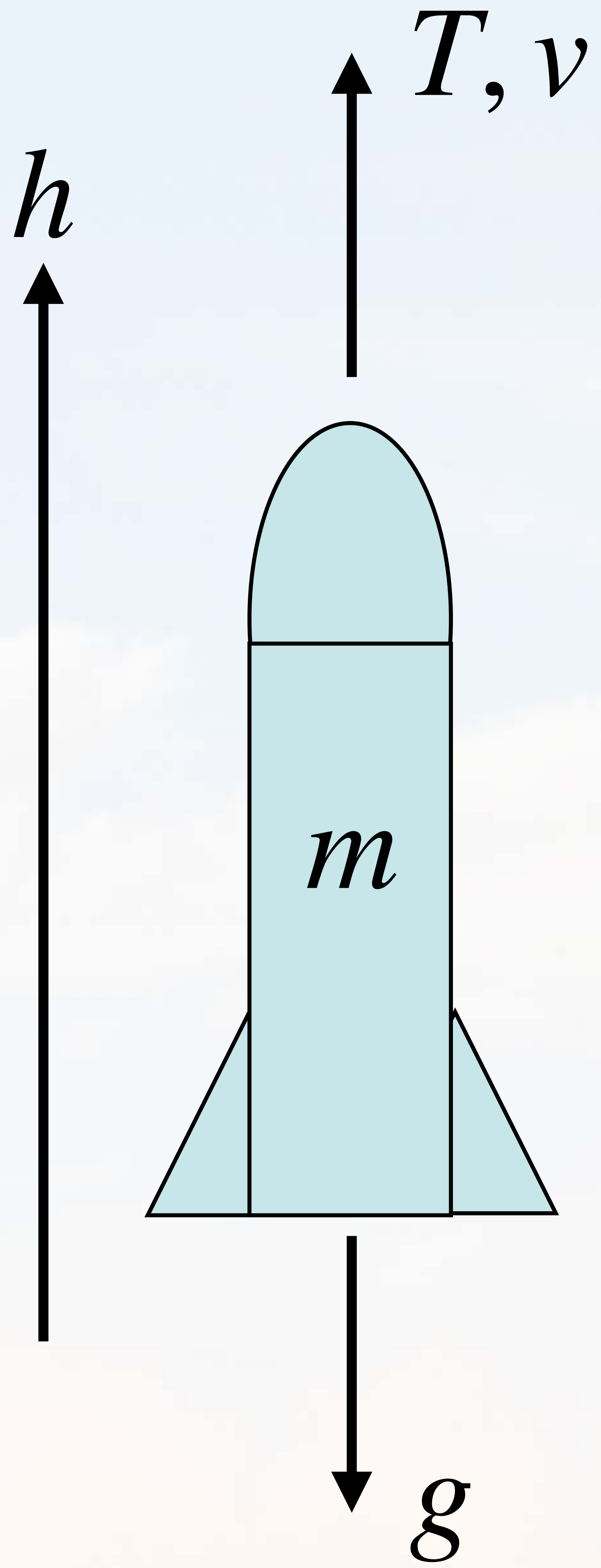
t_{bo} (sec)	v_{bo} (m/sec)	h_{bo} (km)	h_{max} (km)
0	4828	0	1189
10	4730	17	1142
20	4632	34	1095
30	4534	49	1049
40	4436	64	1004
50	4338	77	960
60	4240	90	917

Rocksat-X 2018 (Nike-Terrier)



Rocksat-X Flight 8/14/2018

Sounding Rocket Analysis, This Time with Drag



$$\frac{dh}{dt} = v$$

$$T = \dot{m} v_e$$

$$\dot{m} = -\frac{dm}{dt}$$

$$m\frac{dv}{dt} = T - mg - D$$

$$D = \frac{1}{2}\rho v^2 A c_D$$

$$m\frac{dv}{dt} = -\frac{dm}{dt}v_e - mg - \frac{1}{2}\rho v^2 A c_D$$

$$m\frac{dv}{dt} = -\frac{dm}{dt}v_e - mg - \frac{1}{2}\rho_o e^{-h/h_s} v^2 A c_D$$

No analytical solution possible - numerical integration required