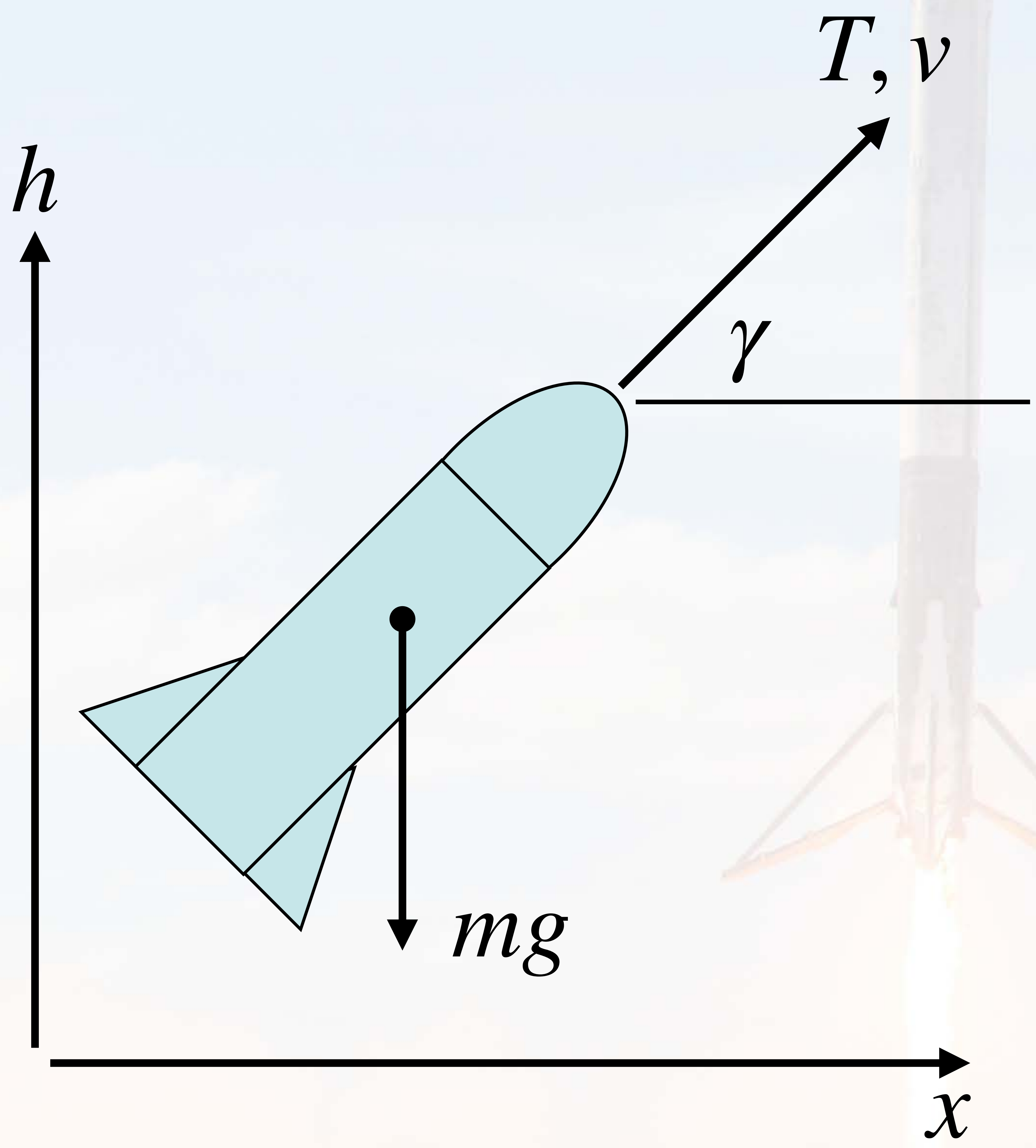
Launch Vehicle Trajectory Analysis

- Gravity turns
- Estimating gravity and other losses
- Full launch trajectories (coming soon!)

