Mass Estimating Relationships

- Review of iterative design approach
- Mass Estimating Relationships (MERs)
- Sample vehicle design analysis
Overview of the Design Process

Program Objectives 🟰 System Requirements

Vehicle-level Estimation (based on a few parameters from prior art)

System-level Estimation (system parameters based on prior experience)

System-level Design (based on discipline-oriented analysis)
Vehicle-Level Preliminary Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - $Isp=430$ sec
  - $\delta=0.08$

\[
\frac{-\Delta V}{gI_{sp}} = 0.1127 \\
\lambda = r - \delta = 0.0327 \\
M_0 = \frac{M_L}{\lambda} = 153,000 \text{ kg} \\
M_i = \delta M_0 = 12,240 \text{ kg} \\
M_p = M_0 (1 - r) = 135,800 \text{ kg}
\]
• Start with propellant tanks (biggest part)
• LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
• Propellant densities

\[
\rho_{\text{LOX}} = 1140 \frac{\text{kg}}{\text{m}^3} \quad \rho_{\text{LH}_2} = 112 \frac{\text{kg}}{\text{m}^3}
\]
LOX Tank MERs

- Mass of Tank
  
  \[ M_{\text{LOX Tank}}(\text{kg}) = 0.0152 M_{\text{LOX}}(\text{kg}) + 318 \]

- Mass of Insulation
  
  \[ M_{\text{LOX Insulation}}(\text{kg}) = 1.123 \frac{\text{kg}}{\text{m}^2} \]
LOX Tank Design

- Mass of LOX = 116,400 kg
  \[ M_{LOX\ Tank}(kg) = 0.0152(116,400) + 318 = 2087\ kg \]

- Need area to find LOX tank insulation mass
  - assume a sphere

\[ V_{LOX\ Tank} = \frac{M_{LOX}}{\rho_{LOX}} = 102.1\ m^3 \]

\[ r_{LOX\ Tank} = \left( \frac{V_{LOX}}{\frac{4\pi}{3}} \right)^{\frac{1}{3}} = 2.90\ m \]

\[ A_{LOX\ Tank} = 4\pi r_{LOX}^2 = 105.6\ m^2 \]

\[ M_{LOX\ Insulation}(kg) = 1.123 \frac{kg}{m^2} \left( 105.6\ m^2 \right) = 119\ kg \]
LH2 Tank MERs

- **Mass of Tank**

\[
M_{LH_2\, Tank}(kg) = 0.0694 M_{LH_2}(kg) + 363
\]

- **Mass of Insulation**

\[
M_{LH_2\, Insulation}(kg) = 2.88 \frac{kg}{m^2}
\]
**LH2 Tank Design**

- **Mass of LH2** = 19,390 kg

\[
M_{LH_2\ Tank}(kg) = 0.0694(19,390) + 363 = 1709\text{ kg}
\]

- **Again, assume LH2 tank is spherical**

\[
V_{LH_2\ Tank} = \frac{M_{LH_2}}{\rho_{LH_2}} = 346.3\ m^3
\]

\[
r_{LH_2\ Tank} = \left(\frac{V_{LH_2}}{\frac{4\pi}{3}}\right)^{\frac{1}{3}} = 3.46\ m
\]

\[
A_{LH_2\ Tank} = 4\pi r_{LH_2}^2 = 150.2\ m^2
\]

\[
M_{LH_2\ Insulation}(kg) = 2.88 \frac{kg}{m^2}(150.2\ m^2) = 433\ kg
\]
Current Design Sketch

- **Masses**
  - LOX Tank 2089 kg
  - LOX Tank Insulation 1709 kg
  - LH2 Tank 119 kg
  - LH2 Tank Insulation 433 kg
Other Tankage MERs

- Storable Propellants (RP-1, N$_2$O$_4$, N$_2$H$_4$)

\[ M_{\text{Storables Tank}}(\text{kg}) = 0.316 [ M_{\text{Storables}}(\text{kg}) ]^6 \]

- Small tank (liquids)

\[ M_{\text{Small Liquid Tank}}(\text{kg}) = 0.1 M_{\text{contents}}(\text{kg}) \]

- Small tank (pressurized gases)

\[ M_{\text{Small Gas Tank}}(\text{kg}) = 2 M_{\text{contents}}(\text{kg}) \]
Boost Module Propellant Tanks

