Orbital Maneuvering

- Fundamentals of attitude dynamics
- Layout of reaction control systems
- Single-axis attitude control design
- Rendezvous maneuvers
- Proximity operations

Derivation of Rotational Motion Equations

- Rotational equivalent of F=ma:

$$\tau = I\ddot{\theta}$$

- But since there are three rotation axes:

$$\vec{\tau} = [I]\ddot{\vec{\theta}}$$

- Instantaneous differential equations of motion:

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \end{bmatrix}$$

Principal Axis System

- Every object has a principal axis system in which all products of inertia are zero

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix} \Rightarrow \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}$$

- Found by solving eigenvalue problem

$$\begin{vmatrix} I_{xx} - A & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} - B & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} - C \end{vmatrix} = 0$$

Euler's Equations

- [x,y,z] is a rotating coordinate frame
- Have to include coriolis, centripetal terms
- Equations of motion about principal axis system become Euler's equations:

$$\tau_x = I_{xx}\dot{\omega}_x + \omega_y\omega_z\left(I_{zz} - I_{yy}\right)$$

$$\tau_y = I_{yy}\dot{\omega}_y + \omega_x\omega_z\left(I_{xx} - I_{zz}\right)$$

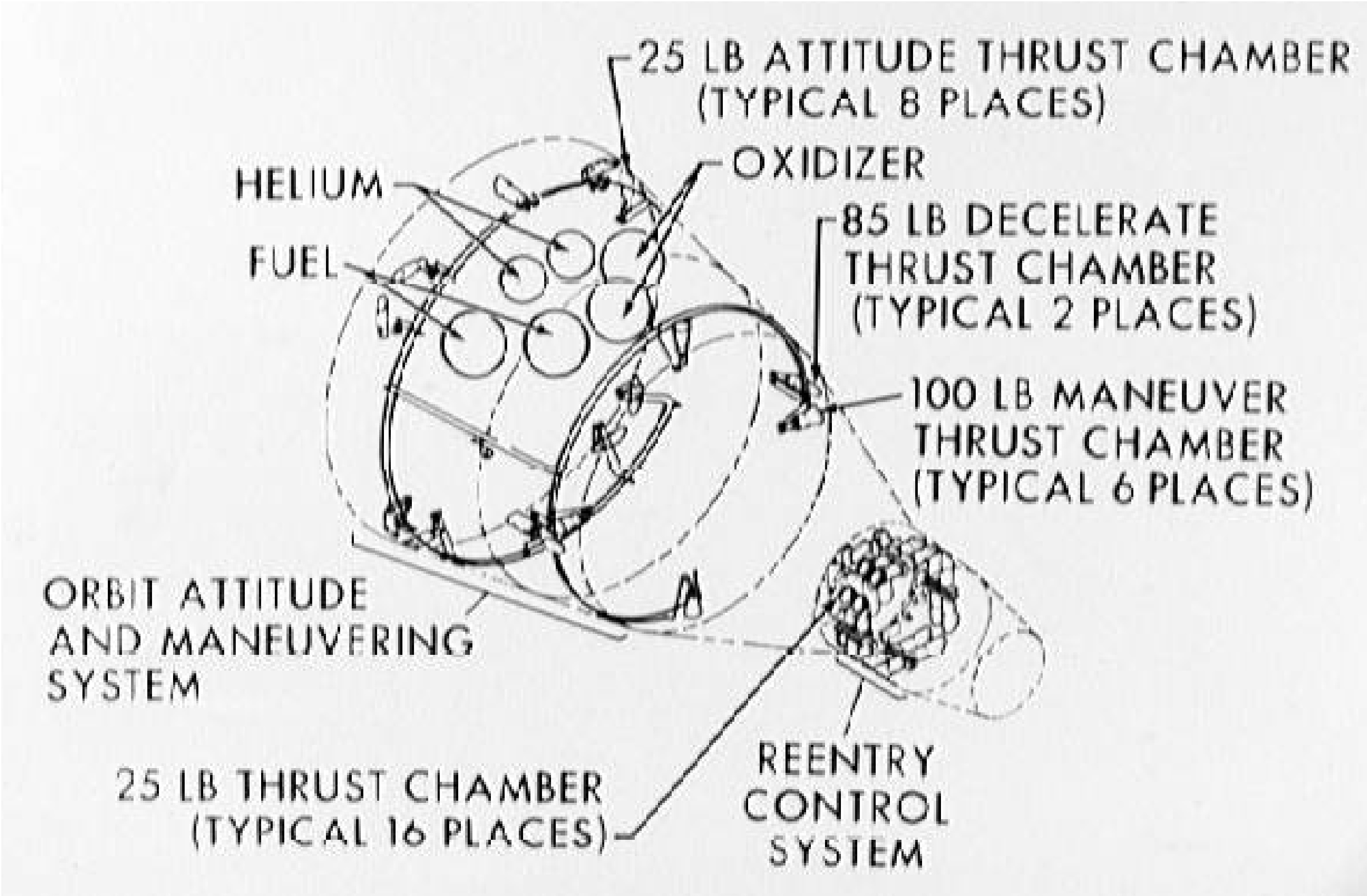
$$\tau_z = I_{zz}\dot{\omega}_z + \omega_x\omega_y\left(I_{yy} - I_{xx}\right)$$