Gravity Turn Trajectories

- Keep thrust vector aligned with velocity vector
- Let gravity affect the trajectory as it will
- Aim for insertion state appropriate to the desired mission

Sounding Rocket Analysis

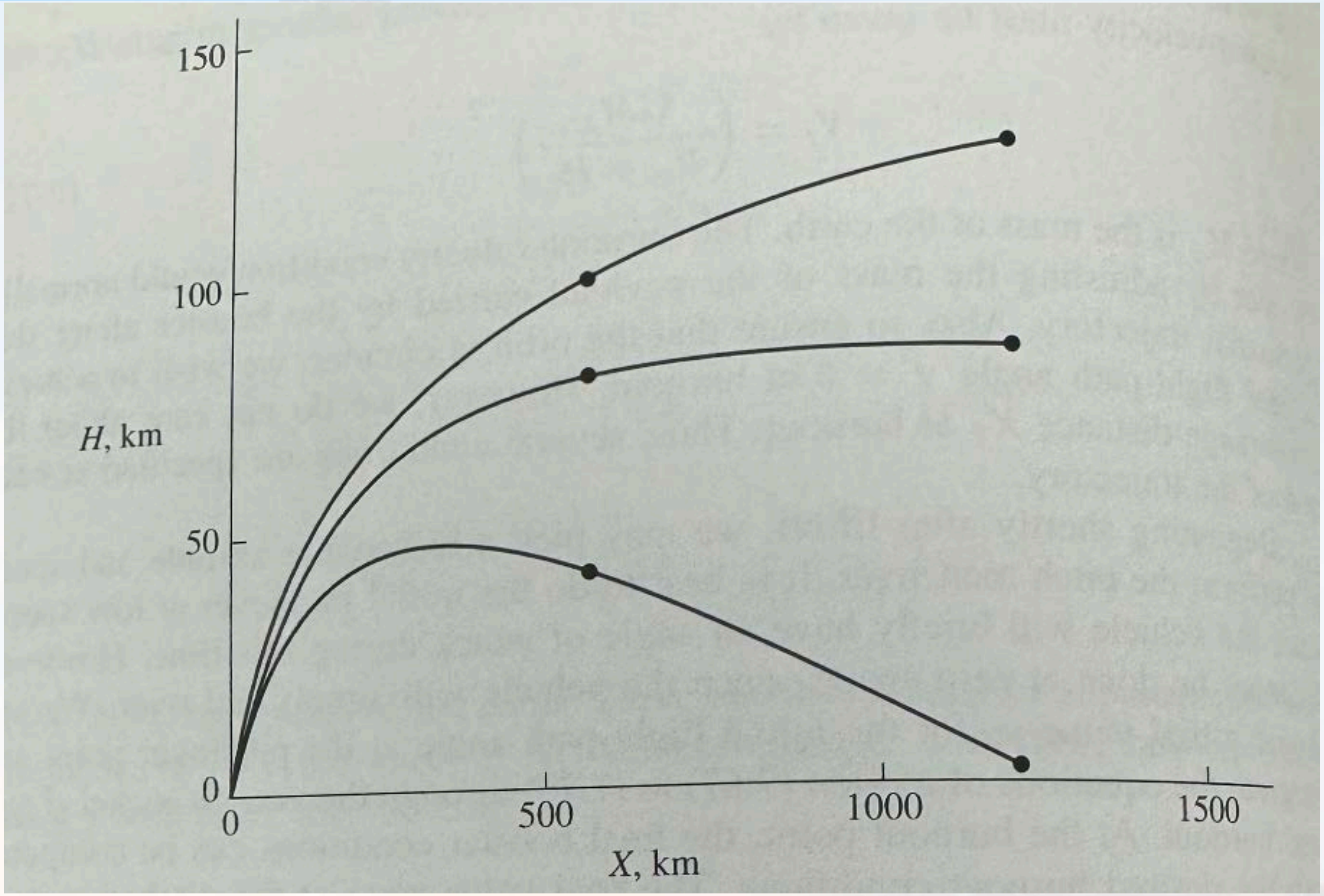


$$\frac{dh}{dt} = v\cos\gamma \qquad \frac{dx}{dt} = v\sin\gamma$$

$$m\frac{dv}{dt} = T - D - \left(mg - \frac{m\dot{x}^2}{r_0 + h}\right)\sin\gamma$$

$$mv\frac{d\gamma}{dt} = -\left(mg - \frac{m\dot{x}^2}{r_0 + h}\right)\cos\gamma$$