• Gross mass 23,000 kg
  – Inert mass 2300 kg
  – Propellant mass 20,700 kg
  – Mixture ratio N₂O₄/A50 = 1.8 (by mass)

• N₂O₄ tank
  – Mass = 13,310 kg
  – Density = 1450 kg/m³
  – Volume = 9.177 m³ --> r_{sphere}=1.299 m

• Aerozine 50 tank
  – Mass = 7390 kg
  – Density = 900 kg/m³
  – Volume = 8.214 m³ --> r_{sphere}=1.252 m
**N₂O₄ Tank Sizing**

- Need total N₂O₄ volume = 9.177 m³
- Single tank
  - Radius = 1.299 m
  - Mass = 94.2 kg
- Dual tanks
  - Radius = 1.031 m
  - Mass = 62.2 kg (x2 = 124.3 kg)
- Triple tanks
  - Radius = 0.900 m
  - Mass = 48.7 kg (x3 = 146.2 kg)
Other Structural MERs

- Fairings and shrouds

\[ M_{\text{Fairing}}(\text{kg}) = 13.3 \frac{\text{kg}}{m^2} \]

- Avionics

\[ M_{\text{Avionics}}(\text{kg}) = 10 \left[ M_0(\text{kg}) \right]^{0.361} \]

- Wiring

\[ M_{\text{Wiring}}(\text{kg}) = 1.058 \sqrt{M_0(\text{kg})} \ell^{0.25} \]
Fairing Analysis

- **Payload Shroud**
  - Area: 51.74 m²
  - Mass: 688 kg

- **Intertank Fairing**
  - Area: 126.1 m²
  - Mass: 1677 kg

- **Aft Fairing**
  - Area: 107.7 m²
  - Mass: 1433 kg
Avionics and Wiring Masses

- Avionics

\[ M_{\text{Avionics}}(\text{kg}) = 10 \times [153,000 \text{ kg}]^{0.361} = 744 \text{ kg} \]

- Wiring

\[ M_{\text{Wiring}}(\text{kg}) = 1.058 \sqrt{153,000} \times (16.95 \text{ m})^{0.25} = 840 \text{ kg} \]
Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass

\[ M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4} T(N) + 3.37 \times 10^{-5} T(N) \sqrt{\frac{A_e}{A_t}} + 59 \]

- Solid Rocket Motor

\[ M_{Motor\ Casing} = 0.135 M_{propellants} \]

- Thrust Structure Mass

\[ M_{Thrust\ Structure}(kg) = 2.55 \times 10^{-4} T(N) \]
Propulsion MERs (continued)

- **Gimbal Mass**

\[ M_{Gimbals} (kg) = 237.8 \left[ \frac{T(N)}{P_0(Pa)} \right]^{0.9375} \]

- **Gimbal Torque**

\[ \tau_{Gimbals} (N \cdot m) = 990,000 \left[ \frac{T(N)}{P_0(Pa)} \right]^{1.25} \]
Propulsion System Assumptions

• Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling

• Number of engines = 6
  - Positive acceleration worst-case after engine out

• Chamber pressure = 1000 psi = 6897 kN
  - Typical for high-performance LOX/LH2 engines

• Expansion ratio $A_e/A_t = 30$
  - Compromise ratio with good vacuum performance
Propulsion Mass Estimates

- Rocket Engine Thrust (each)
  \[ T(N) = \frac{m_0 g (T/W)_0}{n_{\text{engines}}} = 324,900 \text{ N} \]

- Rocket Engine Mass (each)
  \[ M_{\text{Rocket Engine}}(kg) = 7.81 \times 10^{-4} (324,900) + 3.37 \times 10^{-5} (324,900) \sqrt{30} + 59 = 373 \text{ kg} \]

- Thrust Structure Mass
  \[ M_{\text{Thrust Structure}}(kg) = 2.55 \times 10^{-4} (324,900) = 497 \text{ kg} \]
First Pass Vehicle Configuration
## Mass Summary - First Pass

