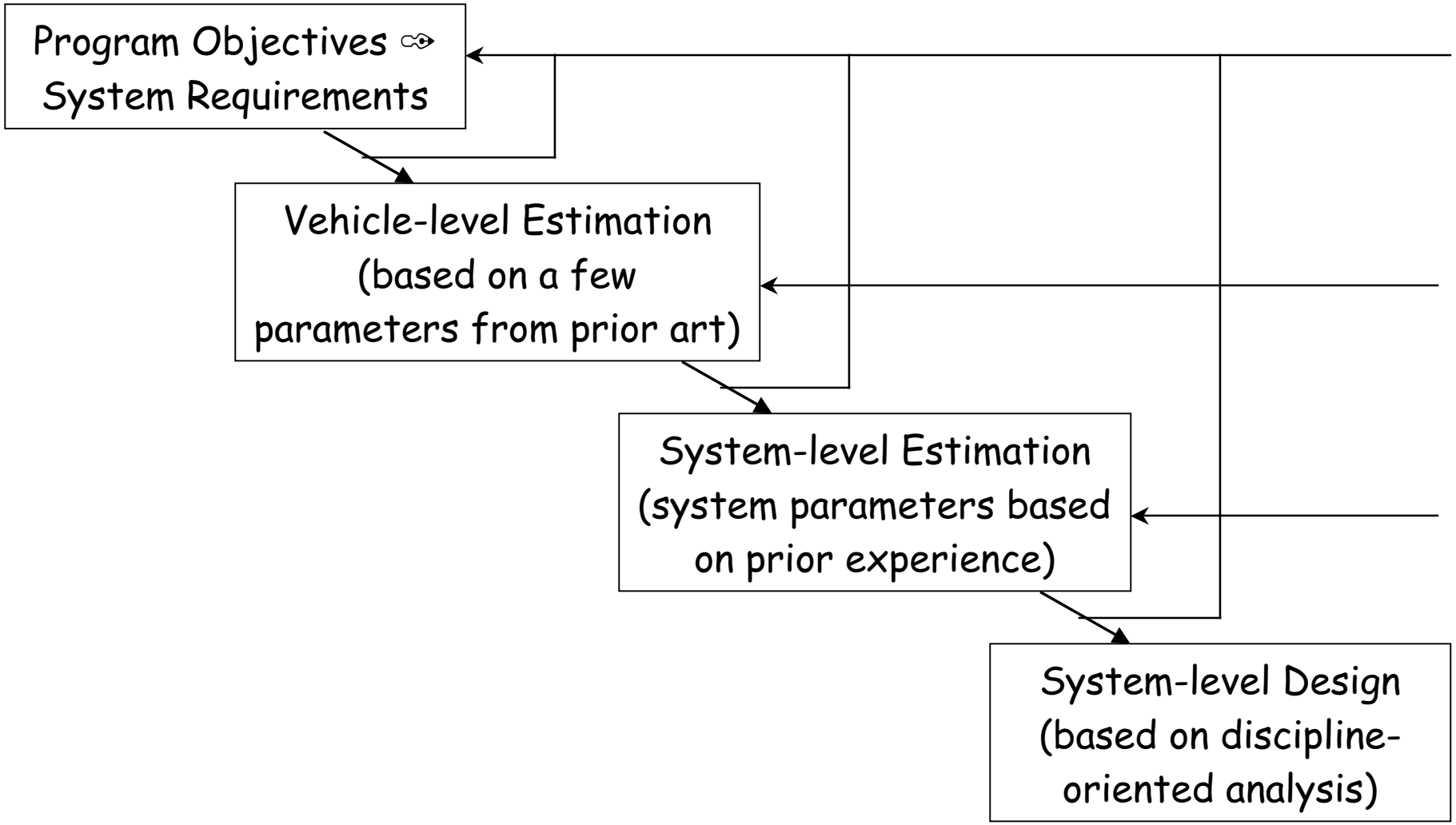
Mass Estimating Relationships

- Review of iterative design approach
- Mass Estimating Relationships (MERs)
- Sample vehicle design analysis

Overview of the Design Process



Vehicle-Level Preliminary Design - 1st Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - δ=0.08

$$r = e^{-\frac{\Delta V}{gI_{sp}}} = 0.1127$$

$$\lambda = r - \delta = 0.0327$$

$$M_0 = \frac{M_L}{\lambda} = 153{,}000\ kg$$

$$M_i = \delta M_0 = 12{,}240\ kg$$

$$M_p = M_0(1 - r) = 135{,}800\ kg$$

System-Level Estimation

- Start with propellant tanks (biggest part)
- LOX/LH2 engines generally run at mixture ratio of 6:1 (by weight)
  - LH2: 19,390 kg
  - LOX: 116,400 kg
- Propellant densities