Gravity Turn Trajectories



from Wiesel, Spaceflight Dynamics

Estimating Gravity Losses

Start from conservation of energy

$$-\frac{\mu}{r_{final}} = \frac{\Delta v^2_{grav\ loss}}{2} - \frac{\mu}{r_{init}}$$

$$\Delta v_{grav\ loss} = \sqrt{2\mu\left(\frac{1}{r_{init}} - \frac{1}{r_{final}}\right)}$$

starting from sea level, $r_{init} \Rightarrow r_0 \quad r_{final} \Rightarrow r_0 + h$

Gravity Loss by Energy Conservation (cont.)

$$\Delta v_{grav\ loss,SL} = \sqrt{2\mu\left(\frac{r_{final} - r_{init}}{r_{init} r_{final}}\right)} = \sqrt{2\mu\left(\frac{h}{r_0(r_0 + h)}\right)}$$

$$\mu = gr^2 = g_0 r_0^2$$

$$\Delta v_{grav\ loss,SL} = \sqrt{\frac{2g_0 r_0 h}{r_0 + h}} = \sqrt{\frac{2g_0 h}{1 + h/r_0}}$$

Gravity Loss by Energy Conservation (cont.)

$$\Delta v_{grav\ loss,SL} = \sqrt{\frac{2g_0 r_0 h}{r_0 + h}} = \sqrt{\frac{2g_0 h}{1 + h/r_0}}$$

Falcon 9 launch into 200 km insertion orbit

$$\Delta v_{grav\ loss,SL} = \sqrt{\frac{2(0.0098\ km/sec^2)\ 200\ km}{1 + (200\ km/6378\ km)}} = 1.950\ \frac{km}{sec}$$

This approach is conservative - a better estimate would be to multiply by 0.8

Gravity Loss by Exact Methods

$$\Delta v_{\text{gravity loss}} = \int_{t_0}^{t_{final}} g \sin \gamma \mathrm{d}t$$

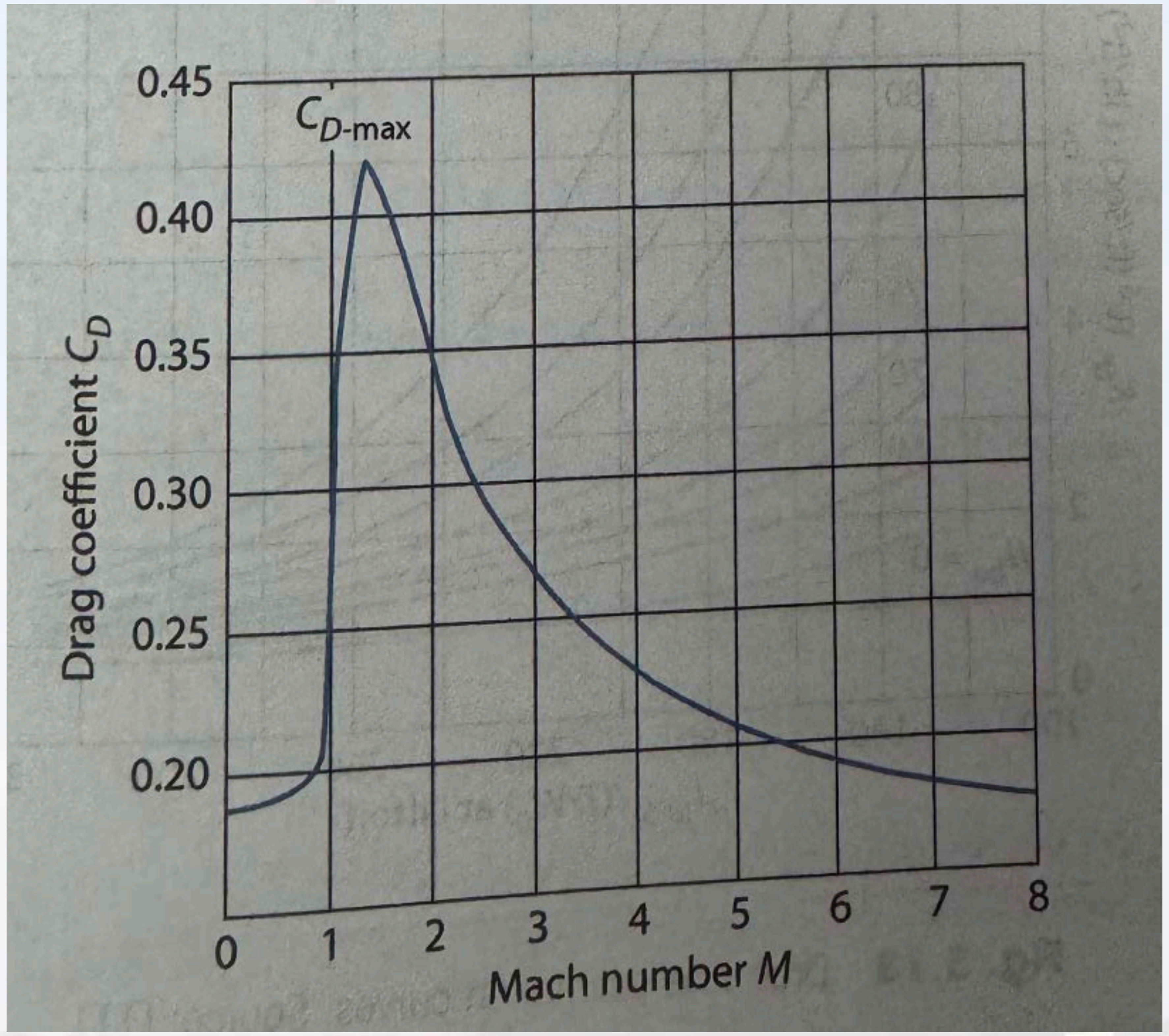
- This equation would be numerically integrated throughout the launch trajectory
- Minimizing gravity loss $\Longrightarrow$ get flight path angle to 0 (horizontal flight) as soon as possible
- But that might conflict with aerodynamic losses!