<table>
<thead>
<tr>
<th>Component</th>
<th>Mass Estimate (kg)</th>
</tr>
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<tbody>
<tr>
<td>Initial Inert Mass Estimate</td>
<td>12,240</td>
</tr>
<tr>
<td>LOX Tank</td>
<td>2087</td>
</tr>
<tr>
<td>LH2 Tank</td>
<td>1709</td>
</tr>
<tr>
<td>LOX Insulation</td>
<td>119</td>
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<tr>
<td>LH2 Insulation</td>
<td>433</td>
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<tr>
<td>Payload Fairing</td>
<td>688</td>
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<tr>
<td>Intertank Fairing</td>
<td>1677</td>
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<tr>
<td>Aft Fairing</td>
<td>1433</td>
</tr>
<tr>
<td>Engines</td>
<td>2236</td>
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<td>Thrust Structure</td>
<td>497</td>
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<td>Gimbals</td>
<td>81</td>
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<tr>
<td>Avionics</td>
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<td>Wiring</td>
<td>840</td>
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<td>Reserve</td>
<td>-</td>
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<tr>
<td><strong>Total Inert Mass</strong></td>
<td>12,543</td>
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<tr>
<td><strong>Design Margin</strong></td>
<td><strong>-2.4%</strong></td>
</tr>
</tbody>
</table>
Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis
Effect of Vehicle Diameter on $\delta$
Effect of Mass-Optimal Diameter Choice

- Vehicle has L/D of 25.2 - severe complications from structural dynamics
- Mass margin goes from -2.4% to +18.3%
- Decreased volume for rocket engines in aft fairing
- Infeasible configuration
Effect of Diameter on Vehicle L/D
Second Pass Vehicle Configuration
# Mass Summary - Second Pass

- **Initial Inert Mass Estimate**: 12,240 kg, 12,240 kg
- **LOX Tank**: 2087 kg, 2087 kg
- **LH2 Tank**: 1709 kg, 1709 kg
- **LOX Insulation**: 119 kg, 133 kg
- **LH2 Insulation**: 433 kg, 547 kg
- **Payload Fairing**: 688 kg, 689 kg
- **Intertank Fairing**: 1677 kg, 669 kg
- **Aft Fairing**: 1433 kg, 585 kg
- **Engines**: 2236 kg, 2236 kg
- **Thrust Structure**: 497 kg, 497 kg
- **Gimbals**: 81 kg, 81 kg
- **Avionics**: 744 kg, 744 kg
- **Wiring**: 840 kg, 985 kg
- **Reserve**: -  -
- **Total Inert Mass**: 12,543 kg, 10,960 kg
- **Design Margin**: -2.4%, +11.7%
Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in $m_0$ and $m_i$, to reach 30% mass margin
Vehicle-Level Preliminary Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - $\delta=0.08655$

\[
\begin{align*}
\frac{-\Delta V}{gI_{sp}} &= 0.1127 \\
\lambda &= r - \delta = 0.0261
\end{align*}
\]

\[
M_0 = \frac{M_L}{\lambda} = 191,300 \text{ kg}
\]

\[
M_i = \delta M_0 = 16,560 \text{ kg}
\]

\[
M_p = M_0(1 - r) = 169,800 \text{ kg}
\]
Third Pass Vehicle Configuration

- Third Pass Vehicle Configuration

Dimensions:
- Height: 32.1 cm
- Diameter: 6.916 cm
- Length: 18.217 cm

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Mass Estimating Relationships
Principles of Space Systems Design
### Mass Summary - Third Pass

<table>
<thead>
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<th>Item</th>
<th>1st Pass</th>
<th>2nd Pass</th>
<th>3rd Pass</th>
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<tr>
<td><strong>Initial Inert Mass Estimate</strong></td>
<td>12,240 kg</td>
<td>12,240 kg</td>
<td>16,560 kg</td>
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<tr>
<td>LOX Tank</td>
<td>2087 kg</td>
<td>2087 kg</td>
<td>2530 kg</td>
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<td>LH2 Tank</td>
<td>1709 kg</td>
<td>1709 kg</td>
<td>2046 kg</td>
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<tr>
<td>LOX Insulation</td>
<td>119 kg</td>
<td>133 kg</td>
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<td>433 kg</td>
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<td>Aft Fairing</td>
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<td>622 kg</td>
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<td>100 kg</td>
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<td>Avionics</td>
<td>744 kg</td>
<td>744 kg</td>
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<tr>
<td>Wiring</td>
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<td><strong>Total Inert Mass</strong></td>
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<td>10,960 kg</td>
<td>12,740 kg</td>
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<tr>
<td><strong>Design Margin</strong></td>
<td>-2.4 %</td>
<td>+11.7 %</td>
<td>+30 %</td>
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# Mass Budgeting

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<td>LH2 Tank</td>
<td>2046 kg</td>
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