$$\rho_{LOX} = 1140 \frac{kg}{m^3} \qquad \rho_{LH_2} = 112 \frac{kg}{m^3}$$

LOX Tank MERs

- Mass of Tank

$$M_{LOX\ Tank}(kg) = 0.0152 M_{LOX}(kg) + 318$$

- Mass of Insulation

$$M_{LOX\ Insulation}(kg) = 1.123 \frac{kg}{m^2}$$

LOX Tank Design

- Mass of LOX=116,400 kg

$$M_{LOX\ Tank}(kg) = 0.0152(116,400) + 318 = 2087\ kg$$

- Need area to find LOX tank insulation mass
  - assume a sphere

$$V_{LOX\ Tank} = M_{LOX} / \rho_{LOX} = 102.1\ m^3$$

$$r_{LOX\ Tank} = \left(\frac{V_{LOX}}{4\pi / 3} \right)^{1/3} = 2.90\ m$$

$$A_{LOX\ Tank} = 4\pi r_{LOX}^2 = 105.6\ m^2$$

$$M_{LOX\ Insulation}(kg) = 1.123 \frac{kg}{m^2} \left(105.6\ m^2 \right) = 119\ kg$$

LH2 Tank MERs

- Mass of Tank

$$M_{LH_2\ Tank}(kg) = 0.0694 M_{LH_2}(kg) + 363$$

- Mass of Insulation

$$M_{LH_2\ Insulation}(kg) = 2.88 \frac{kg}{m^2}$$

LH2 Tank Design

- Mass of LH2=19,390 kg

$$M_{LH_2\ Tank}(kg) = 0.0694(19{,}390) + 363 = 1709kg$$

- Again, assume LH2 tank is spherical

$$V_{LH_2\ Tank} = M_{LH_2} / \rho_{LH_2} = 346.3\ m^3$$

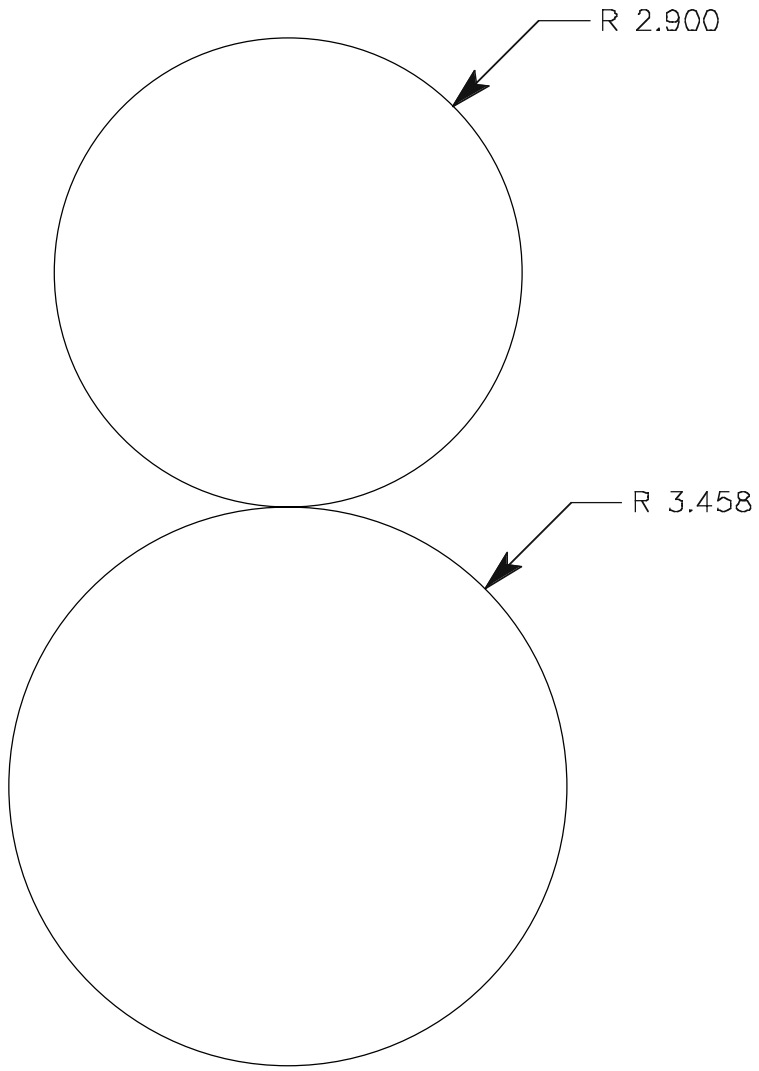
$$r_{LH_2\ Tank} = \left(\frac{V_{LH_2}}{4\pi/3}\right)^{1/3} = 3.46\ m$$

$$A_{LH_2\ Tank} = 4\pi r_{LH_2}^2 = 150.2\ m^2$$

$$M_{LH_2\ Insulation}(kg) = 2.88\frac{kg}{m^2}\left(150.2\ m^2\right) = 433\ kg$$

Current Design Sketch

- Masses
  - LOX Tank 2089 kg
  - LOX Tank Insulation 1709 kg
  - LH2 Tank 119 kg
  - LH2 Tank Insulation 433 kg



Other Tankage MERs

- Storable Propellants (RP-1, N_2O_4, N_2H_4)

$$M_{Storables\ Tank}(kg) = 0.316\left[M_{Storables}(kg)\right]^{.6}$$

- Small tank (liquids)

$$M_{Small\ Liquid\ Tank}(kg) = 0.1M_{contents}(kg)$$

- Small tank (pressurized gases)

$$M_{Small\ Gas\ Tank}(kg) = 2M_{contents}(kg)$$

PROJECT DIANA

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Boost Module Propellant Tanks