Drag Losses

$$D = \frac{1}{2}\rho v^2 S_{ref} c_D$$

$$\Delta v_{drag\ loss} = \int_{t_0}^{t_{burn}} \frac{D(t)}{m(t)} \mathrm{d}t$$

- c_D changes with Mach number, and is usually assumed to be a fraction of $c_{D_{max}}$

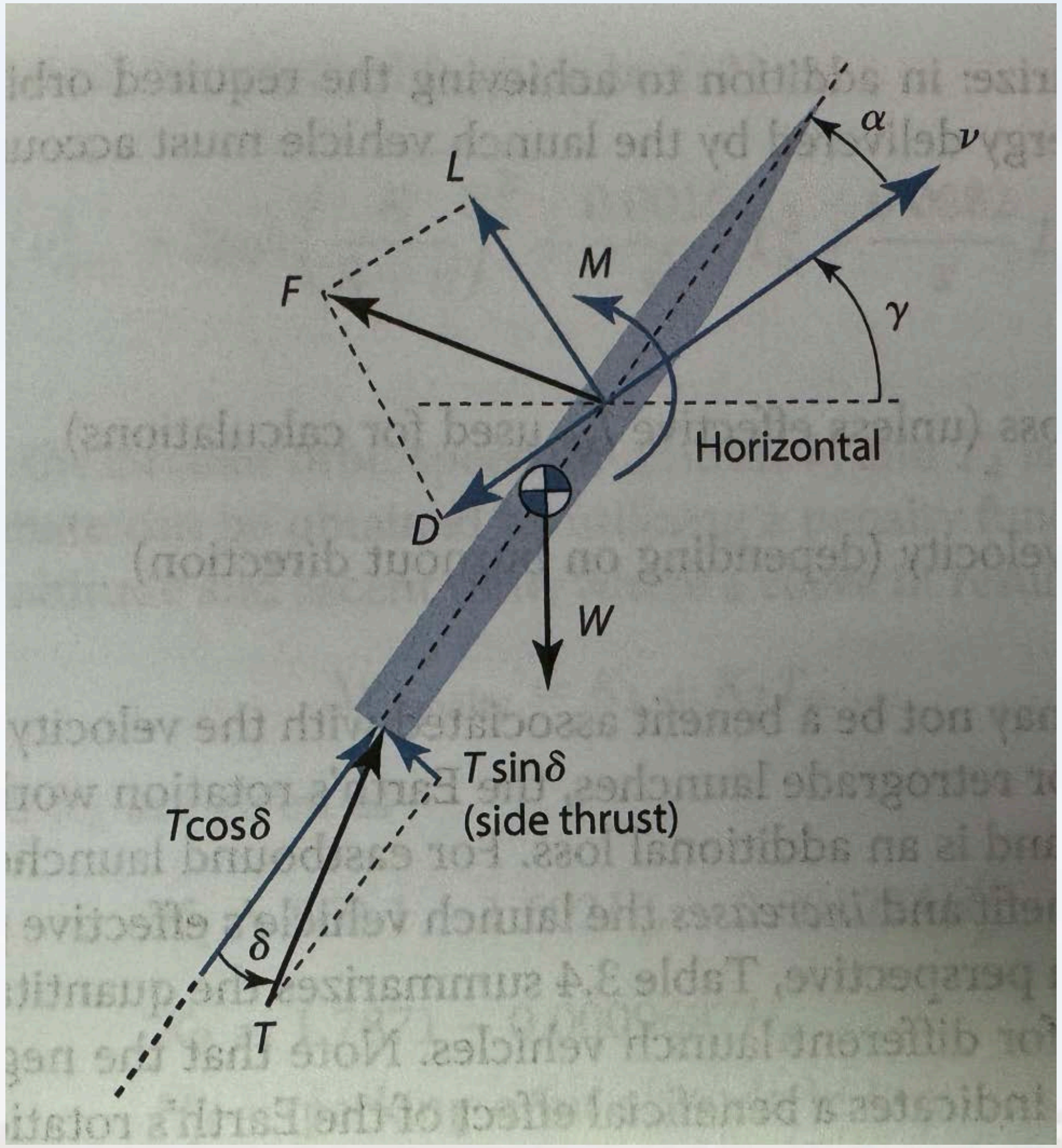


from Edberg and Costa, Design of Rockets and Space Launch Vehicles

Steering Losses

- When thrust is misaligned with velocity vector, there is an impact on Δv
- Sources could be angle of attack α or thrust steering angle δ

$$\Delta v_{steering\ loss} = \int_0^{t_b} T\nu[1 - \cos(\delta + \alpha)]dt$$



from Edberg and Costa, Design of Rockets and Space Launch Vehicles

Δ*v* Summary for Saturn V Launching to LEO

from Edberg and Costa, Design of Rockets and Space Launch Vehicles

Sample Summary of Launch Vehicle Δv Losses

Saturn V Launch to Translunar Injection

Stage	Δv_{ideal}	Gravity Loss		Drag Loss		Steering Loss		Δv_{actual}	$\frac{\Delta v_{actual}}{\Delta v_{ideal}}$
	m/s	m/s	%	m/s	%	m/s	%	m/s	%
1	4,923	1,219	25.0	46	0.9	0	0	3,658	74
2	5,242	335	6.4	0	0	183	3.5	4,724	90