Calculating Vehicle Inertia Tensor

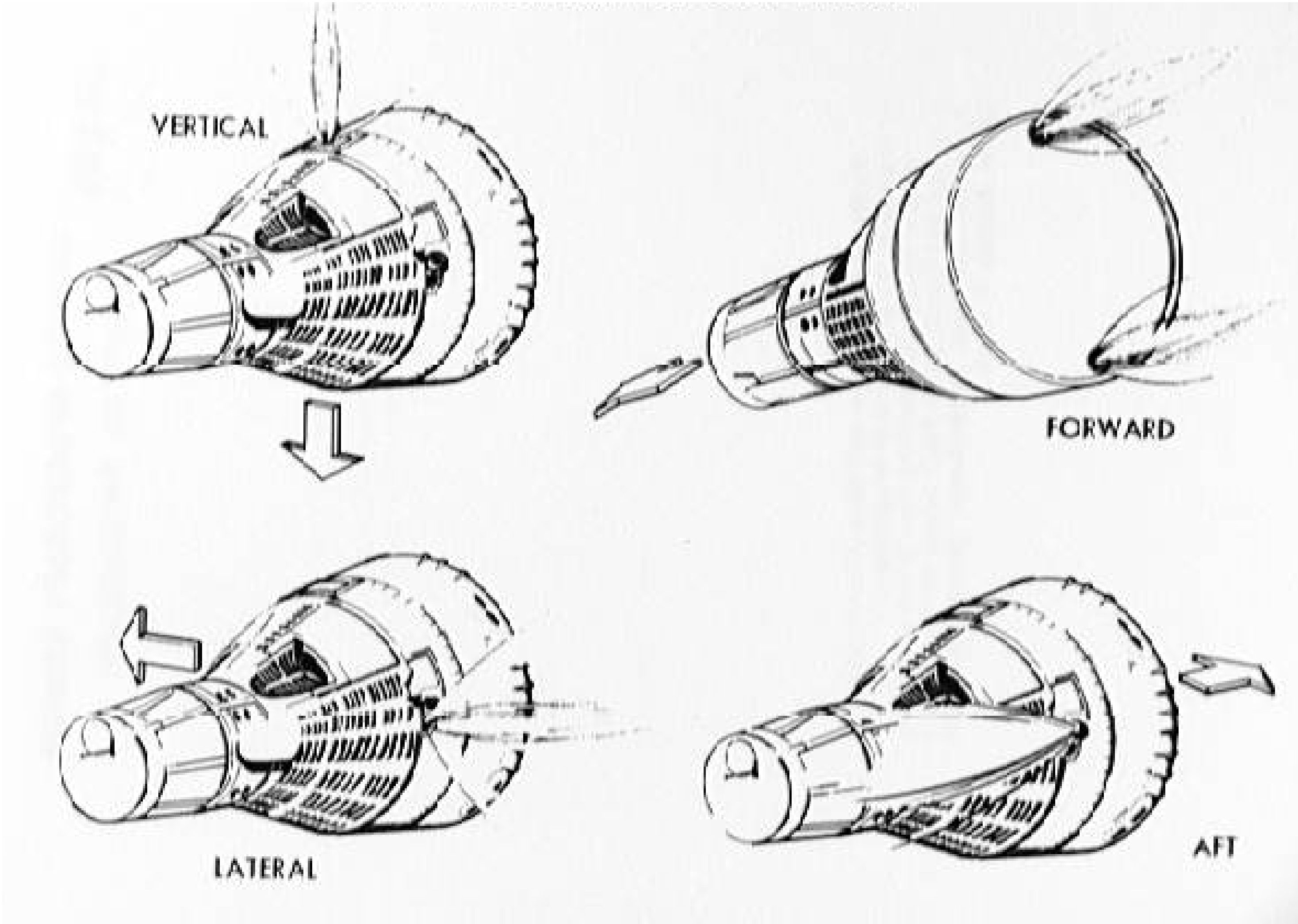
- Component with mass m and inertia tensor [I] in vehicle axis system
- Located at position [x,y,z] w.r.t. vehicle CG
- Contribution to vehicle inertia is ("parallel axis theorem")

$$\begin{bmatrix} I'_x \\ I'_y \\ I'_z \end{bmatrix}_{vehicle} = \begin{bmatrix} I_x \\ I_y \\ I_z \end{bmatrix}_{component} + m \begin{bmatrix} y^2 + z^2 & -xy & -xz \\ -xy & x^2 + z^2 & -yz \\ -xz & -yz & x^2 + y^2 \end{bmatrix}$$