- **Gross mass 23,000 kg**
  - **Inert mass 2300 kg**
  - **Propellant mass 20,700 kg**
  - **Mixture ratio N_2O_4/A50 = 1.8 (by mass)**
- **N_2O_4 tank**
  - **Mass = 13,310 kg**
  - **Density = 1450 kg/m^3**
  - **Volume = 9.177 m^3 --> r_{sphere}=1.299 m**
- **Aerozine 50 tank**
  - **Mass = 7390 kg**
  - **Density = 900 kg/m^3**
  - **Volume = 8.214 m^3 --> r_{sphere}=1.252 m**

PROJECT DIANA

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N_2O_4 Tank Sizing

- **Need total N_2O_4 volume = 9.177 m^3**
- **Single tank**
  - **Radius = 1.299 m**
  - **Mass = 94.2 kg**
- **Dual tanks**
  - **Radius = 1.031 m**
  - **Mass = 62.2 kg (x2 = 124.3 kg)**
- **Triple tanks**
  - **Radius = 0.900 m**
  - **Mass = 48.7 kg (x3 = 146.2 kg)**

Other Structural MERs

- Fairings and shrouds

$$M_{Fairing}(kg) = 13.3 \frac{kg}{m^2}$$

- Avionics

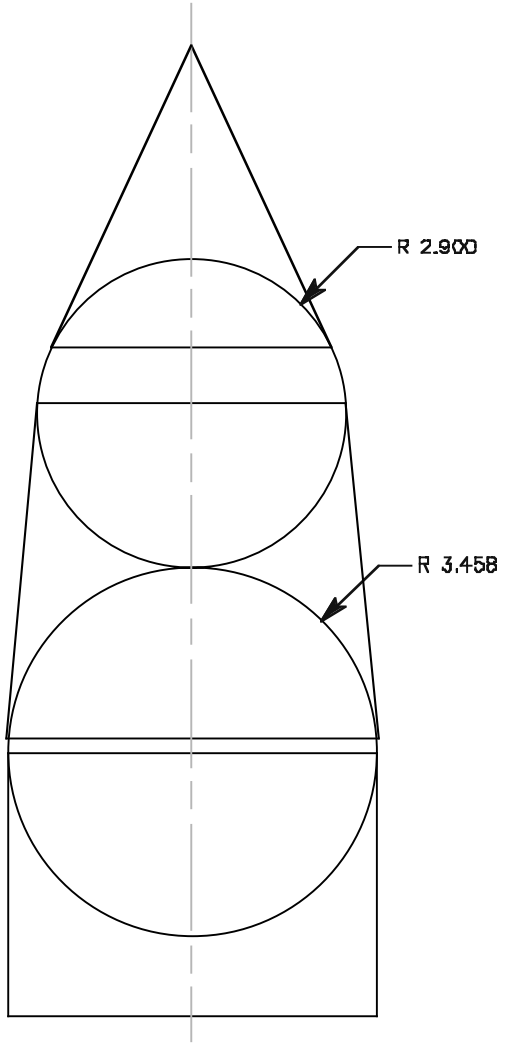
$$M_{Avionics}(kg) = 10\left[M_0(kg)\right]^{0.361}$$

- Wiring

$$M_{Wiring}(kg) = 1.058\sqrt{M_0(kg)}\ \ell^{0.25}$$

Fairing Analysis

- Payload Shroud
  - Area 51.74 m^2
  - Mass 688 kg
- Intertank Fairing
  - Area 126.1 m^2
  - Mass 1677 kg
- Aft Fairing
  - Area 107.7 m^2
  - Mass 1433 kg



Avionics and Wiring Masses

- Avionics

$$M_{Avionics}(kg) = 10\left[153{,}000\ kg\right]^{0.361} = 744\ kg$$

- Wiring

$$M_{Wiring}(kg) = 1.058\sqrt{153{,}000}\,(16.95\ m)^{0.25} = 840\ kg$$

Propulsion MERs

- Liquid Pump-Fed Rocket Engine Mass

$$M_{Rocket\ Engine}(kg) = 7.81\times10^{-4}T(N) + 3.37\times10^{-5}T(N)\sqrt{\frac{A_e}{A_t}} + 59$$

- Solid Rocket Motor

$$M_{Motor\ Casing} = 0.135M_{propellants}$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55\times10^{-4}T(N)$$

Propulsion MERs (continued)

- Gimbal Mass

$$M_{Gimbals}(kg) = 237.8\left\lceil \frac{T(N)}{P_0(Pa)} \right\rceil^{.9375}$$

- Gimbal Torque

$$\tau_{Gimbals}(N \cdot m) = 990{,}000\left\lceil \frac{T(N)}{P_0(Pa)} \right\rceil^{1.25}$$

Propulsion System Assumptions

- Initial T/mg ratio = 1.3
  - Keeps final acceleration low with reasonable throttling
- Number of engines = 6
  - Positive acceleration worst-case after engine out
- Chamber pressure = 1000 psi = 6897 kN
  - Typical for high-performance LOX/LH2 engines
- Expansion ratio A_e/A_t=30
  - Compromise ratio with good vacuum performance

