Problem 3 Solutions
Fall, 2002

a) Initial orbit r_1=500+6378=6878 km; r_2=332,600 km

$$\Delta v_1 = \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}}-1\right) = \sqrt{\frac{398604}{6878}}\left(\sqrt{\frac{2\cdot 332600}{6878+332600}}-1\right)\text{=3.044 km/sec}$$

$$\Delta v_2 = \sqrt{\frac{\mu}{r_2}}\left(1-\sqrt{\frac{2r_1}{r_1+r_2}}\right) = \sqrt{\frac{398604}{332600}}\left(1-\sqrt{\frac{2\cdot 6878}{6878+332600}}\right)\text{=0.8744 km/sec}$$

$\Delta v_{total}=\Delta v_1+\Delta v_2=$ **3.918 km/sec**

b) ΔV_1 is unchanged from (a); Velocity at apogee is

$$\Delta v_a = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1+r_2}} = \sqrt{\frac{398604}{332600}}\sqrt{\frac{2\cdot 6878}{6878+332600}}\text{=0.2204 km/sec}$$

Final orbital velocity is $\Delta v_{c2} = \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{398604}{332600}}$ =1.095 km/sec

$$\Delta v_2 = \sqrt{v_a^2+v_{c2}^2-2v_a v_{c2}\cos(\Delta i_2)}\text{=0.9719 km/sec}$$

$\Delta v_{total}=\Delta v_1+\Delta v_2=$ **4.016 km/sec**

c) Velocity values for the first maneuver: initial circular velocity is

$\Delta v_{c1} = \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{398604}{6878}}$ =7.613 km/sec; velocity at perigee is

$$\Delta v_p = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1+r_2}} = \sqrt{\frac{398604}{6878}}\sqrt{\frac{2\cdot 332600}{6878+332600}}\text{=10.66 km/sec}$$

$\Delta v_1 = \sqrt{v_p^2+v_{c1}^2-2v_p v_{c1}\cos(\Delta i_1)}$; $\Delta i_{total} = \Delta i_1 + \Delta i_2 = 51.2°$

Applying Solver in Excel to minimize total Δv gives

Δi_1=0.4141°; Δv_1=3.044 km/sec; Δi_2=50.79°; Δv_2=0.9706; Δv_{tot}=4.015 km/sec

d) At the given rotation period, the corresponding angular velocity is

$\omega = \frac{2\pi}{27.322\cdot 24\cdot 3600}$=2.662x$10^{-6}$ rad/sec. Since v=wr, the L1 point is actually moving at a velocity of 0.8734 km/sec, instead of 1.095 km/sec as in (b). Using the new velocity for v_{c2} and taking Δi_2=46.2° (using the 5° lunar orbit inclination to minimize Δv) gives Δv_2=0.7382 km/sec and **Δv_{tot}=3.782 km/sec**

e) The Earth and the Moon both orbit around the center of mass of the system, which can be located by the weighted average of the two masses $\frac{\mu_M r_{E-M}}{\mu_E \mu_M}$ =4449 km from the center of the Earth (inside the Earth itself!) The L1, L2, and L3 points are locations where static equilibrium exists between the gravitation attractions of the Earth and the Moon and the centripetal acceleration of the spacecraft orbiting around the Earth-moon system CG at the angular velocity stated in (d). Setting these equations up, and solving for distance from the center of the Earth, the results obtained are L1=328,100 km (56,280 km from the moon), L2=448,300 km (63,940 km from the moon), and L3=382,100 km (on the side opposite the moon). [Okay, so I missed by 4000 km when I listed the L1 location above, but the techniques are the same, and the answers given are appropriate for the L1 distance I gave in (a).]