3	4,242	122	2.9	0	0	5	0.1	4,115	97
Total	14,407	1,676	34.3	46	0.9	188	3.6	12,497	86.7

from Edberg and Costa, Design of Rockets and Space Launch Vehicles

Δv Summaries for Several Launch Vehicles

Vehicle	$h_p \times h_a$	i, deg	v_{LEO}	Δv_{grav}	Δv_{steer}	Δv_{drag}	Δv_{rot}*	$\Sigma(\Delta v)$
Atlas I	149 × 607	7.0	7,946	1,395	167	110	−345†	9,243
Delta 7925	175 × 319	33.9	7,842	1,150	33	136	−347	8,814
Space Shuttle	−196 × 278	28.5	7,794‡	1,222	358†	107	−345†	9,086**
Saturn V	176 × 176	28.5	7,798	1,534	243	40	−348	9,267
Titan IV/Centaur	157 × 463	28.6	7,896	1,442	65	156	−352	9,207

Source: [10].

*Negative sign indicates beneficial effect of Earth's rotation.

†Δv_{rot} values in [10] are given as 375 for Atlas I and 395 for the Shuttle, and may be in error. Total Δvs shown include the corrected values for Δv_{rot}.

‡Injection occurs at approximately 111 km.

**Additional $\Delta v = 144$ m/s needed to circularize orbit at apoapsis height $h_a = 278$ km.

from Edberg and Costa, Design of Rockets and Space Launch Vehicles