Propulsion Mass Estimates

- Rocket Engine Thrust (each)

$$T(N) = \frac{m_0 g (T/W)_0}{n_{engines}} = 324,900\ N$$

- Rocket Engine Mass (each)

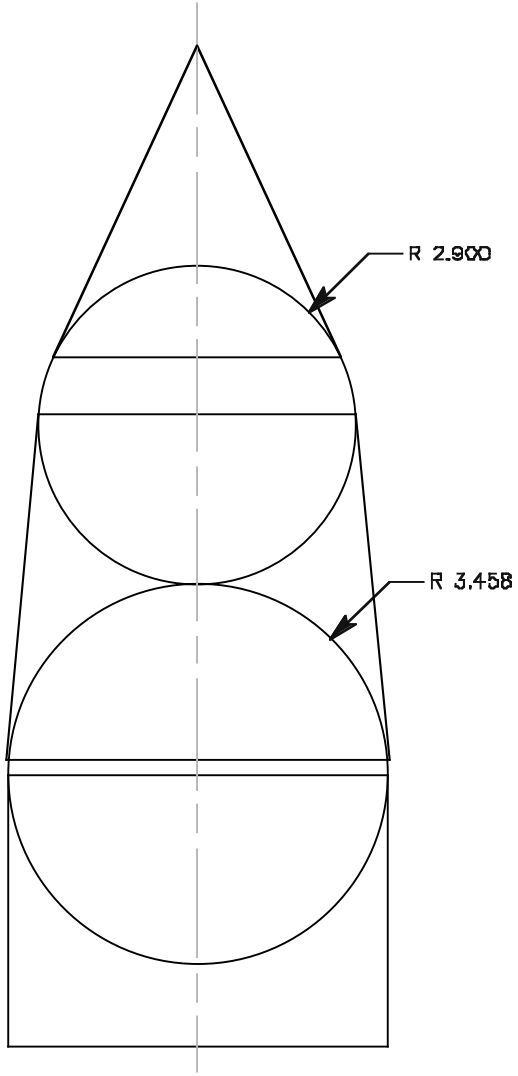
$$M_{Rocket\ Engine}(kg) = 7.81 \times 10^{-4}(324,900) +$$

