Orbital Maneuvering

- Fundamentals of attitude dynamics
- Layout of reaction control systems
- Single-axis attitude control design
- Rendezvous maneuvers
- Proximity operations
Derivation of Rotational Motion Equations

- Rotational equivalent of $F=ma$:
  \[ \tau = I \ddot{\theta} \]

- But since there are three rotation axes:
  \[ \vec{\tau} = [I] \dddot{\theta} \]

- Instantaneous differential equations of motion:
  \[
  \begin{bmatrix}
  \tau_x \\
  \tau_y \\
  \tau_z
  \end{bmatrix} =
  \begin{bmatrix}
  I_{xx} & I_{xy} & I_{xz} \\
  I_{yx} & I_{yy} & I_{yz} \\
  I_{zx} & I_{zy} & I_{zz}
  \end{bmatrix}
  \begin{bmatrix}
  \ddot{\theta}_x \\
  \ddot{\theta}_y \\
  \ddot{\theta}_z
  \end{bmatrix}
  \]
Principal Axis System

- Every object has a principal axis system in which all products of inertia are zero

\[
\begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{xy} & I_{yy} & I_{yz} \\
I_{xz} & I_{yz} & I_{zz}
\end{bmatrix} \Rightarrow 
\begin{bmatrix}
A & 0 & 0 \\
0 & B & 0 \\
0 & 0 & C
\end{bmatrix}
\]

- Found by solving eigenvalue problem

\[
\begin{vmatrix}
I_{xx} - A & I_{xy} & I_{xz} \\
I_{xy} & I_{yy} - B & I_{yz} \\
I_{xz} & I_{yz} & I_{zz} - C
\end{vmatrix} = 0
\]
Euler’s Equations

- \([x,y,z]\) is a rotating coordinate frame
- Have to include coriolis, centripetal terms
- Equations of motion about principal axis system become Euler’s equations:

\[
\tau_x = I_{xx} \ddot{\omega}_x + \omega_y \omega_z (I_{zz} - I_{yy}) \\
\tau_y = I_{yy} \ddot{\omega}_y + \omega_x \omega_z (I_{xx} - I_{zz}) \\
\tau_z = I_{zz} \ddot{\omega}_z + \omega_x \omega_y (I_{yy} - I_{xx})
\]
**Calculating Vehicle Inertia Tensor**

- Component with mass $m$ and inertia tensor $[I]$ in vehicle axis system
- Located at position $[x,y,z]$ w.r.t. vehicle CG
- Contribution to vehicle inertia is ("parallel axis theorem")

\[
\begin{bmatrix}
I'_x \\
I'_y \\
I'_z
\end{bmatrix}_{\text{vehicle}} = \begin{bmatrix}
I_x \\
I_y \\
I_z
\end{bmatrix}_{\text{component}} + m \begin{bmatrix}
y^2 + z^2 & -xy & -xz \\
-xy & x^2 + z^2 & -yz \\
-xz & -yz & x^2 + y^2
\end{bmatrix}
\]
Gemini OAMS/RCS Systems Layout
Gemini OAMS Translational Control

[Diagram of Gemini spacecraft showing translational control in vertical, forward, lateral, and aft directions]
Gemini OAMS Attitude Control

- Pitch
- Roll
- Yaw
Gemini Reentry Control System
Apollo CSM RCS Configuration

APOLLO COMMAND AND SERVICE MODULES
ENGINE LOCATIONS

CM RCS ENGINE (12)

SM RCS ENGINES (16)

SPS

Orbital Maneuvering
Principles of Space Systems Design
Lunar Module Reaction Control System
LM RCS Schematic
Orbiter Forward RCS Pod
Orbiter Aft RCS/OMS Pod
Design Issues for Reaction Control Systems

• Thruster placement
  - Number of thrusters
  - Attempt to minimize cross-coupling
  - Plumbing runs and vehicle integration

• Thruster sizing
  - Required rotational performance
  - Minimize propellant usage in attitude hold
  - Perturbing torques (e.g., main engine firings, gravity gradients, aerodynamic torques, docking)
Single-Axis Motion

- Assume single-axis motion with constant torque (all cross-coupling treated as noise)
  \[ \tau = I \ddot{\theta} \Rightarrow \ddot{\theta} = \frac{I}{\tau} \]

- Integrate over time to get
  \[ \dot{\theta} = \frac{I}{\tau} t + \dot{\theta}_0 \]

- Integrate again to get
  \[ \theta = \frac{1}{2} \frac{I}{\tau} t^2 + \dot{\theta}_0 t + \theta_0 \]
Shuttle Standard Rendezvous Approach
Transition to R-Bar Proximity Operations
V-Bar Approach Technique

INITIATE V-BAR APPROACH
(~1.0 FPS FOR INITIAL
R = 1000 FT)

USE PRCS +X THRUSTERS
FOR INTERMEDIATE
CORRECTIONS TO
MAINTAIN V-BAR
POSITION

INTERMEDIATE
CORRECTION

FINAL BRAKING
FORCE (~) 0 FPS
FOR INITIAL
1000 FT RANGE
AND MAY REQUIRE
SMALL BRAKING
GATES)

ORBITER TRAJECTORY
IF NO CORRECTION
APPLIED AT V-BAR CROSSING

TARGET

R-BAR

APPROACH FROM V-BAR
REQUIRES ΔV'S IN OPPOSITE
DIRECTION FROM V-BAR
APPROACH
R-Bar Approach Technique

A
Initialize with zero relative rates

B
Obtain closing rate

C
Correct for -V-BAR drift
R-Bar Velocity Scheduling Curves