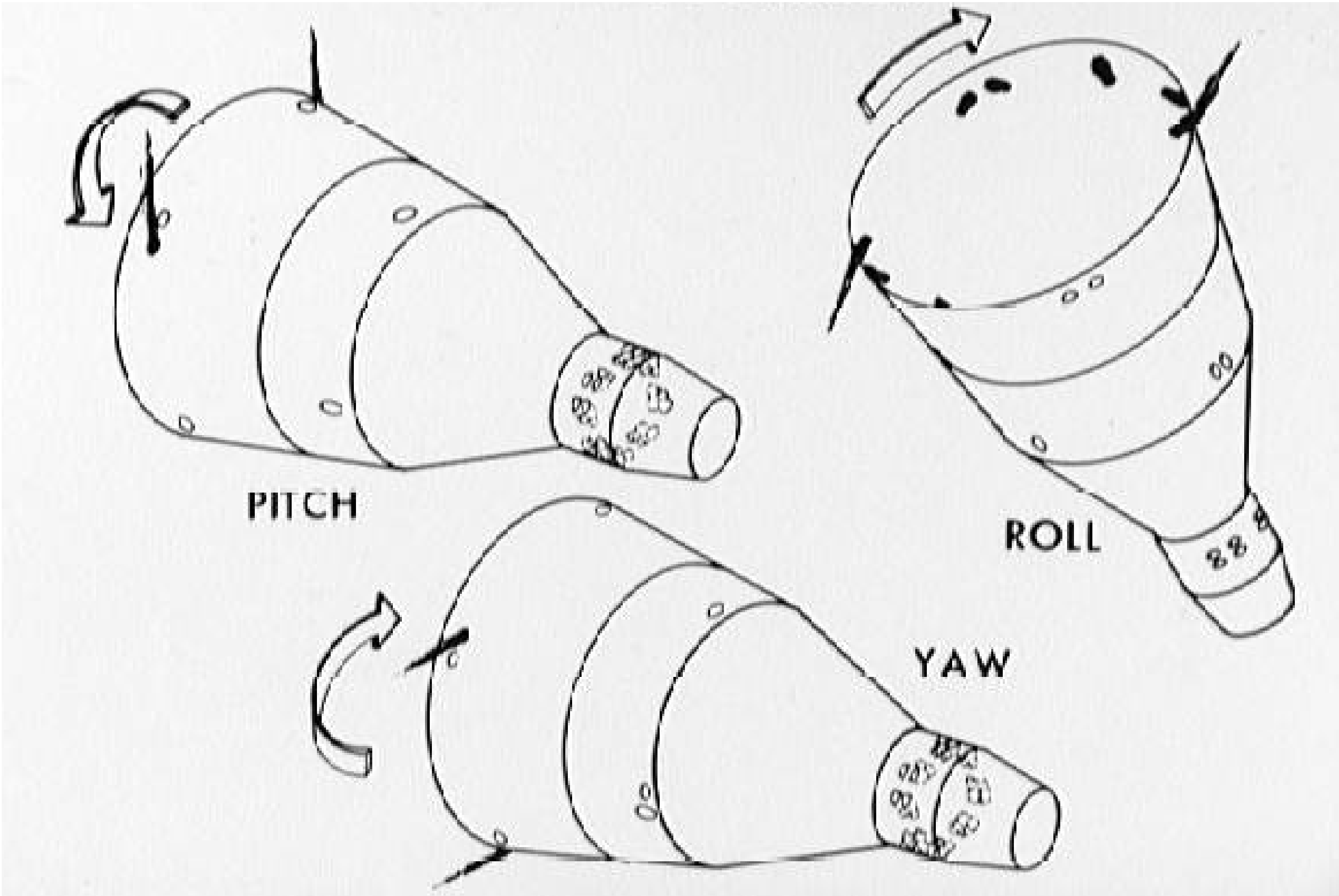
Gemini OAMS/RCS Systems Layout



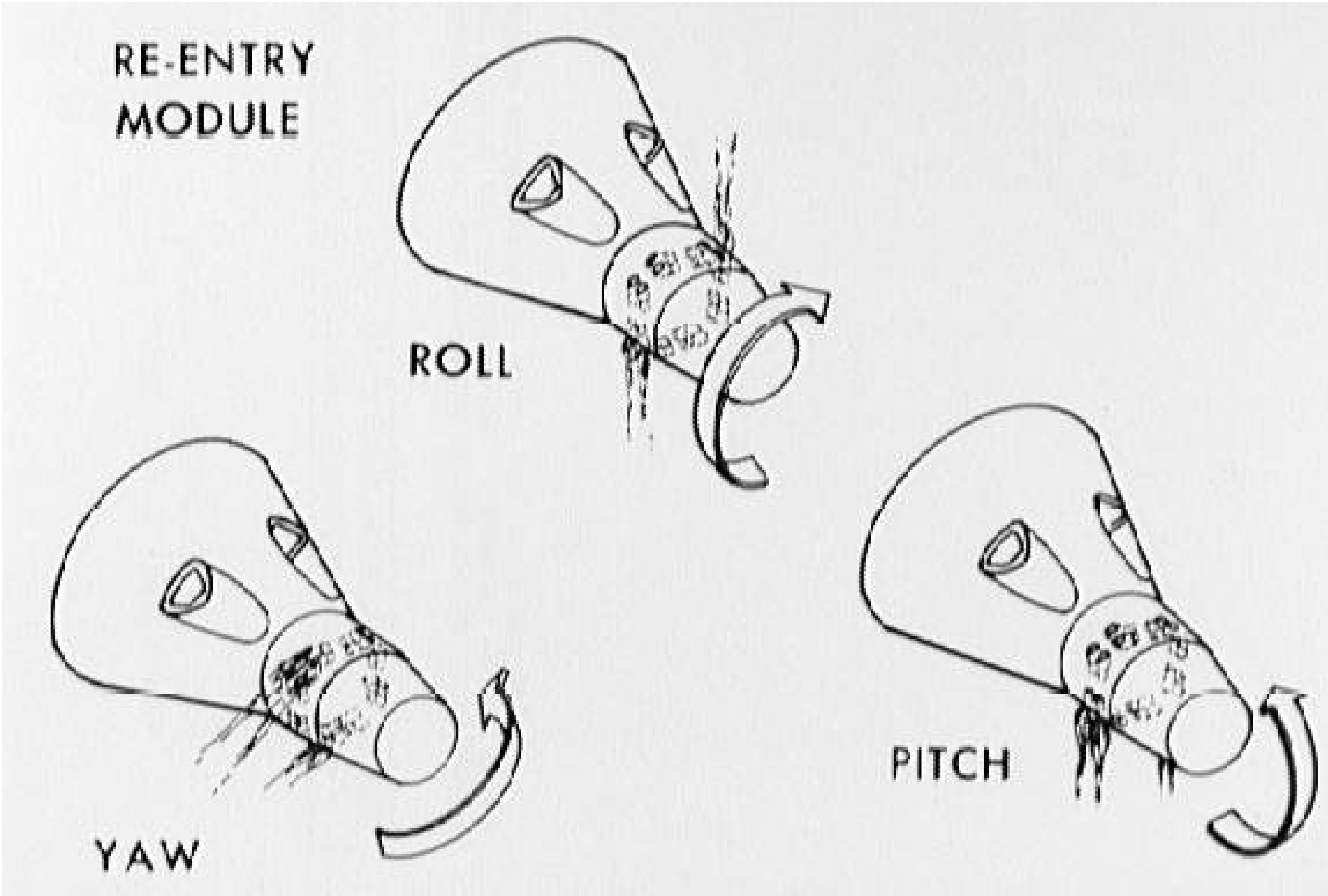
Gemini OAMS Translational Control



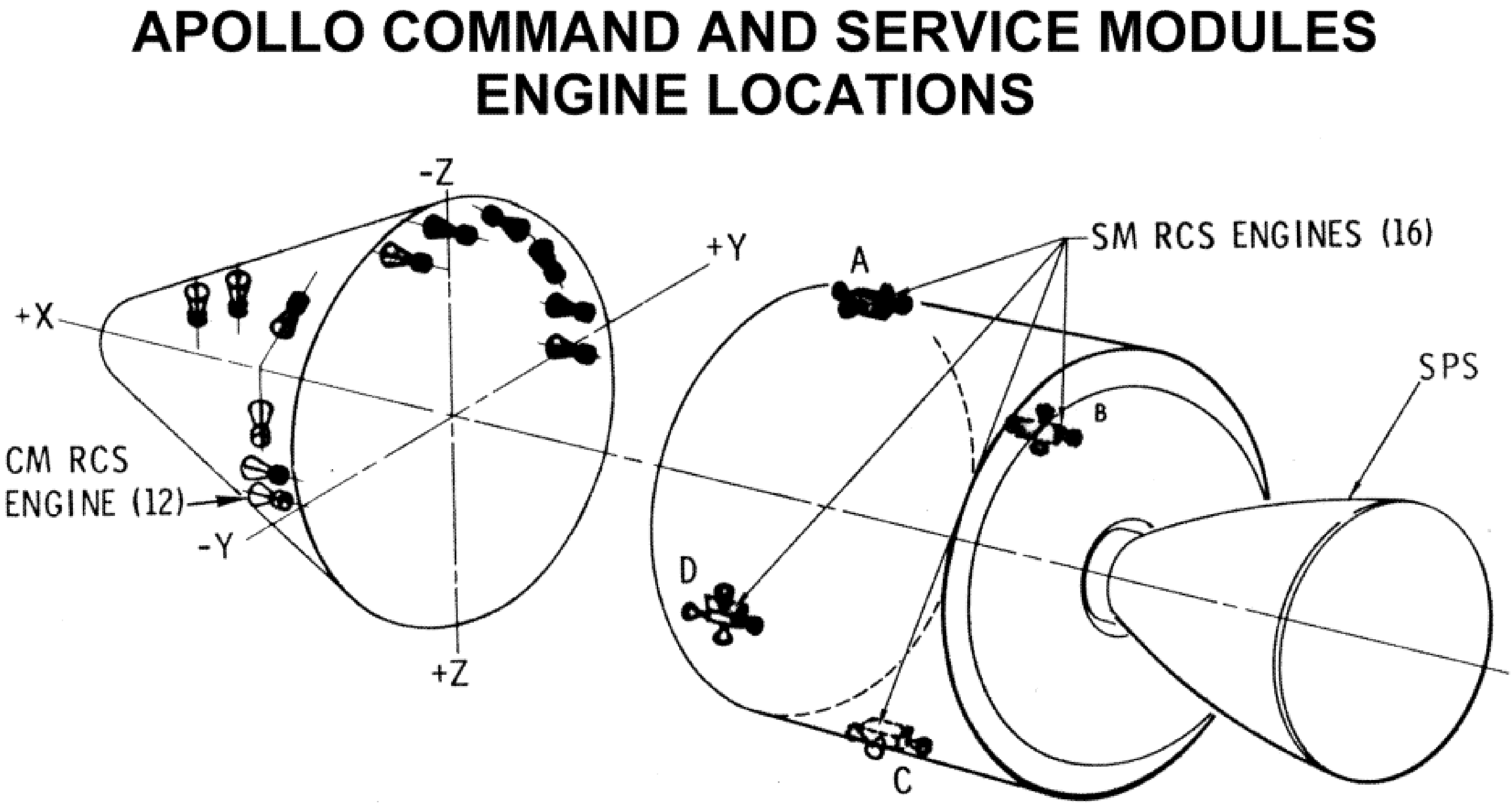
Gemini OAMS Attitude Control



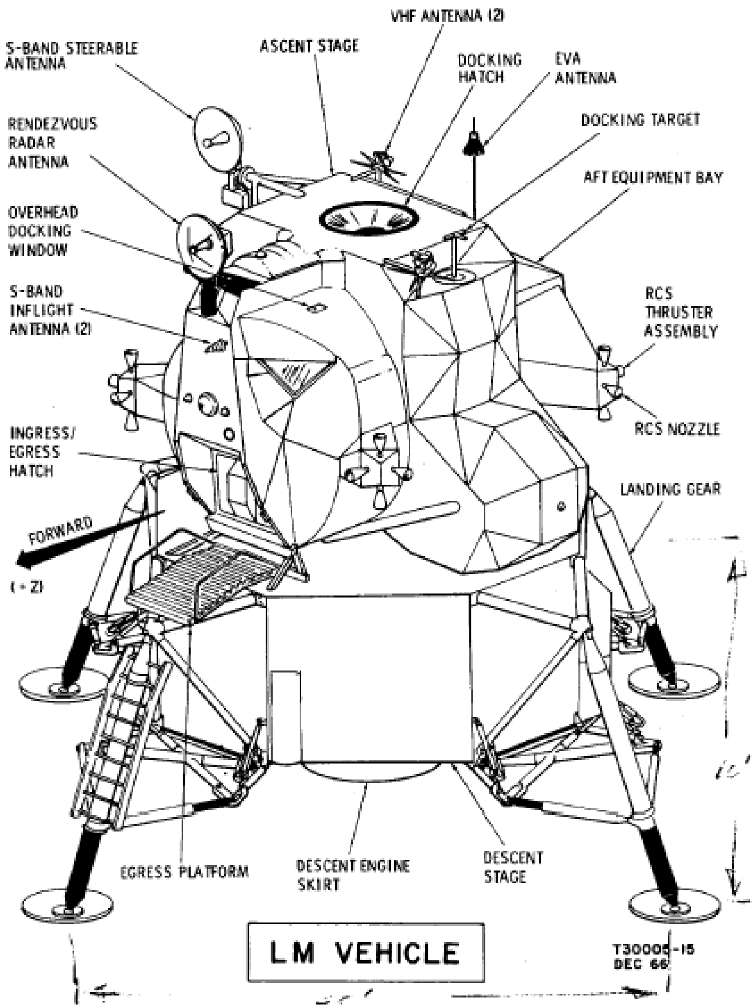