$$3.37 \times 10^{-5}(324,900)\sqrt{30} + 59 = 373\ kg$$

- Thrust Structure Mass

$$M_{Thrust\ Structure}(kg) = 2.55 \times 10^{-4}(324,900) = 497\ kg$$

First Pass Vehicle Configuration

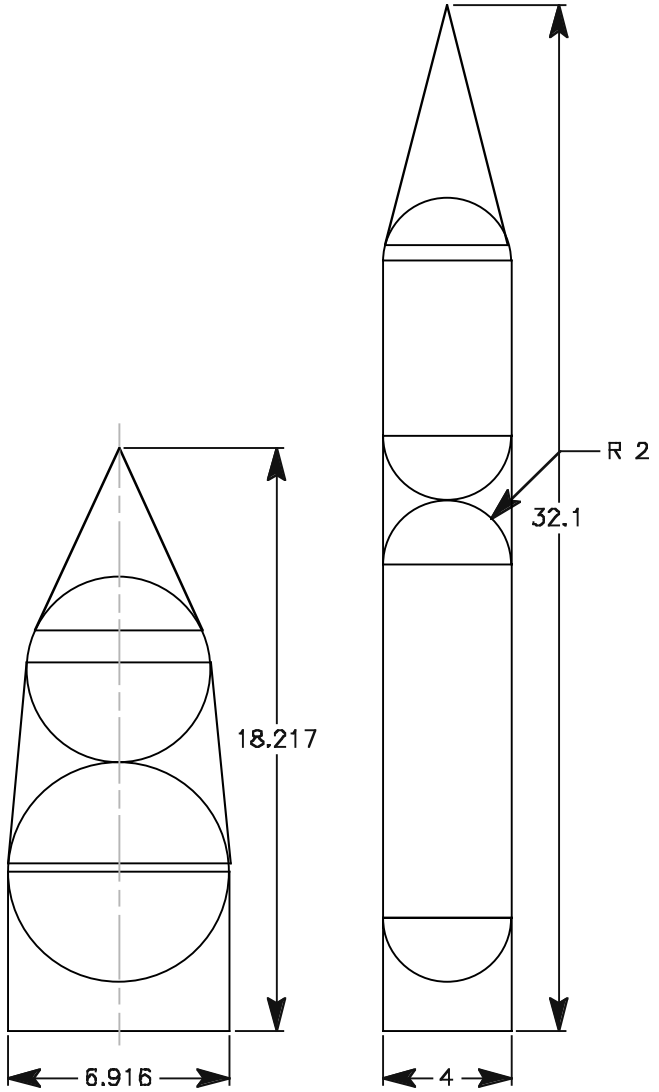


Mass Summary - First Pass

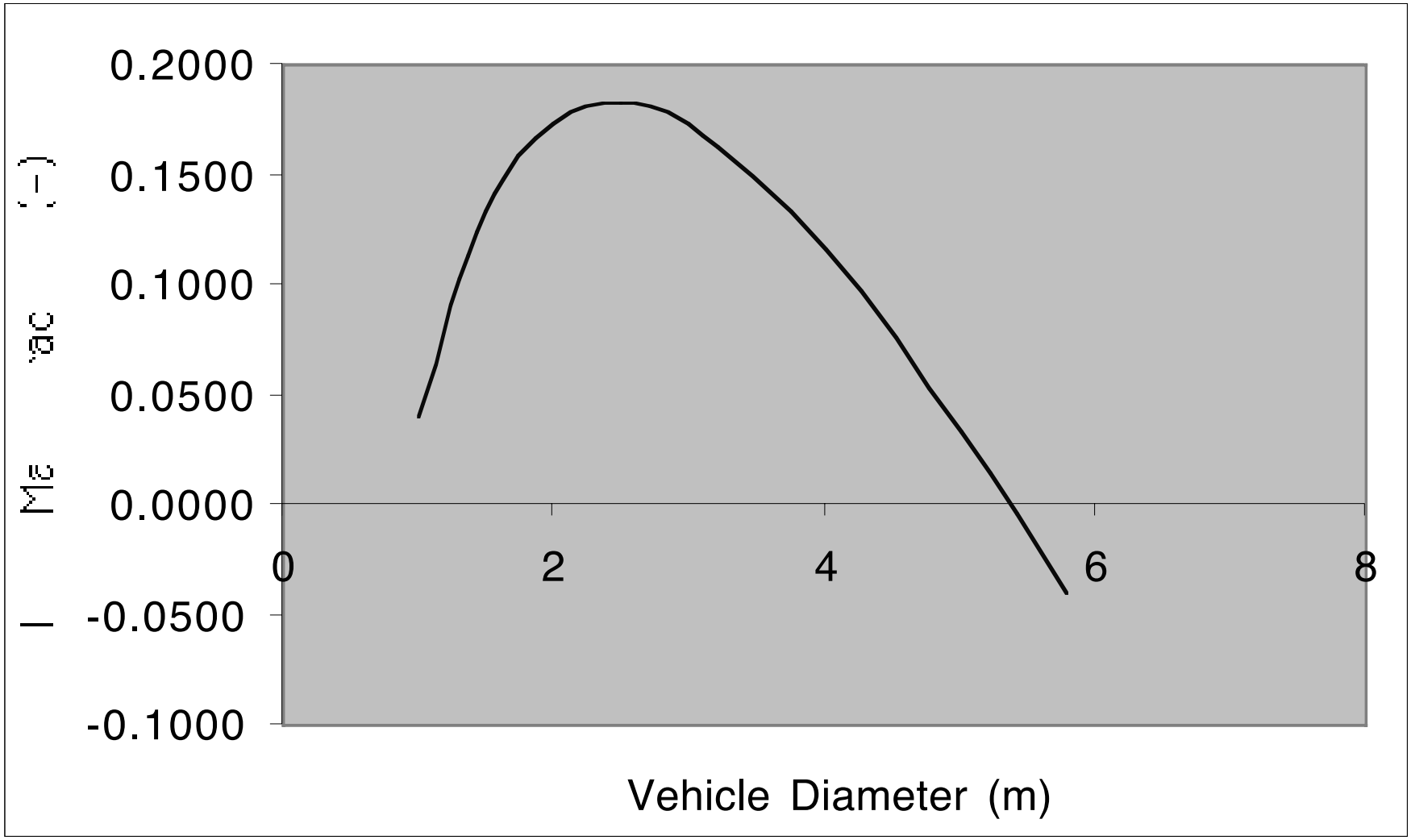
• Initial Inert Mass Estimate	12,240 kg
• LOX Tank	2087 kg
• LH2 Tank	1709 kg
• LOX Insulation	119 kg
• LH2 Insulation	433 kg
• Payload Fairing	688 kg
• Intertank Fairing	1677 kg
• Aft Fairing	1433 kg
• Engines	2236 kg
• Thrust Structure	497 kg
• Gimbals	81 kg
• Avionics	744 kg
• Wiring	840 kg
• Reserve	-
• Total Inert Mass	12,543 kg
• Design Margin	-2.4 %

Modifications for Second Pass

- Keep all initial vehicle sizing parameters constant
- Pick vehicle diameter and make tanks cylindrical to fit
- Redo MER analysis

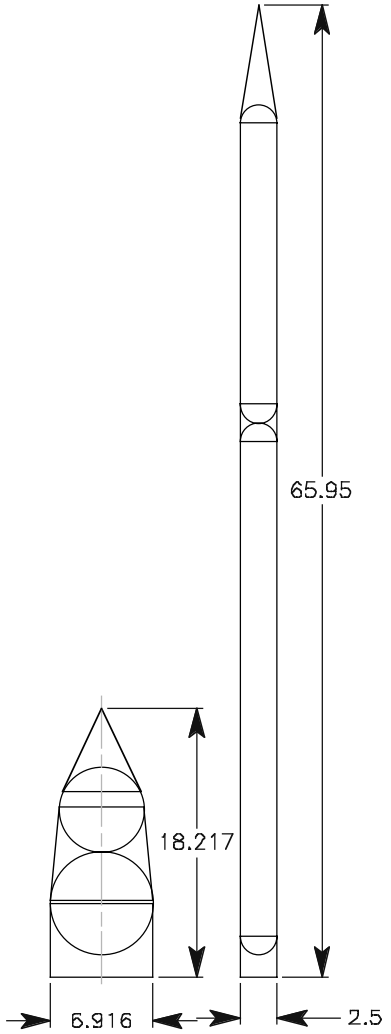


Effect of Vehicle Diameter on δ

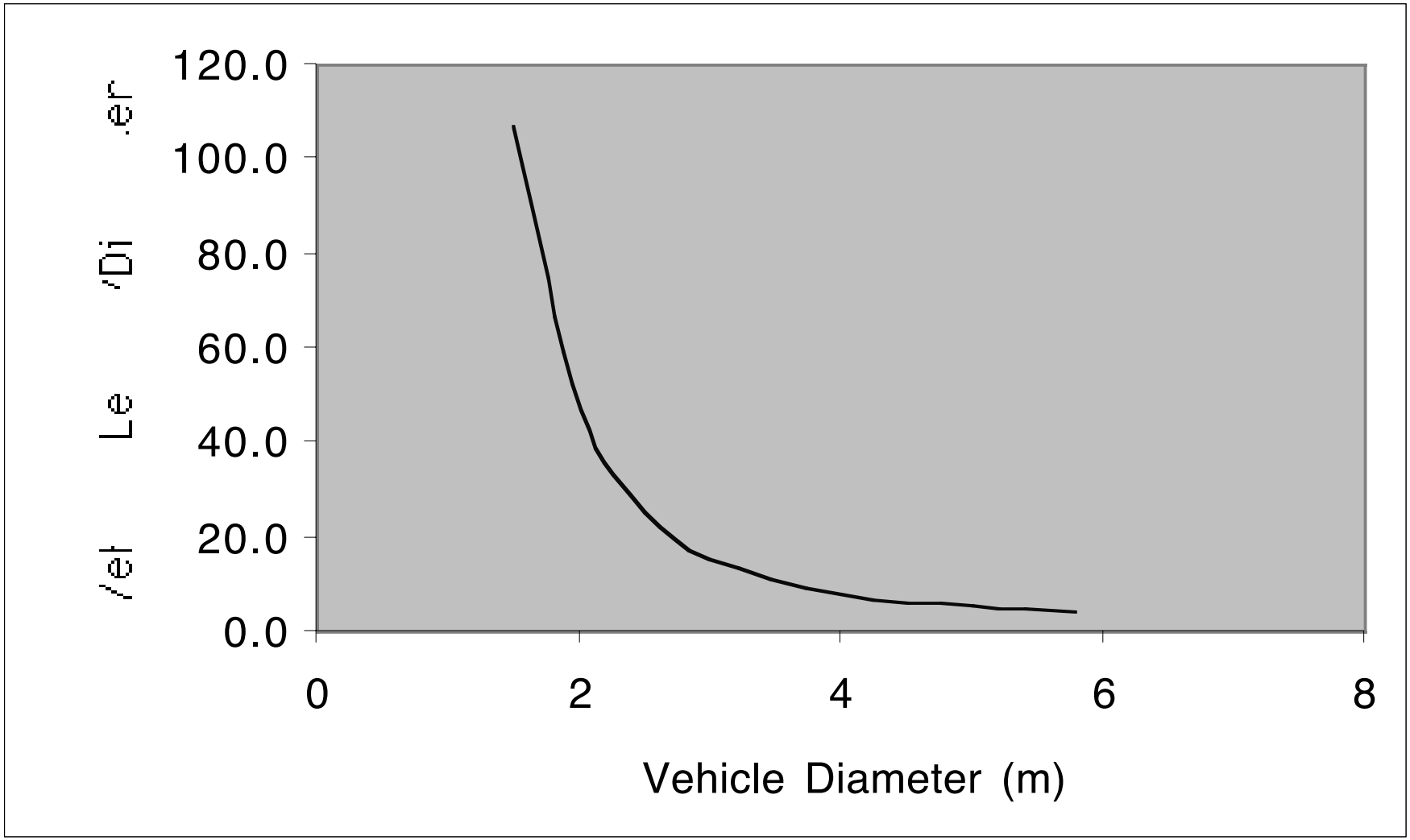


Effect of Mass-Optimal Diameter Choice

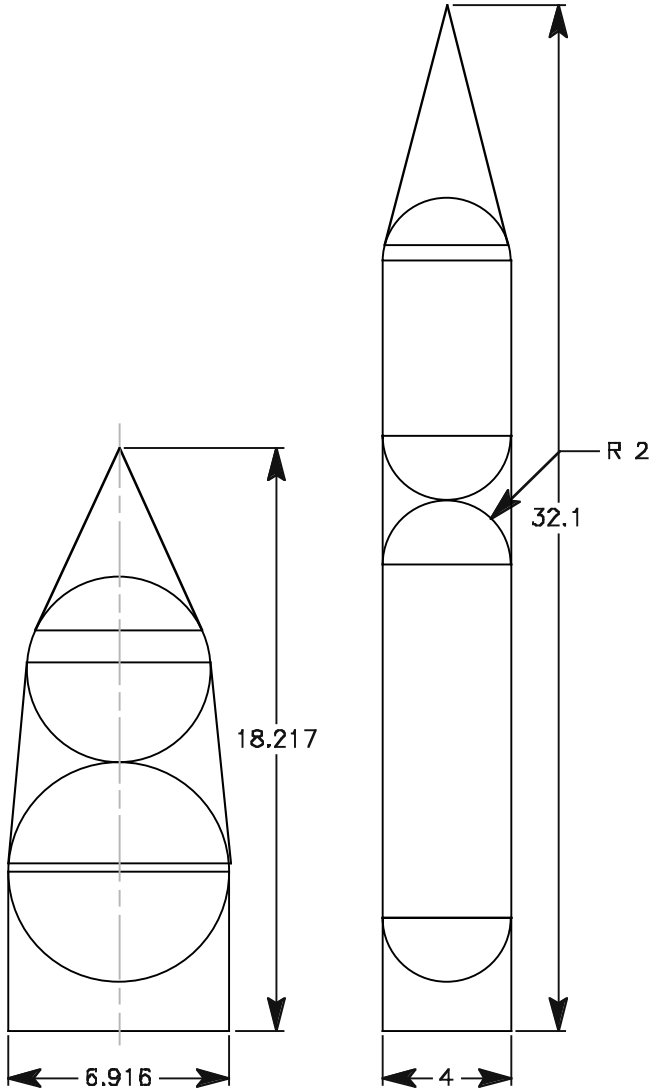
- Vehicle has L/D of 25.2 - severe complications from structural dynamics
- Mass margin goes from -2.4% to +18.3%
- Decreased volume for rocket engines in aft fairing
- Infeasible configuration



Effect of Diameter on Vehicle L/D



Second Pass Vehicle Configuration

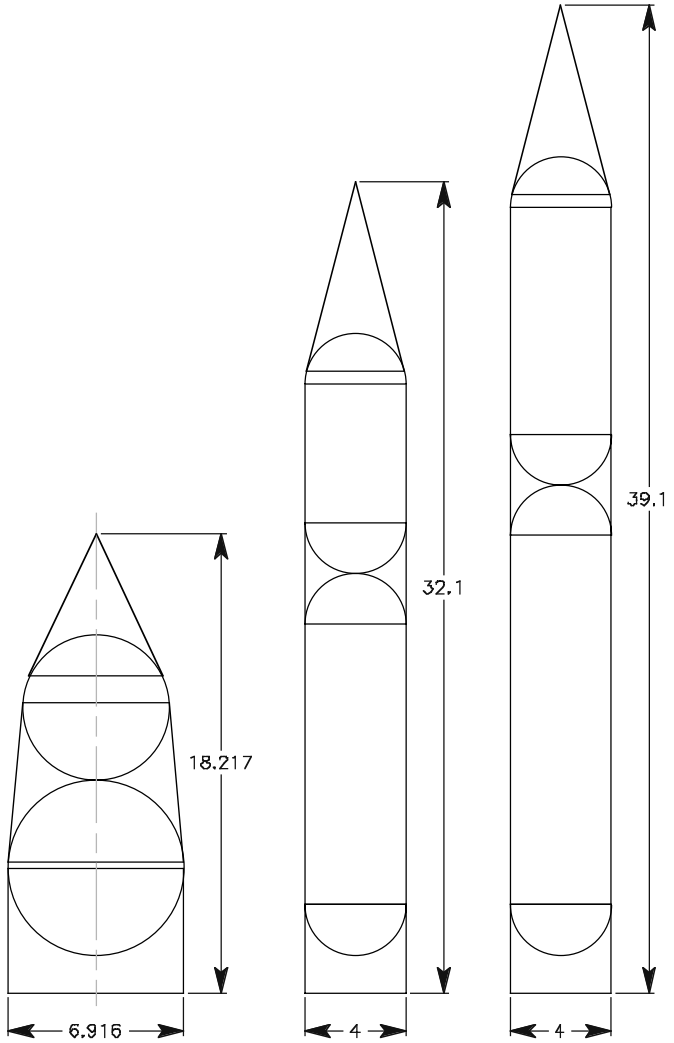


Mass Summary - Second Pass

• Initial Inert Mass Estimate	12,240 kg	12,240 kg
• LOX Tank	2087 kg	2087 kg
• LH2 Tank	1709 kg	1709 kg
• LOX Insulation	119 kg	133 kg
• LH2 Insulation	433 kg	547 kg
• Payload Fairing	688 kg	689 kg
• Intertank Fairing	1677 kg	669 kg
• Aft Fairing	1433 kg	585 kg
• Engines	2236 kg	2236 kg
• Thrust Structure	497 kg	497 kg
• Gimbals	81 kg	81 kg
• Avionics	744 kg	744 kg
• Wiring	840 kg	985 kg
• Reserve	-	-
• Total Inert Mass	12,543 kg	10,960 kg
• Design Margin	-2.4 %	+11.7 %