Gemini Reentry Control System

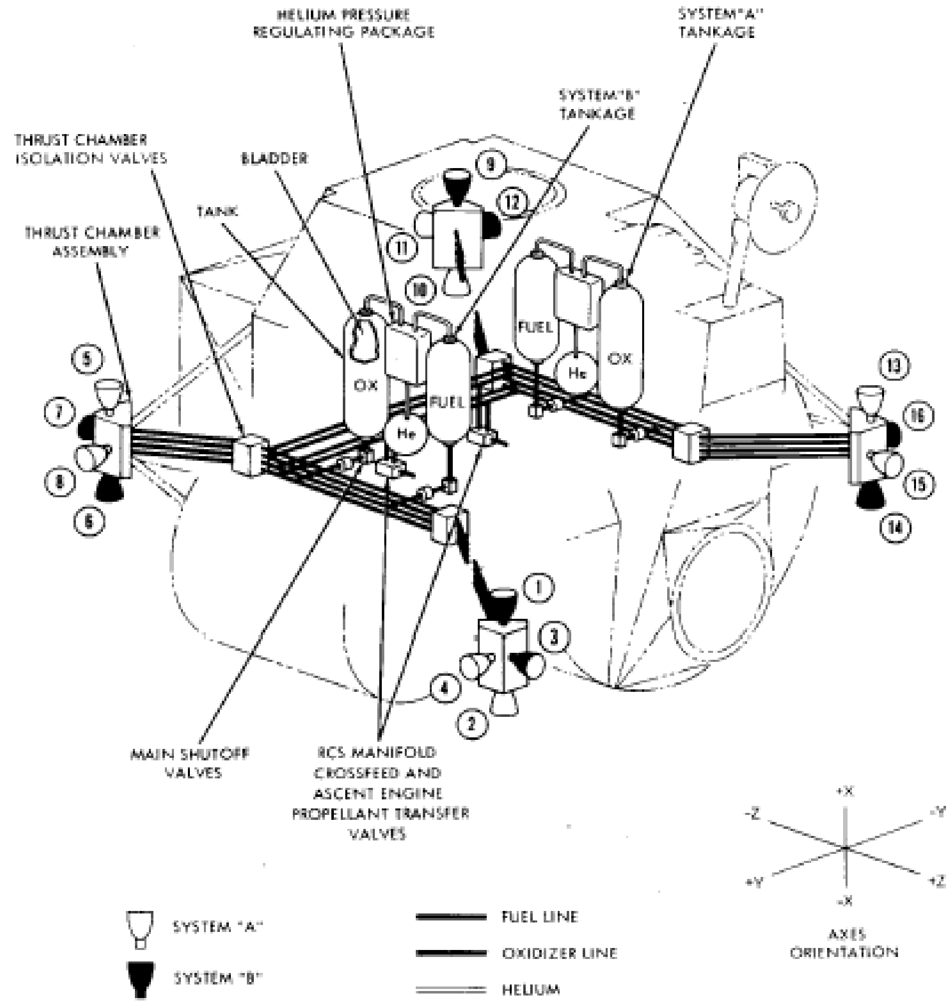


Apollo CSM RCS Configuration

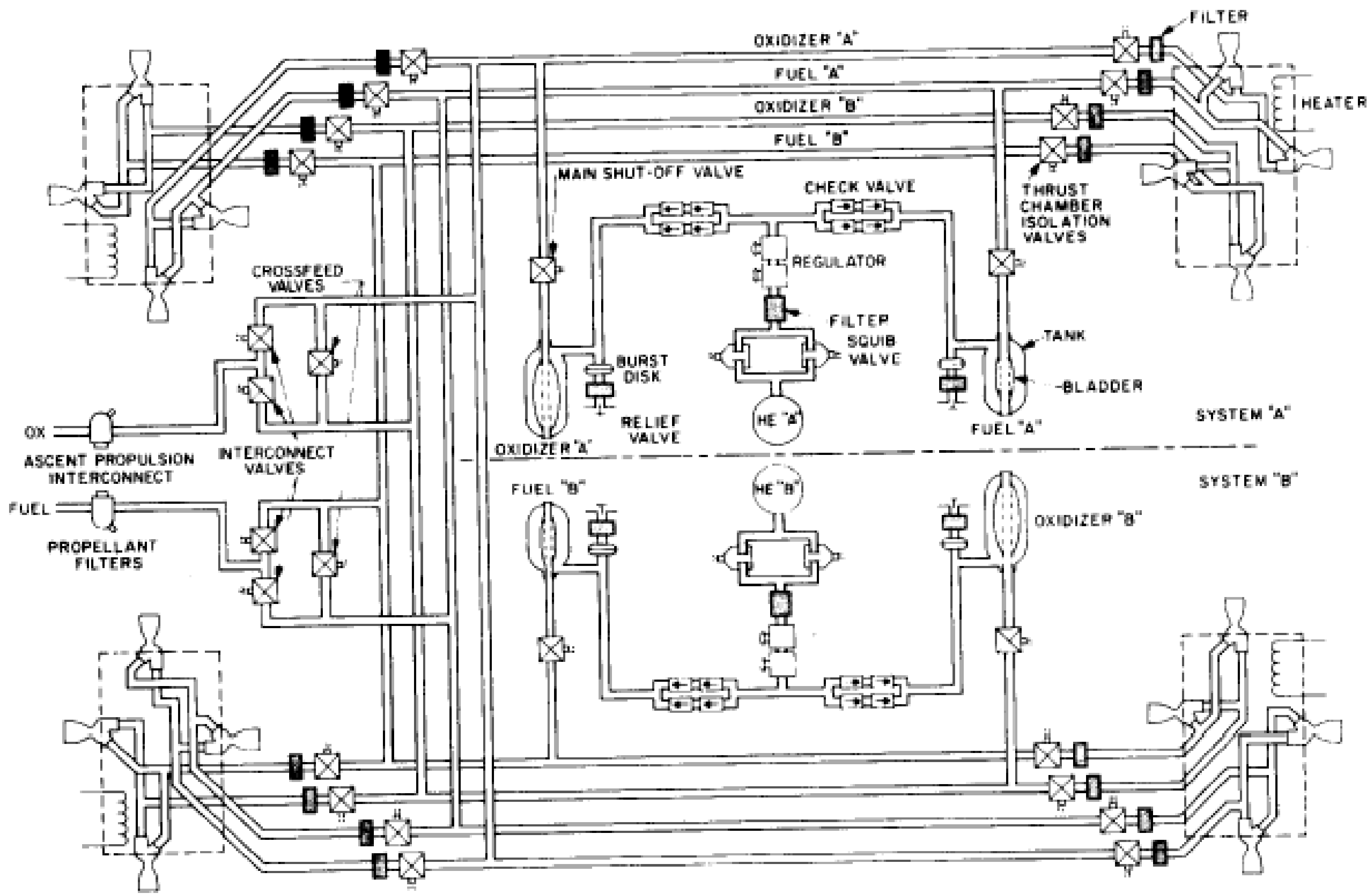


Lunar Module Reaction Control System

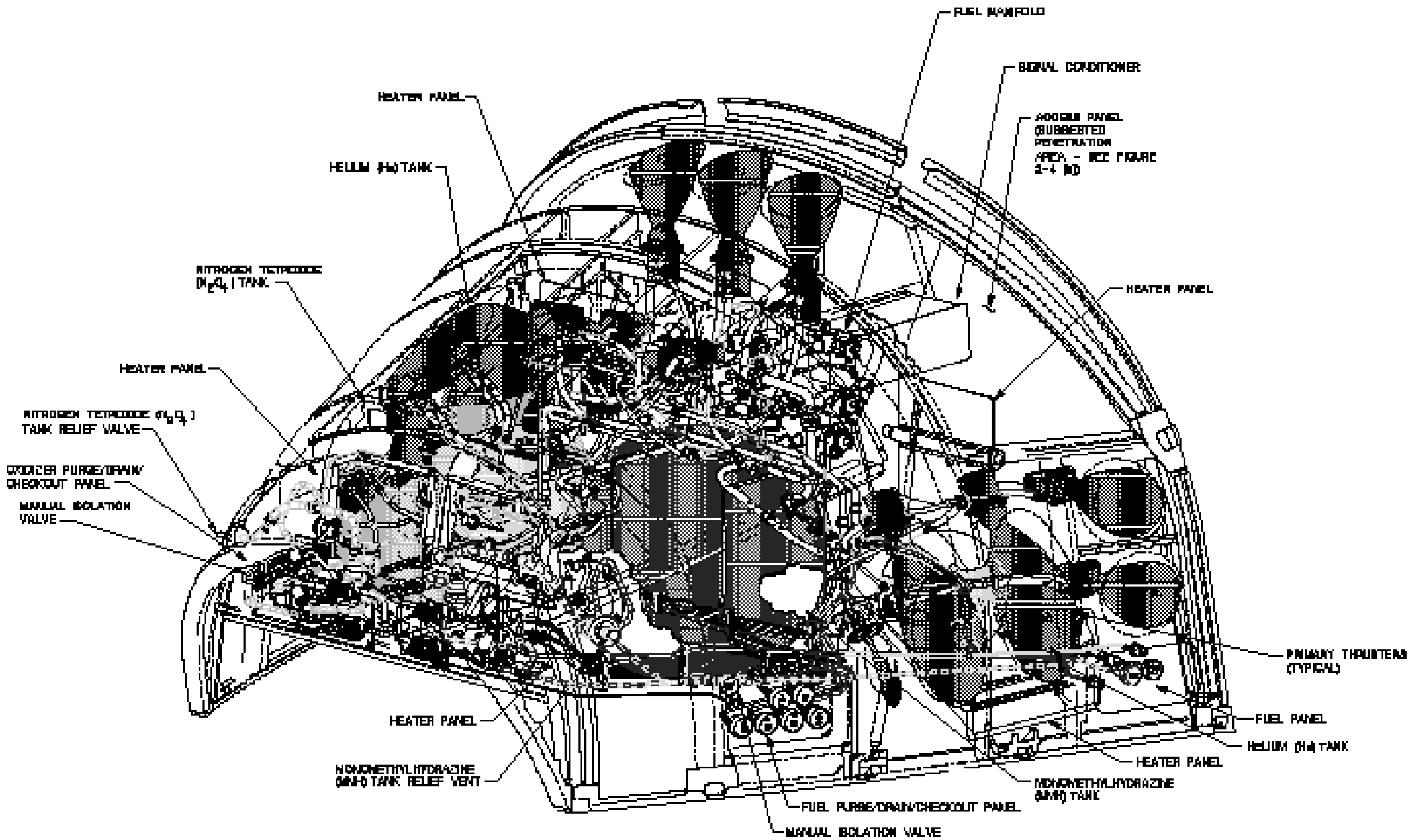




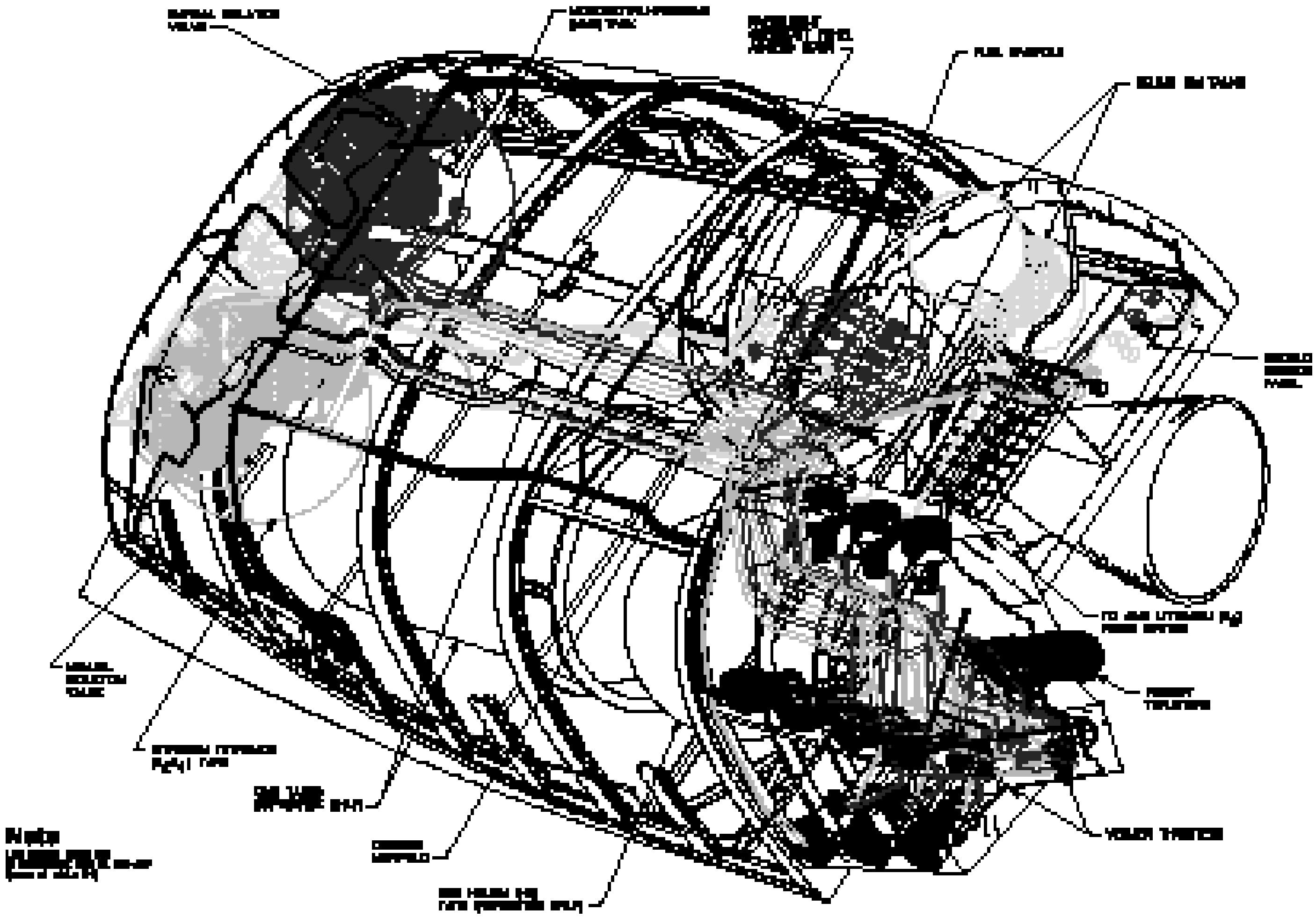
LM RCS Schematic



Orbiter Forward RCS Pod



Orbiter Aft RCS/OMS Pod



Design Issues for Reaction Control Systems

- Thruster placement
  - Number of thrusters
  - Attempt to minimize cross-coupling
  - Plumbing runs and vehicle integration
- Thruster sizing
  - Required rotational performance
  - Minimize propellant usage in attitude hold
  - Perturbing torques (e.g., main engine firings, gravity gradients, aerodynamic torques, docking)

Single-Axis Motion

- Assume single-axis motion with constant torque (all cross-coupling treated as noise)