Modifications for Iteration 3

- Keep 4 m tank diameter
- Change initial assumption of δ iteratively, with resulting changes in m_0 and m_i, to reach 30% mass margin



Vehicle-Level Preliminary Design - 3rd Pass

- Single Stage to Orbit (SSTO) vehicle
- 5000 kg payload
- LOX/LH2 propellants
  - Isp=430 sec
  - δ=0.08655

$$r = e^{-\frac{\Delta V}{gI_{sp}}} = 0.1127$$

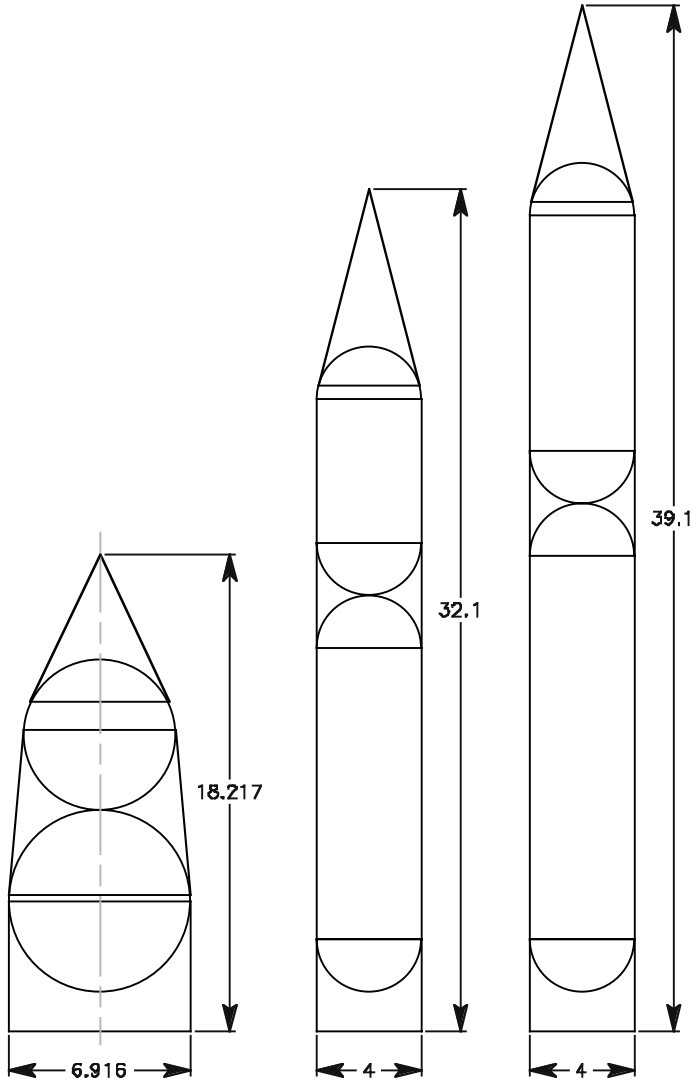
$$\lambda = r - \delta = 0.0261$$

$$M_0 = \frac{M_L}{\lambda} = 191{,}300\ kg$$

$$M_i = \delta M_0 = 16{,}560\ kg$$

$$M_p = M_0(1 - r) = 169{,}800\ kg$$

Third Pass Vehicle Configuration



Mass Summary - Third Pass

• Initial Inert Mass Estimate	12,240 kg	12,240 kg	16,560 kg
• LOX Tank	2087 kg	2087 kg	2530 kg
• LH2 Tank	1709 kg	1709 kg	2046 kg
• LOX Insulation	119 kg	133 kg	162 kg
• LH2 Insulation	433 kg	547 kg	672 kg
• Payload Fairing	688 kg	689 kg	689 kg
• Intertank Fairing	1677 kg	669 kg	669 kg
• Aft Fairing	1433 kg	585 kg	585 kg
• Engines	2236 kg	2236 kg	2708 kg
• Thrust Structure	497 kg	497 kg	622 kg
• Gimbals	81 kg	81 kg	100 kg
• Avionics	744 kg	744 kg	807 kg
• Wiring	840 kg	985 kg	1146 kg
• Reserve	-	-	-
• Total Inert Mass	12,543 kg	10,960 kg	12,740 kg
• Design Margin	-2.4 %	+11.7 %	+30 %

Mass Budgeting

	Estimates	Budgeted	Margins
• Initial Inert Mass Estimate	16,560 kg	16,560 kg	3826 kg
• LOX Tank	2530 kg	2910 kg	380 kg
• LH2 Tank	2046 kg	2354 kg	307 kg
• LOX Insulation	162 kg	187 kg	24 kg
• LH2 Insulation	672 kg	773 kg	101 kg
• Payload Fairing	689 kg	793 kg	104 kg
• Intertank Fairing	669 kg	769 kg	100 kg
• Aft Fairing	585 kg	673 kg	88 kg
• Engines	2708 kg	3115 kg	407 kg
• Thrust Structure	622 kg	715 kg	93 kg
• Gimbals	100 kg	115 kg	15 kg
• Avionics	807 kg	928 kg	121 kg
• Wiring	1146 kg	1318 kg	172 kg
• Reserve	-	1913 kg	1913 kg
• Total Inert Mass	12,740 kg	16,560 kg	