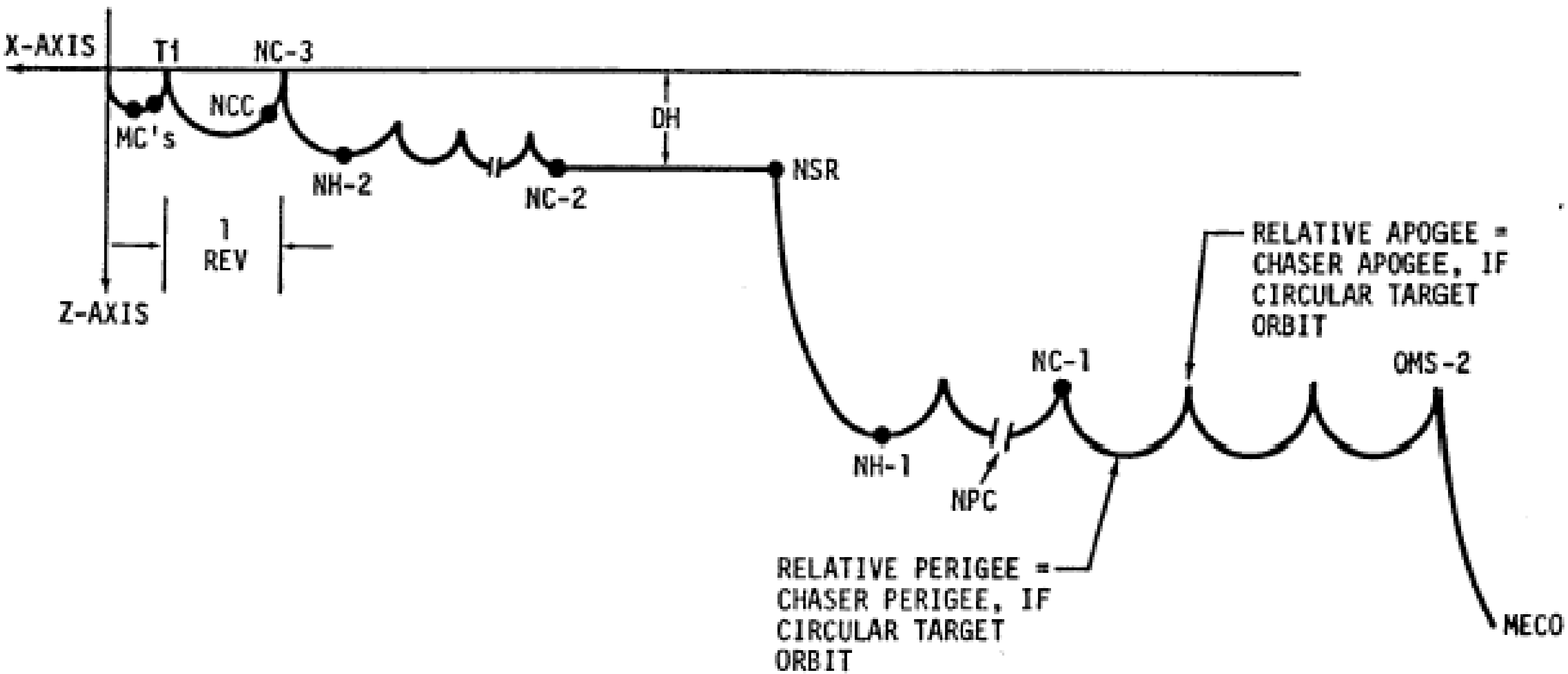
$$\tau = I\ddot{\theta} \Rightarrow \ddot{\theta} = \frac{I}{\tau}$$

- Integrate over time to get

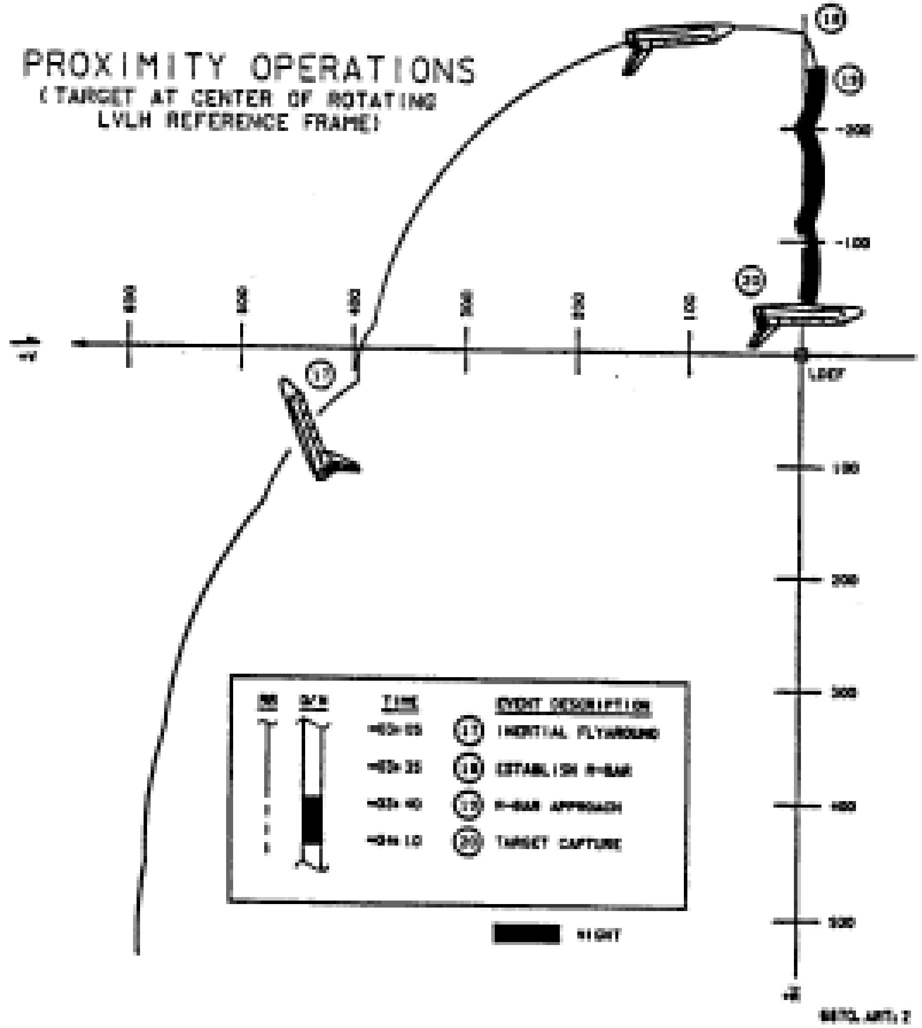
$$\dot{\theta} = \frac{I}{\tau}t + \dot{\theta}_0$$

- Integrate again to get $\theta = \frac{1}{2}\frac{I}{\tau}t^2 + \dot{\theta}_0 t + \theta_0$

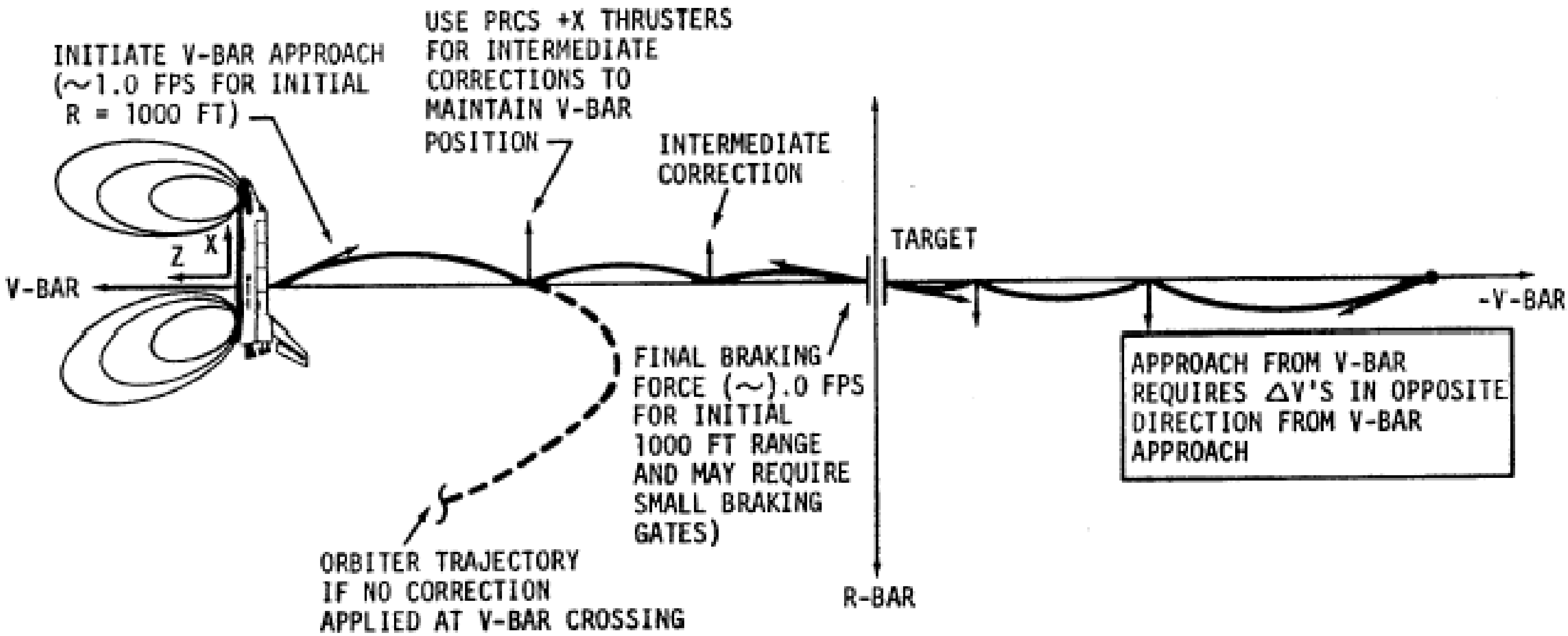
Shuttle Standard Rendezvous Approach



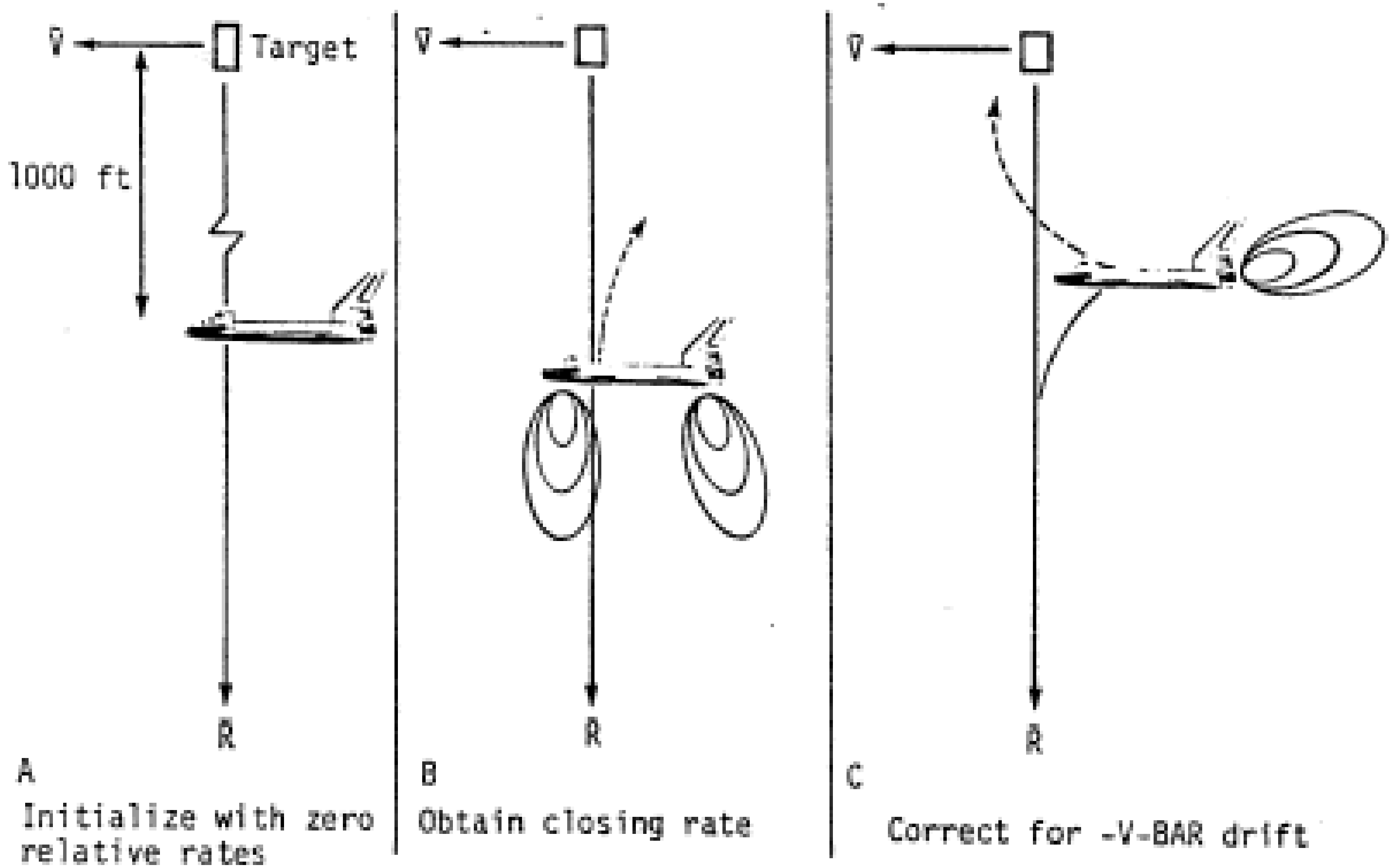
Transition to R-Bar Proximity Operations



V-Bar Approach Technique



R-Bar Approach Technique



R-Bar Velocity Scheduling Curves